UNITED NATIONS UNIVERSITY
UNU-WIDER

WIDER Working Paper 2016/126

Technical note

Proposed model for regional power sector integration in Africa

Amy Rose[1] and Ignacio Pérez-Arriaga[2]

November 2016

Abstract: Regional power pools present a significant and potentially defining opportunity for African power systems to develop domestic energy resources, improve system reliability, and contribute to overall economic development. Hydropower is expected to play a significant role in many regional power pools in Africa. Feasible power transmission highways from Grand Inga in the Democratic Republic of the Congo and the Grand Renaissance Dam in Ethiopia to other regions on the continent create the possibility of a pan-African electricity grid. However, in the medium and long term, global climate change is expected to cause major variations in Africa's hydrological resources and it is not known how these changes may impact the value of regional power sector integration. This paper presents a model developed to study the value of different levels of regional integration in sub-Saharan Africa and how this value may change in the face of climate change. This work builds on previous studies by incorporating the ability to trade between different regional pools, co-optimisation of generation and transmission, the ability to share reserves, and detailed simulation of the major hydropower basins in Africa. Numerical results of the analysis will be presented in a parallel paper.

Keywords: climate change, capacity expansion planning, power pools, Africa
JEL classification: C61, O21, P28, Q40, R58

Acknowledgements: The work was supported by the United Nations University World Institute for Development Economics Research as part of the 'African Energy Futures (AEF)' project. The authors gratefully acknowledge this as well as additional support provided by the MIT Energy Initiative. We benefitted from discussions and feedback from other AEF researchers including Yohannes Gebrestadik, Chas Fant, Alyssa McClusky, Richard Swanson, and Kenneth Strzepek.

[1] Institute for Data, Systems and Society, Massachusetts Institute of Technology (MIT), Cambridge, MA, United States, corresponding author: amrose@mit.edu; [2] Center for Energy and Environmental Policy Research, MIT, Cambridge, MA, United States and Institute for Research in Technology, Comillas Pontifical University, Madrid, Spain.

This study has been prepared within the UNU-WIDER project on 'Africa' s energy futures', implemented in collaboration with MIT Joint Program on the Science and Policy of Global Change.

Information and requests: publications@wider.unu.edu

ISSN 1798-7237 ISBN 978-92-9256-170-3

Typescript prepared by Ayesha Chari.

The United Nations University World Institute for Development Economics Research provides economic analysis and policy advice with the aim of promoting sustainable and equitable development. The Institute began operations in 1985 in Helsinki, Finland, as the first research and training centre of the United Nations University. Today it is a unique blend of think tank, research institute, and UN agency—providing a range of services from policy advice to governments as well as freely available original research.

The Institute is funded through income from an endowment fund with additional contributions to its work programme from Denmark, Finland, Sweden, and the United Kingdom.

Katajanokanlaituri 6 B, 00160 Helsinki, Finland

1 Introduction

Despite an abundance of energy resources, electric power systems in Africa have been slow to develop and a large portion of the population in sub-Saharan Africa (SSA) remains without access to electricity (United Nations, 2003). A key challenge to providing electricity throughout SSA is that energy resources, although plentiful, are not uniformly distributed in form or location. The Congo River alone has an estimated potential to produce 1400 TWh per year, equivalent to over three times the annual consumption in all of SSA in 2010 (OECD/IEA, 2013; World Bank, 2015a). Whereas abundant coal resources are concentrated in the south, most of the natural gas and oil reserves are found in West Africa and North Africa. Developing these resources will require substantial capital investments, on the order of US$27 billion per year (ICA, 2011), and sufficient consumer demand to guarantee investors can recover their costs through revenues. However, in over half of SSA countries, national demand is less than the size of a typical utility-scale power plant. By pooling demand across multiple countries, regional power pools provide a larger consumer base in which to sell power, making projects that would be oversized and risky for a single country economically feasible for a regional market.

Regional power pools present a significant and potentially defining opportunity for African power systems to develop domestic energy resources, improve system reliability, and contribute to overall economic development (Resource Planning Associates, 1980). Regional trade can enable resource sharing among countries, allowing resource-rich countries to export power to countries with limited resources and greater diversity in the fuels used for electricity generation. Power trading between neighbouring countries has existed for many years. Bilateral trade agreements between Zambia and the Democratic Republic of the Congo were established in the 1950s. Additional agreements in the region eventually led to the formation of the 12-nation Southern African Power Pool (SAPP) in 1995 (ESMAP, 2010). Today there are three other regional power pools in SSA. These are the West African Power Pool (WAPP), Central Africa Power Pool (CAPP), and East Africa Power Pool (EAPP), all in various stages of development.

A second related development is the continued progress to build several major hydropower plants that could potentially dominate regional electricity trade and impact decision-making for investments in other generation and network infrastructure across the continent. Feasible power transmission highways – from Grand Inga in the Democratic Republic of the Congo and the Grand Renaissance Dam in Ethiopia to other regions on the continent – create the possibility of a pan-African electricity grid with hydropower as the principal energy source.

Global climate change is expected to cause major variations in Africa's hydrological resources, resulting in increased seasonal and inter-annual variations in water availability (IPCC, 2014:1132). These changes could increase the benefits of regional or pan-African integration, whereby additional trade compensates for seasonal differences in water availability across multiple hydropower basins. Alternatively, hydrological changes could expose a hydro-based system to greater risk if the variations across the major hydropower basins become synchronised.

Given the significant opportunities and uncertainty surrounding the potential benefits of regional power sector integration in Africa, the model presented in this paper is designed to investigate the advantages of greater inter-regional integration of the African power system and how climate change impacts on river basin flows may impact viability of pan-African trading.

2 Method

The goal of this study is to estimate the value of different levels of regional integration in SSA and how this value may change in the face of climate change. In pursuit of these objectives, we use a capacity expansion planning model covering both generation and transmission investments to simulate how the electric power systems in SSA may develop under different climate change scenarios.

2.1 Model design

The model is represented as a bottom-up, dynamic, multi-year optimisation problem, applying linear programming techniques to solve for the 'optimal' mix of generation and transmission infrastructure. We assume the perspective of a central planner under a traditional regulatory framework.[1] From this perspective, the objective of the planning problem is to maximise global welfare (i.e. meet total demand at lowest cost, including the cost of non-served energy) assuming perfect competition and considering:

- demand projections,
- existing and committed generation and transmission infrastructure,
- resource availability,
- available generation technologies and investment costs,
- fuel prices, and
- operating constraints (i.e. trade rules, emissions policies).

The planning problem consists of an investment problem to determine what plants or transmission lines to build and when to build them and an operations problem to determine which plants are used to meet demand at any given time. The results provide technical and cost information on how the system changes over the planning horizon for both components (see Table 1 for information list on each country, technology, and time period).

Table 1: Results of the expansion planning model

Investments	New transmission lines
	New power plants
	Investment costs
	Size, date and location of investments
Operations	Fuel consumption
	Production
	Emissions
	Trade flows
	Operating costs

Source: Authors' compilation.

[1] Under this assumption, all investment and operating decisions are made by a central entity and investors are guaranteed to recover their costs plus a rate of return. An alternative framework is to have multiple planners, each seeking to maximise their private profits. This framework would require very difficult long-term predictions about the ownership structure of national power sectors.

The model design contains the following key characteristics:

- Single node per country: Each country is represented as a single demand and supply node. This results in a loss of locational information about specific projects or intra-national networks but captures key trade-offs and benefits related to regional trade and coordinated planning on a pan-African scale.[2]
- Partially stochastic: The largest source of uncertainty is due to seasonal and long-term variations in hydropower inflows. These variations are modelled stochastically to generate a single optimal solution that minimises the expected cost over a set of possible hydropower outcomes and their respective probabilities. The robustness of this solution against other sources of uncertainty such as fuel prices and demand growth can be tested using scenario and sensitivity analysis.
- Time slices: Each planning year is divided into a series of time slices with a fixed load level and duration.
- Economic dispatch: The operating costs are based on least-cost economic dispatch to determine the output from each generator to meet demand at lowest cost.
- Generic and committed investments: New generation and transmission investments include both committed projects (with specified cost, capacity, and schedule parameters) and generic power plants and transmission lines that are not currently scheduled but might be added for economic or reliability reasons. The model does not include options for extending the life cycle for existing plants through refurbishments or fuel switching.

2.2 Trade scenarios

In order to evaluate the benefits of different levels of regional integration, we test three different trading scenarios: base case, minimum trade (MinTrade), and full trade (FullTrade) (Table 2).

Table 2: Trading scenarios used to explain varying levels of inter-regional integration

Trade scenario	Investments	Trade	Reserves
Base case	Only committed new transmission projects	No inter-regional trade	Countries supply their own firm capacity margin and operating reserves
MinTrade	Committed and economic transmission investments	Limited inter-regional trade	Countries supply their own firm capacity margin and operating reserves
FullTrade	Committed and economic transmission investments	Unlimited inter-regional trade	Countries can share firm capacity reserves

Source: Authors' compilation based on study criteria.

For all scenarios, 'trade' refers to exchanges of electricity or reserves, not fuels. The base case serves as a reference scenario where there is no trade between the four power pools and only countries within the same power pool can trade. Intra-pool trade is limited by the existing transmission capacity. The MinTrade and FullTrade scenarios explore the impacts of moderate or unlimited trading between power pools with coordinated investments in transmission infrastructure. In addition to power trading, countries also have the opportunity to share reserves. Firm capacity margins and operating reserves are reliability requirements to ensure countries have sufficient capacity installed to meet growing demand in the long term and can respond to emergencies in the short term. In the base case and MinTrade scenarios, countries operate in a 'self-sufficiency' mode where requirements of reserves of firm and operating

[2] In some exceptional cases, such as Mozambique, the national network currently exists as disconnected grids.

capacities must be met domestically. In the FullTrade scenario, countries can share reserves. (More information on these reserves is provided in the discussion that follows.)

2.3 Hydropower scenarios

Climate change is expected to have the largest impact on water resources available for hydropower generation. Hydropower generation is modelled externally on a plant-by-plant basis using the Water Evaluation and Planning (WEAP) simulation tool (Stockholm Environment Initiative, 2016). WEAP captures factors such as precipitation, runoff, water demand by other economic sectors, and river basin characteristics in greater detail than traditional power sector representations. For each plant and climate change scenario, WEAP produces an expected monthly generation after considering agricultural, environmental, and other external demands for water. These monthly values are fed in as upper bounds on the available hydropower generation for each country and month in the power sector model. After the optimisation problem is solved, two sets of output data are used to rerun the WEAP simulation tool. The first is the dual variable of the energy balance equation. This provides the *value of water*, or value of having one more unit of hydropower generation in a particular time slice. The second output is the amount of available hydropower used each month. If all of the hydropower that was available was not used to meet demand in a given month, it may be possible to store the water and use it during a month when more hydropower generation is needed. These values are returned to WEAP as inputs and the water simulation tool is rerun to provide new monthly generation values. We iterate between the electricity model and WEAP until they converge on an optimal solution. New hydropower investments are based on the long-term investment plan outlined by the Programme for Infrastructure Development in Africa (African Development Bank, 2010).

3 Input parameters

3.1 Planning horizon and time slices

The model covers the period from 2010 to 2030 with actual simulations extending to the year 2035 to avoid any abnormal 'edge' effects. The planning horizon is divided into:

- periods (i.e. 1 year),
- sub-periods (i.e. 1 month), and
- day types (i.e. L1, peak; L2, shoulder; L3, off-peak).

The 21-year planning horizon, therefore, is represented by 21×12×3, or 756 time slices. A shortcoming of reducing hourly demand into a subset of time slices is the loss of chronological information. As a result, the model does not account for ramping needs, start-up and shut-down costs, or other intra-day time-dependent interactions. For the purposes of this study, representing the planning horizon as time slices will still provide sufficient detail to examine long-term trends. Intra-day variations in demand and supply that drive ramping and start-up and shut-down decisions are used for decision-making on a monthly or weekly timeframe, not for long-term investment decisions, although the associated costs and operational restrictions may impact the optimal generation mix. Frequently, the costs associated with ramping and start-ups for fossil fuel plants are significantly less than total investment or operating costs and can be ignored for the purposes of exploring possible development trajectories over a long planning horizon. However, generally this is not the case if the future generation mix contains significant intermittent generation from wind and solar sources, as short-term operations may impact investment decisions to guarantee there is sufficient flexibility in the system (Palmintier, 2013). However, given the significant levels of storage in the form of reservoir hydropower in SSA, we assume the system has sufficient flexibility to accommodate high levels of wind and solar penetration and that the detailed representation of short-term effects can be avoided.

A key benefit of regional power pools is the ability to reduce total installed capacity needed to meet demand and reserve requirements, particularly in cases where peak demand is not occurring at the same time across multiple countries. Therefore, it is important to retain some chronological information about demand levels in each country. To do this, we will use the same time slice divisions for all countries. Therefore, a given time slice such as L1 will correspond to a particular time, 17–20 hours, for example, which may not correspond to peak demand in all countries.

3.2 Model regions

The model includes 39 SSA countries and seven river basins divided into four regional power pools. Each country and river basin is assigned to a single power pool and island nations are not included. Table 3 shows how the countries and river basins are grouped into power pools.

Table 3: Grouping of countries and river basins into regional power pools

	Southern African Power Pool	East African Power Pool	West African Power Pool	Central African Power Pool
Member countries	Angola Botswana Lesotho Malawi Mozambique Namibia South Africa Swaziland Zambia Zimbabwe	Burundi Djibouti Egypt Ethiopia Kenya Rwanda Sudan* Tanzania Uganda	Burkina Faso Cote d'Ivoire Gambia Ghana Guinea Guinea Bissau Liberia Mali Niger Nigeria Senegal Sierra Leone Togo/Benin**	Cameroon Central African Republic Chad Congo Democratic Republic of Congo Equatorial Guinea Gabon
River basin	Orange Zambezi	Nile	Volta Senegal Niger	Congo

Notes: *Sudan and South Sudan are represented as a single country in order to be consistent with most of the available data. **Togo and Benin are also represented as a single country because they share a common utility company and most data group these countries together.

Source: Authors' compilation based on study data.

3.3 Demand data

Demand data for each time slice consists of a load level (megawatts) and duration (hours). Demand growth is modelled as an annual growth rate specific to each country and year.

3.4 Generation technologies

This model only considers centralised grid-connected generation technologies presented in Table 4. Generation plants considered in the model include all *existing* plants, *committed* plants, and *generic* technologies. Generic projects are technology-specific capacity additions with associated investment and operating costs not tied to a specific project currently being considered. Each generation technology is characterised by the following techno-economic input parameters:

- Installed capacity (megawatts)
- Availability factor (percentage)
- Heat rate (million British thermal units per megawatt-hour
- Operational life (years)
- Variable operation and maintenance cost (US dollars per megawatt-hour)
- Fuel cost (US dollars per million British thermal unit)
- Fixed cost (US dollars per megawatt per year)
- Capital cost (US dollars per megawatt)
- Monthly generation limit (megawatt-hours) (for hydropower)
- Emissions rate (metric ton per megawatt-hour)
- Year online (for committed plants)
- (If applicable) Maximum capacity additions (megawatts)

Table 4: Technology options for new generation investments

Thermal non-renewable	Renewable
Distillate	Geothermal
Natural gas	Hydropower: reservoir, pumped storage, run-of-river
Coal	Wind (onshore)
Nuclear	Solar: photovoltaic
	Biomass

Source: Authors' compilation based on study data.

3.5 Transmission network

The representation of the network includes only cross-border interconnections. A full alternating current load flow problem is non-linear and the solution generally involves an iterative process to solve for the magnitude and angles of all node voltages and flows of active and reactive power across all lines. For this model, the network flows are represented using a 'transportation model', a simplifying approximation that only accounts for Kirchoff's first law that the sum of power flows entering a node must equal the sum of flows exiting the node (including injections from generators and withdrawals from load). Each 'line', therefore, is a representation of the cumulative transfer capacity between nodes. Table 5 presents options for new transmission investments.

Table 5: Technology options for new transmission investments

Candidate lines
132 kV
220 kV
330 kV
400 kV

Source: Authors' compilation.

These lines are chosen because they are common to all regions of the continent.[3] The total transfer capacity across each line is included as a constraint to power trading, losses are applied equally between the two nodes, and the quadratic expression is approximated as a fraction of active power flows across the line.[4]

For the base case, the network is composed of only existing and committed interconnections. The MinTrade and FullTrade scenarios include the option to build economic lines between countries and regions. The network is characterised by the following input parameters:

- Existing transfer capacities (megawatt)
- Investment cost (US dollars per kilometre)
- Line transfer reference capacity[5] (megawatts)
- Distance between nodes (kilometres)
- Losses (percentage)
- Year online (for committed lines)

3.6 Resource availability

For each country, fossil fuel resources are characterised by fuel cost and availability projected over the planning period. Renewable fuels include solar, wind, biomass, geothermal, and water. Wind and solar resources are represented as country-specific capacity factors for each time slice. For countries with geothermal resources, a maximum capacity limit (in megawatts) is imposed on the basis of the most recent resource assessments for each country. Biomass is modelled similar to fossil fuels with a fixed heat rate and per unit cost with no cap on available biomass resources. Hydropower is modelled on a project-by-project basis separately using the WEAP simulation tool. Hydropower generation, therefore, is included as a fixed input parameter consisting of a maximum available generation (in megawatt-hours) per month for each country and hydropower scenario.

3.7 Reliability

Reliability rules are included for all countries in the form of reserve requirements and a penalty for non-served energy. Two types of reliability requirements are included in this model: a firm capacity margin and operating reserves. The firm capacity margin, calculated as a percentage above estimated peak demand, mandates the minimum amount of firm capacity margin that must be available for each country. Operating reserve requirements are used to allocate a portion of installed generating capacity that must remain unused for contingency events. This reserve level must be sufficient to cover the highest production from a single power plant at any given moment plus a fraction of demand. This is approximated as the size of the largest plant in each country plus a margin based on demand. By sharing firm capacity across multiple countries, the total amount of firm capacity margin required for all countries remains the same but there are large potential savings as countries with excess capacity or less expensive generation sources can

[3] For more information on the method used to approximate the total transfer capacity for each type of line and pair of countries, see Appendix A.

[4] A piecewise approximation can also be used, but this comes at the cost of many additional variables.

[5] The reference capacity is calculated using the St. Clair approximation. More information on this method and the lines used for this model can be found in Appendix A.

supply firm capacity for neighbouring countries. This is one of the major recognised advantages of power pools.

In the case of operating reserves, sharing reserves *can* reduce the total amount that must be supplied. Interconnected countries need only supply sufficient reserves to cover the failure of the largest plant from the entire interconnected system (instead of the largest plant from each country) plus a fraction of peak demand.[6]

The cost of non-served energy is used as a penalty for not having sufficient generating capacity to meet demand. Although the actual value of non-served energy depends on the level of economic development and varies for each country, we use a single value for all countries for the purposes of this study.

3.8 Financial parameters

A uniform discount rate is applied to all countries to capture the time value of investment capital over the planning period.

4 Model formulation

This section contains the algebraic formulation of the optimisation problem. The problem consists of an objective function being minimised and a series of linear equations that form the constraints.

Tables 6–8 define the indices, input parameters, and decision variables. As a matter of nomenclature, all input parameters are designated with the letter *p* before the name and decision variables with the letter *v*.

Table 6: Indices

Variable	Definition
y	Year
m	Month
l	Day type (i.e., morning, evening)
r	Region (SAPP, EAPP, WAPP, CAPP)
c	Country
$CtoReg_{c,r}$	Assign each country to one region
g	Candidate technologies (i.e. coal, wind)
t_g	Subset of thermal candidate technologies
h_g	Subset of hydropower candidate technologies
r_g	Subset of renewable candidate technologies
$Lines_{c,c}$	Interconnection between countries
$RegLines_{c,c}$	Interconnection between countries from different regions
li	Line type (i.e. 132 kV, 330 kV)
e	Types of emissions (CO_2, NO_X)
CSC	Climate scenario

Note: In cases where we compare different values from the same set (i.e. power flowing from country A to country B or total costs in year 1 compared to year 2), we use different aliases to avoid confusion. For example, trade between country A and country B will be referenced as $Trade_{ci,cf}$ instead of $Trade_{c,c}$.

Source: Authors' compilation.

[6] See Appendix B for more detail on shared operating reserves.

Table 7: Input parameters

Parameter	Definition	Unit
Time periods		
pYearVal$_y$	Number of years since the base year	
pYearSplit$_{m,l}$	Duration of each day type in a given month	Hours
pDiscountRate	Discount rate	%
pAccumDiscountRate$_y$	Discount rate in a given year relative to the base year	%
Demand and reserves		
pDemandLevel$_{c,m,l}$	Demand in a given country, month and day type	MW
pPkDemand$_c$	Annual peak demand in base year	MW
pDemandGrowth$_{c,y}$	Rate of demand growth	%
pAccumDemandGrowth$_{c,y}$	Demand growth in a given year relative to the base year	%
pCostNonservedEnergy	Cost of non-served energy	US$/MWh
pFirmRes$_{c,y}$	Firm capacity margin requirement	MW
pOpRes$_{c,y}$	Operating reserve requirement	MW
pOpResMin$_c$	Minimum domestic operating reserve requirement	MW
pResCoef$_{c,c}$	Operating reserve coefficient between two connected countries	
Generation		
pExCapacity$_{c,y,g}$	Existing generation units including retirements and committed plants	MW
pAvailabilityFactor$_{g,y}$	Fraction of time plants are available due to maintenance	%
pOperationalLife$_g$	Plant lifetime	Years
pHeatRate$_{g,y}$	Fuel consumption	MMBTU/MWh
pRenewCF$_{c,g,l,m}$	Capacity factor for renewable plants	%
pFuelLimit$_{c,g}$	Available fossil fuel supplies in each country	MMBTU
pEmissionsRate$_{g,e}$	Rate of emissions for each technology type	Ton/MMBTU
Transmission		
pDistance$_{c,c}$	Distance between adjacent countries	km
pExLineCapacity$_{c,c,y}$	Maximum transfer capacity for existing and committed lines	MW
pLosses	Line losses	%
pMaxTransferCapacity$_{c,c,li}$	Maximum transfer limit for each line type and country pair	MW
Technology costs		
pCapCost$_{g,y}$	Investment cost for new plants	US$/MW
pVarCost$_{g,y}$	Variable operation and maintenance cost for generation	US$/MWh
pFixedCost$_{g,y}$	Annual fixed plant costs	US$/MW/year
pFuelCost$_{c,g}$	Fuel costs	US$/MMBTU
pInvestCap$_{c,g}$	If applicable, cap on allowable installed capacity for a given technology in each country	MW
pLineCapCost$_{li}$	Investment cost for new lines	US$/km
pAnnLineCapCost$_{li}$	Annualised investment cost for new lines	US$/km/year
Trade		
pMinTrade$_{r,y,m,l}$	Limit on inter-regional power trade (for MinTrade scenario)	W
Climate scenarios		
pProbCSC$_{csc}$	Probability of each climate scenario	%
pHydroLimit$_{c,h,csc,y,m}$	Maximum hydropower generation in each country, month, and climate scenario	MWh

Source: Authors' compilation.

Table 8: Decision variables

Variable	Definition	Unit
Generation capacity		
$vNewCapacity_{c,y,g}$	New plant additions in year y	MW
$vAccumNewCapacity_{c,y,g}$	New additions minus retirements from year 0 to year y	MW
$vTotCapacity_{c,y,g}$	Total capacity	MW
$vFirmCapacity_{c,y,m,l,g}$	Firm capacity margin supplied	MW
$vConCapacity_{csc,c,y,m,l,g}$	Connected capacity, plants that are online	MW
Transfer capacity		
$vTotNewLines_{c,c,y}$	New transfer capacity (committed and economic)	MW
$vTotTransferCapacity_{c,c,y}$	Total transfer capacity including new and existing lines	MW
$vNewLines_{c,c,y,li}$	Number of new lines built	Integer
Production		
$vProduction_{csc,c,y,m,l,g}$	Production level from each technology	MW
$vSpill_{csc,c,y,m,l,h}$	Unused ('spilled') hydropower generation	MW
$vENS_{csc,c,y,m,l}$	Non-served energy	MW
Costs		
$vTotGxCost_{c,y,g}$	Total costs of generation	US$
$vFixedGxCost_{c,y,g}$	Annual fixed generation costs	US$
$vVarGxCost_{c,y,g}$	Annual variable operation and maintenance generation costs	US$
$vCapGxCost_{c,y,g}$	Annual capital cost for new plants	US$
$vCapTxCost_{y}$	Annual capital costs for new lines	US$
$vDiscountedTotCost_{y}$	Discounted total costs for generation and transmission	US$
$vENSCost_{c,y}$	Cost of non-served energy	US$
Emissions		
$vAnnEmissions_{csc,c,y,t,e}$	Annual emissions by technology and climate scenario	Ton
Trade		
$vTrade_{csc,y,m,c,c,l}$	Power trade in each day type	MW
$vTradeFirmCap_{y,m,l,c,c}$	Shared firm reserves	MW

Source: Authors' compilation.

4.1 Objective function

The objective function is the minimisation of all discounted investment and operating costs over the planning period.

$$
\begin{aligned}
&Min\sum_{y} vCapxCost_{y} \times pAccumDiscountRate_{y} \\
&+\sum_{c,y,g}\left[vFixedGxCost_{c,y,g} + vVarGxCost_{c,y,g} + vCapGxCost_{c,y,g}\right]. \\
&\times pAccumDiscountRate_{y} + \sum_{c,y} vENSCost_{c,y}
\end{aligned}
\tag{1}
$$

4.1.1 *Capital costs*

The capital cost of new generation investments in each year, y, is based on the addition of new generic power plants.[7] The installed capacity of generic plants is designated by the continuous

[7] The capital costs associated with committed and existing projects are included outside of the optimisation problem as these costs are fixed inputs and do not affect the final solution.

variable *vNewCapacity*.[8] The capital cost is equal to the installed capacity (megawatts) times a per-unit capital cost, *pCapCost* (US dollars per megawatt), which may vary by technology and year.

$$vCapGxCost_{c,y,g} = vNewCapacity_{c,y,g} \times pCapCost_{g,y}. \tag{2}$$

For transmission investments, the capital cost in each year, *y*, is equal to the number of lines built of each type, *li*, times the annualised capital cost for that type of line (US dollar per kilometre) and the distance (kilometres) between the origin, *ci*, and destination, *cf*. The annualised line cost is based on an interest rate of 1.5% (World Bank, 2015b) and payback period of 40 years.

$$\begin{aligned} vCapTxCost_{y} = & \sum_{yy \in (y-yy>0)} \sum_{Lines(ci,cf)} vNewLines_{ci,cf,y,li} \\ & \times pDistance_{ci,cf} \times pAnnLineCapCost_{li} \end{aligned}. \tag{3}$$

We use the *annualised* capital cost for transmission because of the 'lumpy' nature of transmission investments. As we cannot build a fraction of a line, the number of new lines built, *vNewLines*, is an integer variable. Because we assume generation investments are continuous (i.e. they can be built incrementally), the capital cost for new plants can be applied entirely in the year the plant is built.

4.1.2 Fixed costs

Fixed operating costs apply only to generation plants and are calculated as a fixed per unit cost (US dollars per megawatt per year) times the total installed capacity. The fixed cost can vary according to year and technology.

$$vFixedGxCost_{c,y,g} = pFixedCost_{g,y} \times vTotCapacity_{c,y,g}. \tag{4}$$

4.1.3 Variable costs

Variable costs apply only to generation plants and consist of costs associated with (a) operation and maintenance, (b) fuel consumption, (c) unit commitment, and (d) non-served energy. Operation and maintenance costs are a fixed parameter, *pVarCost* (US dollars per megawatt-hour), times the total generation in each time slice. The total generation is equal to the production level in each time slice, *vProduction* (megawatts), times the number of hours in that time slice, *pYearSplit* (hours). Fuel consumption is based on the total production in each time slice times the technology specific heat rate and fuel costs. The cost for connected units is based on the total capacity that is 'connected' (running but not necessarily producing power) in each time slice, *vConCapacity*, times the number of hours connected and the plant's variable cost, *pVarCost*. For a full unit commitment problem, this would be represented as the start-up cost, but unit commitment models require a significant number of binary variables ('on'/'off' decisions) for each technology, which can significantly slow down the solution time.

[8] In order to maintain clarity, the subscripts are not included for parameters and variables referenced in the text. They have been included only in equations for accuracy.

Variable costs will vary with each climate scenario as the generation capacity of hydropower changes. The final variable cost is calculated as the expected value of the variable costs over all climate scenarios. The parameter *pProbCSC* represents the probability of each climate scenario.

$$vVarGxCost_{c,y,g} = \sum_{csc} pProbCSC_{csc} \sum_{m,l} pYearSplit_{m,l} \begin{pmatrix} \sum_{l,y} vProduction_{csc,c,y,m,l,g} \times pVarCost_{g,y} + vProduction_{csc,c,y,m,l,g} \\ \times pHeatRate_{g,y} \times pFuelCost_{c,g} + vConCapacity_{csc,c,y,m,l,g} \\ \times pYearSplit_{m,l} \times pVarCost_{g,y} \end{pmatrix}. \tag{5}$$

The costs of non-served energy are calculated separately because they cannot be assigned to a particular generation technology, *g*. This cost, *vENSCost*, is calculated as the expected value over all climate scenarios of the amount of non-served energy in each time slice times a fixed cost, *pCostNonservedEnergy*.

$$vENSCost = \sum_{csc} pProbCSC_{csc} \sum_{m,l} pYearSplit_{m,l} \times vENS_{csc,c,y,m,l} \times pCostNonservedEnergy. \tag{6}$$

4.1.4 Discounting

To account for the time value of money and investments, all costs are discounted to the start year 2010. The discount rate in the active year is given by:

$$pAccumDiscountRate_y = \frac{1}{(1 + pDiscountRate)^{n-1}}, \tag{7}$$

where *n* is the number of years since the base year.

4.2 Constraints

4.2.1 Generation capacity adequacy

The total installed generating capacity in each country is updated yearly to account for existing plants, retirements, and new installations. The parameter *pExCapacity* includes committed investments and retirements for existing units.

$$vTotCapacity_{c,y,g} = pExCapacity_{c,y,g} + vAccumNew. \tag{8}$$

New generic plant installations are equal to the sum of all new generic plants from previous years minus those plants that are retired when their operational life has ended.

$$vAccumNewCapacity_{c,y,g} = \sum_{yy \in (y-yy<OperationalLife) \in (y-yy>0)} vNewCapacity_{c,yy,g}. \tag{9}$$

The total installed capacity is not equal to the capacity available to meet demand. Power plants use some portion of their power internally and must go offline occasionally for maintenance. With renewable generators, the plant's output also depends on resource availability, which may vary throughout the day and year. We use the variable *vFirmCapacity* to represent the maximum available capacity that each technology can contribute to meet demand in each time slice.

$$vFirmCapacity_{c,y,m,l,g} \leq vTotCapacity_{c,y,g} \times pAvailabilityFactor_{g,y} \times pRenewCF_{c,g,l,m}. \quad (10)$$

The parameters *pAvailabilityFactor* and *pRenewCF* take on values between 0 and 1 to account for the fraction of installed capacity available in each month and time slice due to internal use and renewable resources, respectively. For example, solar plants may have a *pRenewCF* value of 0.8 during the day, indicating that a 100 W plant could generate up to 80 W during this time and a value of 0 at night when there is no sunlight. For all non-renewable plants, the value of *pRenewCF* is set to 1.

The variable *vConCapacity* indicates the capacity of each technology that is connected (or running). Connected plants serve two purposes: (a) they produce electricity to meet demand; and (b) all or a fraction of their capacity is left unused to serve as operating reserves. The maximum capacity that can be connected is limited by the total firm installed capacity.

$$vConCapacity_{CSC,c,y,m,l,g} \leq vFirmCapacity_{c,y,m,l,g}. \quad (11)$$

In this formulation, we ignore the minimum technical operating limits of individual plants. The total production from each technology, therefore, is limited by its connected capacity.

$$vProduction_{CSC,c,y,m,l,g} \leq vConCapacity_{CSC,c,y,m,l,g}. \quad (12)$$

As discussed in the calculation of variable costs, there is a cost associated with maintaining connected capacity. Therefore, the cost-minimising solution will 'shut down' or disconnect capacity when demand is low and 'start up' or connect additional capacity when demand is high. However, as *vConCapacity* is a continuous variable, this approximation avoids the need to run a full unit commitment model and introduce binary start-up and shut-down decision variables for each technology.

4.2.2 Resource limits

Unlike solar and wind energy, which must be used or wasted when the resource is available, fossil fuel resources can be stored for use only when the energy is needed. However, these resources have a finite domestic supply, which limits their usage over the planning horizon. To a lesser extent, hydropower resources can also be stored by retaining water in reservoirs. Hydropower resources are also limited by a finite supply that varies seasonally and year to year.

Production from hydropower plants is limited by a maximum monthly production limit, *pHydroLimit*, determined externally from water simulation models for each river basin and climate change scenario, *CSC*. The dummy variable *vSpill* is introduced as a slack variable for instances where the system cannot use all of the available hydropower, equivalent to 'spilling water' from the reservoir.

$$\sum_{l}\left(vProduction_{CSC,c,y,m,l,h} + vSpill_{CSC,c,y,m,l,h}\right) \times pYearSplit_{m,l} = pHydroLimit_{c,h,CSC,y,m}. \quad (13)$$

Fossil fuel resources do not have annual or monthly usage limits in this model. They can be used gradually over time or all in one year subject to the constraint that the total usage over the planning horizon does not exceed the available fuel resources in the host country, *pFuelLimit*.

$$\sum_{y,l,m} vProduction_{CSC,c,y,m,l,t} \times pYearSplit_{m,l} \times pHeatRate_{t,y} \leq pFuelLimit_{c,t}. \tag{14}$$

Finally, for technologies such as geothermal, investments are limited by an estimated capacity that can be developed and not by fuel availability. These limits may be related to issues such as geography, geology, or land availability. For those countries and technologies with identified capacity limits, the following constraint is applied:

$$\sum_{y} vTotCapacity_{c,y,g} \leq pInvestCap_{c,g}. \tag{15}$$

4.2.3 Transmission adequacy

Similar to generation, the total transfer capacity between countries is updated annually. The total transfer capacity is equal to the contributions from existing lines and committed lines, represented by *pExLineCapacity*, plus the sum of new line capacities added during the active year.

$$vTotTransferCapacity_{ci,cf,y} = pExLineCapacity_{ci,cf,y} + vTotNewLines_{ci,cf,y}. \tag{16}$$

New transmission investments are represented by the variable *vNewLines*, an integer number representing the number of each type of candidate line that is added. Each type of line has a maximum carrying capacity, *pMaxTransferCapacity*, based on the distance between the countries.[9] The transfer capacities added for each type of line are summed to yield the total new transfer capacity.

$$vTotNewLines_{ci,cf,y} = \sum_{li} vNewLines_{ci,cf,y,li} \times pMaxTransferCapacity_{ci,cf,li}. \tag{17}$$

Constraint (18) ensures that an investment between countries *ci* and *cf* also increases the transfer capacity in the opposite direction:

$$vNewLines_{ci,cf,y,li} = vNewLines_{cf,ci,y,li}. \tag{18}$$

4.2.4 Energy balance

The energy balance constraint maintains that total supply must equal demand in all time slices. Supply can take the form of domestic production, non-served energy, and energy imports. Demand comes from electricity consumption and energy exports. Each year, demand is expected to increase by some country-specific factor, *pDemandGrowth*. Therefore, the demand after *n* years is the original demand in year 1 times an accumulated demand growth which is equal to $(1+pDemandGrowth)^{n-1}$. The energy balance equation can therefore be summarised as:

---

[9] See Appendix A for more information on the method to derive these values.

$$Production + Nonserved\ energy + Imports = Demand + Exports. \tag{19}$$

Using the model notation, this becomes:

$$\begin{aligned}&\sum_{g} vProduction_{CSC,c,y,m,l,g} + vENS_{CSC,c,y,m,l} + \sum_{Lines(ci,c)} vTrade_{CSC,y,m,ci,c,l} \times pLosses \\ &= pDemandLevel_{c,m,l} \times pAccumDemandGrowth_{c,y} \\ &+ \sum_{LINES(c,cf)} vTrade_{CSC,y,m,c,cf,l} \times pLosses\end{aligned} . \tag{20}$$

For imports and exports, the total electricity delivered between importing and exporting countries is reduced by a loss factor, *pLosses*, to account for the thermal losses that occur on transmission networks. These losses are divided equally between the importing and exporting countries.

4.2.5 Trade

The variable *vTrade* indicates the volume of electricity trading between countries in each time slice. Total trade is limited by the transmission interconnections between countries.

$$vTrade_{CSC,y,m,ci,cf,l} \le vTotTransferCapacity_{ci,cf,y}. \tag{21}$$

4.2.6 Trading scenarios: Base case

In the base case, there is no electricity trading between different power pools. This constraint is imposed by mandating that all flows over lines identified as *regional lines*, because they connect countries belonging to different power pools, are 0.

$$\sum_{LINES \in RegLines(c,cf)} vTrade_{CSC,y,m,c,cf,l} = 0. \tag{22}$$

Exclusively in the base case, we assume there is no coordinated transmission planning between power pools. To represent this constraint, we mandate that no transmission lines between regions (i.e. *RegLines*) be built.

$$vNewLines_{ci,cf,y,li} = 0 \quad \forall Lines \in RegLines. \tag{23}$$

Generation capacity must be sufficient to meet demand plus some firm capacity margin and operating reserve requirements. For the base case and MinTrade scenarios, countries operate in a self-sufficiency mode where they supply their own firm capacity and reserves. The firm capacity reserve margin for each country, *pFirmRes*, is an input parameter equal to 10% of that country's annual peak demand. We assume that peak demand grows at the same rate as the country's overall demand and use the peak demand from the base year, *pPkDemand*, times the accumulated demand growth to estimate the peak demand for any future year.

$$pFirmRes_{c,y} = pPkDemand_{c} \times pAccumDemandGrowth_{c,y}. \tag{24}$$

Under the self-sufficiency mode, each country is responsible for meeting its firm capacity margin using domestic plants.

$$\sum_{g} vFirmCapacity_{c,y,m,l,g} \geq pFirmRes_{c,y}. \qquad (25)$$

Operating reserve requirements, *pOpRes*, denote the amount of unused connected capacity that must be available to meet demand during contingency events (e.g. a plant or transmission line fails). *pOpRes* is calculated as the size of the largest plant in each country plus 5% of peak demand.

$$\sum_{g} vConCapacity_{CSC,c,y,m,l,g} - \sum_{g} vProduction_{CSC,c,y,m,l,g} \geq pOpRes_{c,y}. \qquad (26)$$

4.2.7 Trading scenarios: MinTrade case

In the MinTrade case, there is a cap placed on the volume of inter-regional trading. The cap is set exogenously as a fraction of total regional demand in each time slice.

$$\begin{aligned} \sum_{ri} \sum_{c \in CtoReg(c,r)} \sum_{ci \in CtoReg(ci,ri)} \sum_{LINES \in RegLines(c,ci)} & vTrade_{CSC,y,m,ci,c,l} \\ & \times pYearSplit_{m,l} \leq pMinTrade_{r,y,m,l} \end{aligned}. \qquad (27)$$

Firm capacity margin and operating reserve requirements remain the same as in the base case.

4.2.8 Trading scenarios: FullTrade case

In the FullTrade case, electricity trading is unlimited and countries have the ability to share reserves [Equations (22)–(27) are inactive]. For firm capacity margins, countries can either supply them domestically or trade firm capacity with neighbouring countries. The variable *vTradeFirmCap* indicates the level of firm reserves supplied to and from neighbouring countries. Similar to the equation for energy, the balance equation for maintaining firm capacity (*FC*) margins can be represented as:

$$Domestic\ FC - Exported\ FC + Imported\ FC \geq Firm\ Capacity\ Requirement. \qquad (28)$$

Using the model notation, this becomes

$$\begin{aligned} & \sum_{g} vFirmCapacity_{c,y,m,l,g} - \sum_{LINES(c,cf)} vTradeFirmCap_{y,m,l,c,cf} \\ & \quad + \sum_{LINES(ci,c)} vTradeFirmCap_{y,m,l,ci,c} \geq pFirmRes_{c,y} \end{aligned}. \qquad (29)$$

The maximum firm capacity that can be traded is constrained by the interconnection capacity between countries.

$$vTradeFirmCap_{y,m,l,ci,cf} \leq vTotTransferCapacity_{ci,cf,y}. \qquad (30)$$

For operating reserve requirements, sharing can reduce the amount of operating reserves that must be supplied by each country. The size of this decrease depends on the interconnection capacity between countries and the characteristics of each country pair (e.g. a small country sharing reserves with a large country or sharing between two similarly sized countries). The parameter *pResCoef* captures this relationship between any country pair.

$$\begin{aligned}&\sum_{g} vConCapacity_{CSC,c,y,m,l,g} - \sum_{g} vProduction_{CSC,c,y,m,l,g}\\ &\geq pOpRes_{c,y} + \sum_{cf} pResCoef_{c,cf} \times vTotTransferCapacity_{c,cf,y}\end{aligned}, \tag{31}$$

where *pOpRes* is the amount of reserves that country *c* needs when considered in isolation. The last term in Equation (31) corresponds to the support that neighbouring countries can provide with imperfect transmission.

In accordance with international practices, we assume each county must supply some minimum amount of operating reserves domestically, *pOpResMin*.

$$\sum_{g} vConCapacity_{CSC,c,y,m,l,g} - \sum_{g} vProduction_{CSC,c,y,m,l,g} \geq pOpResMin_{c}. \tag{32}$$

Equation (32) limits the amount of reserves that country *c* can import from other countries.[10]

4.2.9 Emissions

Emission factors, *pEmissionsRate* (ton per million British thermal unit), are used to calculate the total emissions produced in each time step, country, and technology type. The model currently includes NO_X and CO_2 emissions with the option to add other types as needed.

$$\begin{aligned}vTotalAnnEualEmissions_{CSC,c,y,g,e} = &\sum_{m,l} vProduction_{CSC,c,y,m,l,g} \times pYearSplit_{m,l}\\ &\times pHeatRate_{c,g} \times pEmissionsRate_{g,e}\end{aligned}. \tag{33}$$

While the model does not contain emissions caps, this could be added as a constraint.

5 Limitations of the model

As with any model, it is important to note that this model cannot accurately capture all the dynamic interactions that occur in an electric power system, which is subject to technical, economic, and behavioural influences. Instead, the model is designed to provide insight into how systems may evolve over time under different climate conditions. In this effort, several simplifications have been made regarding the representation of time, operation characteristics, investment decisions, demand, and resource availability. In addition to these simplifications, expansion and operational decisions are made under an implicit assumption of perfect foresight,

---

[10] More information on the method used to calculate *pResCoef* and *pOpResMin* as well as numerical examples of this approximation method are provided in Appendix B.

where future hydrological, solar, and wind resources over the entire period are known, and perfect competition. In actual system operations, decisions must be made in the face of uncertainty and, in general, the structure of the sector is such that agents' behaviour deviates from that of perfect competition. This paper presents the design and formulation of the model. Numerical results will be presented in a separate paper.

References

African Development Bank, 2010. 'Programme for infrastructure development in Africa (PIDA): Interconnecting, integrating, and transforming a continent'. African Development Bank, Tunis. http://www.afdb.org/fileadmin/uploads/afdb/Documents/Project-and-Operations/PIDA%20note%20English%20for%20web%200208.pdf Accessed 31 October 2016.

Donohoo-Vallett, PE, 2014. Design of wide-area electric transmission networks under uncertainty: Methods for dimensionality reduction. PhD thesis in Engineering systems: Technology, management and policy. Massachusetts Institute of Technology, Cambridge, MA. http://globalchange.mit.edu/files/document/Donohoo_PhD_2014.pdf Accessed 31 October 2016.

ESMAP (Energy Sector Management Assistance Program), 2010. Regional power sector integration: Lessons from global case studies and a literature review'. ESMAP briefing note 004/10, June. ESMAP, World Bank, Washington, DC. https://openknowledge.worldbank.org/bitstream/handle/10986/17507/773070v10ESMAP0ISP0Lessons0BN004010.pdf?sequence=1&isAllowed=y Accessed 31 October 2016.

ICA (Infrastructure Consortium for Africa), 2011. Regional power status in African power pools. ICA report, November. ICA Secretariat, African Development Bank, Tunis. http://www.icafrica.org/fileadmin/documents/Knowledge/Energy/ICA_RegionalPowerPools_Report.pdf Accessed 31 October 2016.

IPCC (Intergovernmental Panel on Climate Change), 2014. Climate change 2014: Impacts, adaptation, and vulnerability. Part A: Global and sectoral aspects. Contributions of working group II to the fifth assessment report of the IPCC. : Cambridge University Press, Cambridge. http://ipcc-wg2.gov/AR5/images/uploads/WGIIAR5-PartA_FINAL.pdf Accessed 31 October 2016.

Liu, A, Hobbs, BH, McCalley, JD, Shahidehpour, M, & Zheng, QP, 2013. Co-optimization of transmission and other supply resources. Report prepared for Eastern Interconnection States' Planning Council (EISPC) and National Association of Regulatory Utility Commissioners (NARUC). EISPC and NARUC, Washington, DC.

Miketa, A, & Merven, B, 2013a. 'Southern African power pool: Planning and prospects for renewable energy. International Renewable Energy Agency (IRENA) report. IRENA, Bonn. http://www.irena.org/documentdownloads/publications/sapp.pdf Accessed 31 October 2016.

Miketa, A, & Merven, B, 2013b. 'West African power pool: Planning and prospects for renewable energy. International Renewable Energy Agency (IRENA) Report. IRENA, Bonn. https://www.irena.org/DocumentDownloads/Publications/WAPP.pdf Accessed 31 October 2016.

Mwangi, D, 2012. Regional interconnectivity & cross border trade networks: An EAPP update. East African Power Industry Convention, 10–13, September. Dar es Salaam, Tanzani.

Nexant, 2007. SAPP regional generation and transmission expansion plan study. Draft report. Volume 2: Main report. Technical report submitted to Southern Africa power pool coordination centre. Nexant, San Francisco.

OECD/IEA (Organisation for Economic Co-operation and Development/and International Energy Agency), 2013. Key world energy statistics. OECD/IEA, Paris. http://www.observatoire-du-nucleaire.org/IMG/pdf/2013-keyworld-aie.pdf Accessed 31 October 2016.

Palmintier, B, 2013. Incorporating operational flexibility into electric generation planning: Impacts and methods for system design and policy analysis. PhD thesis in Engineering systems: Technology, management and policy. Massachusetts Institute of Technology, Cambridge, MA. http://bryan.palmintier.net/pdf/PalmintierDissertation.pdf Accessed 31 October 2016.

Resource Planning Associates, 1980. 'Power pooling issues and approaches. Contractor report DOE/ERA/6385-1, contract EE-78-C-01-6385, National Technical Information Service, January. US Department of Energy, Washington, DC.

Rosnes, O, Vennemo, H, & Pöyry, E, 2009. Powering up: Costing power infrastructure spending needs in Sub-Saharan Africa. The International Bank for Reconstruction and Development and The World Bank, Washington, DC.

S&P Global Platts, 2010. World electric power plants (WEPP) database. At: http://www.platts.com/products/world-electric-power-plants-database Accessed 31 October 2016.

SAPP (Southern African Power Pool), 2016. SAPP transfer limits – 2015 (North to South wheeling). http://www.sapp.co.zw/tlimits.html Accessed 15 August 2015.

SNC Lavalin International & Brinckerhoff, P, 2011. Regional power system master plan and grid code study. In final master plan report, May. Prepared for East Africa power pool (EAPP) and East Africa community (EAC).

St Clair, HP, 1953. Practical concepts in capability and performance of transmission lines. *Transactions of the American Institute of Electrical Engineering* December, 1152–7.

Stockholm Environmental Institute, 2016. WEAP: Water evaluation and planning'.. http://www.weap21.org/ Accessed 15 February 2016).

Tractebel Engineering, 2011. Update of the ECOWAS Revised Master Plan for the generation and transmission of electrical energy. Economic Community of West African States, Accra.

United Nations, 2003. Assessment of power pooling arrangement in Africa. Ad Hoc Expert Group Meeting on Assessment of Power Pooling Arrangements in Africa, 24–26 June, Economic Commission for Africa, Addis Ababa. http://repository.uneca.org/handle/10855/21403 Accessed 31 October 2016.

WAPP (West African Power Pool), 2011. Status of priority projects/2013. http://www.ecowapp.org/?page_id=168 Accessed 15 August 2015.

World Bank, 2015a. World development indicators: Electricity production, sources, and access. World Bank, Washington, DC. http://wdi.worldbank.org/table/3.7 Accessed 31 October 2016.

World Bank, 2015b. Average interest on new external debt commitments, official (%). World development indicators, International debt statistics. World Bank, Washington, DC. http://data.worldbank.org/indicator/DT.INR.OFFT?end=2014&start=2014&view=map Accessed 31 October 2016.

Appendix A: Incorporating discrete transmission investments into a transportation model

Regional and inter-regional power trading cannot be realised without significant investments in cross-border transmission lines. Therefore, we must consider the expansion of the regional grid in order to investigate the benefits of regional integration. Previous regional studies have ignored transmission expansion (Miketa & Merven, 2013a, 2013b) or used an iterative approach to solving the generation and transmission expansion problems separately (Nexant, 2007; SNC Lavalin & Brinckerhoff, 2011). The first method limits trade to the small number of existing and committed interconnections whereas the second provides a more detailed representation of the network but is more complex to implement.

Co-optimisation of generation and transmission expansion significantly increases the size of the problem, requiring a trade-off between obtaining a detailed representation of power flows and generation units for a small region or modelling a larger region with approximated representations of aggregate generation and load flows (Liu et al., 2013). To incorporate transmission and generation expansion decisions without the added complexity of doing a full-load flow analysis, we designed a 'transportation' or 'pipe' model for the transmission network. In this representation, we model the net transmission capacity (megawatts) between countries instead of individual lines. Transmission costs depend on the type of line being used, which in turn depends on the topology of the existing network and distance between nodes. Investment decisions are discrete (i.e. you cannot build half a line) and there are significant economies of scale in transmission. This section outlines the method used to incorporate discrete transmission investment decisions into the expansion planning model.

A1 Transmission network in Africa

With 79 possible node pairs considered in the model, the number of investment decisions in cross-border lines is significant. To reduce the size of the problem, we surveyed existing and committed cross-border lines in each power pool to narrow the set of candidate lines that should be considered (Table A1).

Table A1: Survey of cross-border transmission lines used in different regions

Region	Number of lines (existing and committed)									
	70 kV	110 kV	132 kV	161 kV	220 kV	225 kV	330 kV	400 kV	500 kV	533 kV
SAPP		2	6		4		5	7		1
EAPP		1	2		9		1	1	3	
WAPP			2	6		2	1			
CAPP					1		1	3		
CAPP/SAPP					1		1			
CAPP/EAPP	2	1			2					
Total	2	4	10	6	17	2	9	11	3	1
Universal							*x*			

Notes: SAPP, South African Power Pool; EAPP, East Africa Power Pool; WAPP, West African Power Pool; CAPP, Central Africa Power Pool.

Source: Authors' compilation based on Rosnes et al. (2009), SNC Lavalin & Brinckerhoff (2011), Tractebel Engineering (2011), WAPP (2011), Mwangi (2012) and SAPP (2016).

The survey revealed 220, 400, 132, and 330 kV lines are the most common, with only the 330 kV line used in all regions. These four types of lines were chosen as candidate technologies for new transmission investments. There are also several 161 kV lines, but these were excluded as potential candidates as they are only used in West Africa.

Additionally, for each node pair we calculated the distance between substations for cross-border interconnections. In 17 cases, no data were available on existing or planned cross-border interconnections. In these cases, data on the location of major generation stations were used to approximate the distance between substations (S&P Global Platts, 2010). It is important to note that these distances do not include potential intra-country reinforcements that may be needed to transmit power from a substation near the border to a load centre in another part of the country or wheel power between two countries. The resulting transmission investment costs, therefore, may be an underestimation of the actual costs.

A2 Approximate load carrying capability: St Clair curve

The load carrying capability of transmission lines is limited by three factors: thermal limit, voltage limit, and angular stability limit. The thermal limit describes the maximum capacity of a transmission line before it overheats resulting in sag and loss of tensile strength. This limit typically determines the line's carrying capacity for short lines, less than 50 miles. For longer lines, 50–150 miles, power flow is constrained by voltage limits. As the distance between nodes increases, the voltage magnitude and corresponding power flow decrease. To maintain network stability, the maximum allowable voltage drop is generally limited to 5–10% of the bus voltage at the injecting node. As the voltage falls with increasing distance, the resulting voltage drop limit also decreases with distance and can be much lower than the thermal limit for longer lines. Finally, as power flows across a line, imbalances between the line's capacitance and inductance can cause reactive power to be supplied or consumed by the line, resulting in an electrical phase shift. The phase shift increases with distance and can cause stability problems for the network. Stability limits are generally represented by the surge impedance loading (SIL), a product of the operating bus voltages divided by the characteristic impedance of the line. For very long lines, >150 miles, the SIL sets the limit on the maximum allowable power flow.

The St Clair curve is a simple means for estimating the power transfer capabilities for transmission lines using engineering heuristics to capture these three factors. The curve, first developed by H.P. St Clair in 1953, relates the carrying capacity of a particular line in terms of SIL to the line length (see St Clair, 1953). It can be used for any voltage level using literature values for the surge impedance loading of a particular line to determine load carrying capacity in MW at a given distance (Donohoo-Vallett, 2014). Table A2 contains SIL values for the candidate lines used in the model.

Table A2: SIL values for candidate transmission lines

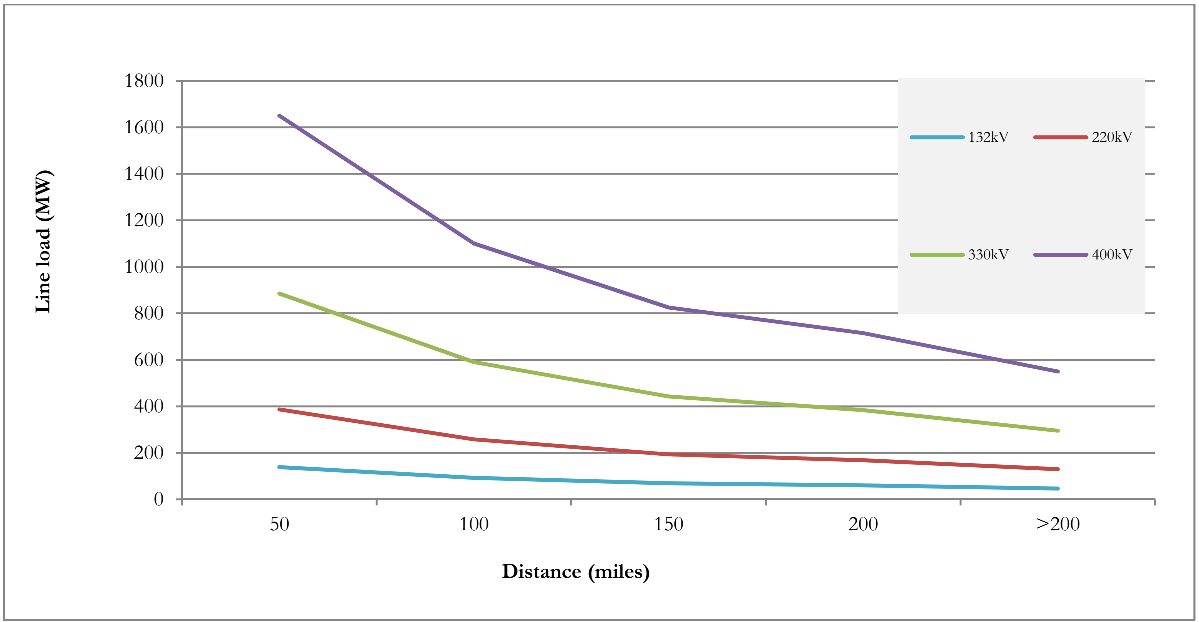
Line	SIL
132 kV*	46
220 kV**	129
330 kV	295
400 kV	410

Notes: *Approximated from 138 kV line (SIL = 48). **Approximated from 230 kV line (SIL = 132).

Source: Authors' compilation based on Donohoo-Vallett (2014).

SIL values are used with the St Clair curve to calculate the maximum line loading for each candidate line and distance range as a piecewise linear function (Figure A1 and Table A3).

Figure A1: Piecewise power transfer function for candidate lines



Source: Authors' representation based on study data.

Table A3: Max line loading for each line type and distance range

Line	<50 miles	50–100 miles	100–150 miles	150–200 miles	>200 miles
132 kV	138	92	69	59.8	46
161 kV	195	130	97.5	84.5	65
220 kV	387	258	193.5	167.7	129
330 kV	885	590	442.5	383.5	295
400 kV	1239	820	615	533	410

Source: Authors' compilation based on study data.

A2.1 Transmission line costs

The carrying capacity is only one factor to consider when choosing whether or not to build a new line and which type of line is the best choice. The other critical factor is the cost of the line. Per-unit line costs vary depending on the voltage rating. Table A4 contains the economic parameters for each candidate line considered in the model. (Other factors such as local geography and land use costs that also impact transmission line costs are not included in this analysis.)

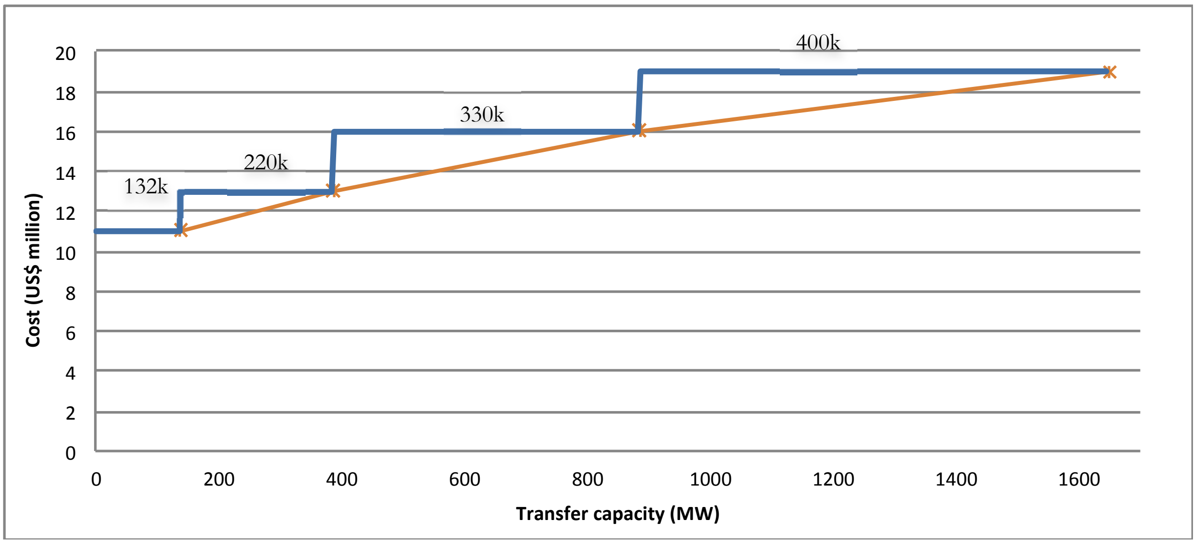
Table A4: Line cost assumptions for candidate lines

Line	Line cost (2010 USD per kilometre)
132 kV	132 923
220 kV	166 154
330 kV	199 384
400 kV	232 616

Source: Reproduced from Nexant (2007).

Using this cost data and the power transfer capacities from Figure A2, we can plot the relationship between investment cost and carrying capacity for each distance range. Figure A2 shows this relationship for the 0–50-mile range.

Figure A2: Relationship between cost and transfer capacity for the 0–50-mile range



Source: Authors' representation based on study data.

The plot is a piecewise relationship, with each segment representing a switch to a higher voltage line. As the desired transfer capacity increases, it becomes cheaper to choose a line with a larger voltage rating even though it may have a higher cost per kilometre. With each step up to a higher voltage line, the slope of the curve decreases, reflecting the economies of scale that exist in transmission investments.

Note that as it is impossible to build a fraction of a line, we cannot add a fraction of a line's transfer capacity. For example, with the candidate lines being considered, it is not possible to add exactly 250 MW of transfer capacity for two nodes 50 miles apart. We could add, for example, one 220-kV line with a carrying capacity of 400 MW (150 MW more than needed) or two 132-kV lines with a total carrying capacity of 280 MW (30 MW more than needed). The discrete nature of transmission investments is represented by the blue step function in Figure A2.

Similar curves can be derived for any node pairs given the distance between them. As the carrying capacity decreases with distance for any given line, the cost to achieve a given carrying capacity increases as the distance between nodes increases.

A3 Implementing the St Clair approximation in the model

The St Clair approximation method simplifies the transmission expansion problem by characterising the physical and economic characteristics of candidate lines before the optimisation run. By estimating the additional transfer capacity a new line will contribute, it permits easy addition and updating of transfer capacities without solving the full-load flow problem. The method is operationalised in the model with three input parameters and one integer variable (Table A5).

Table A5: Transmission parameters and decision variables

Transmission parameters	
$pDistance_{c,c}$	Distance between node pairs (kilometres)
$pLineCapitalCost_{li}$	Capital cost for each line type (US dollars per kilometre)
$pMaxTransferCap_{c,c,li}$	Max transfer limit for each line type and distance range (megawatts)
Transmission decision variables	
$vNewLines_{c,c,y,li}$	Integer number of lines built for each line type and node pair

Source: Authors' compilation based on study data.

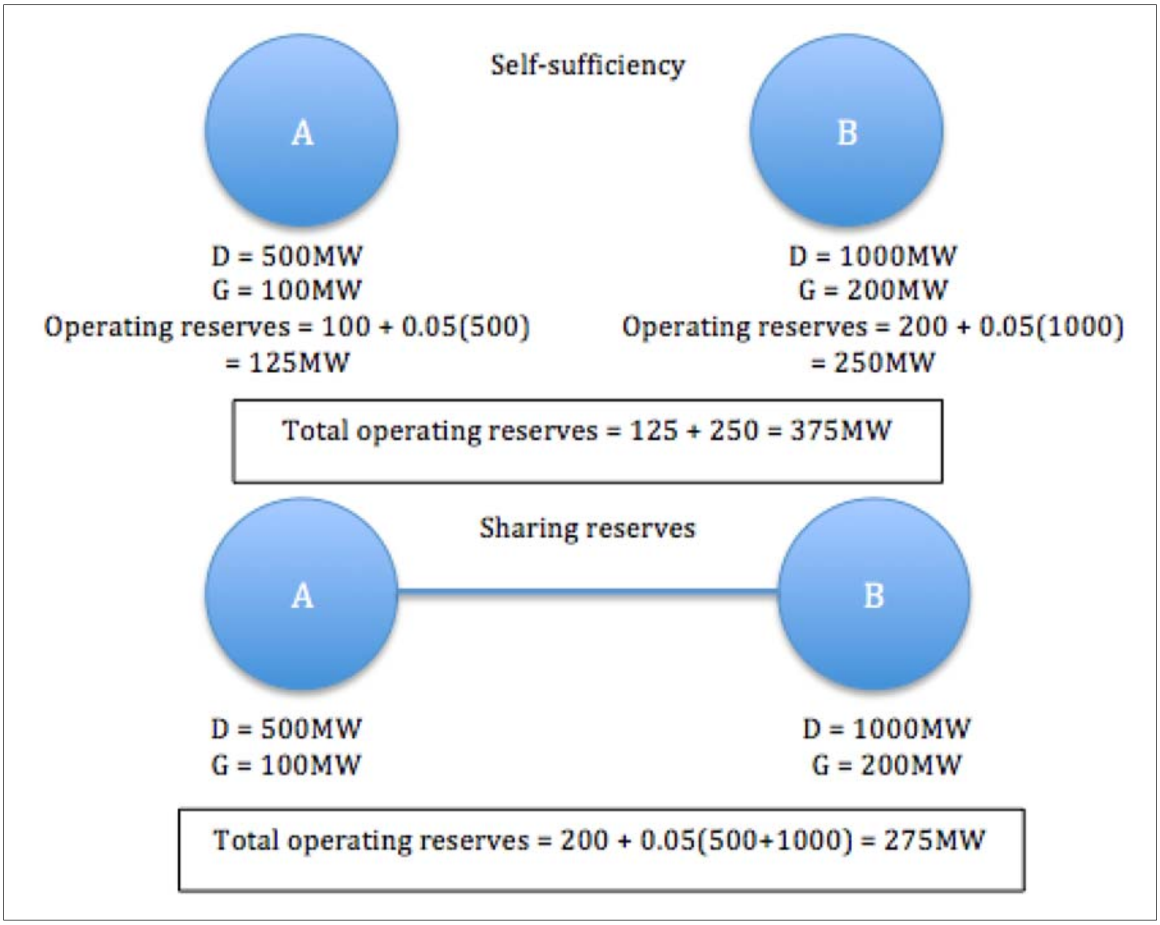
To avoid unrealistic solutions, we impose a constraint that no more than five new lines can be built for a particular node pair and line type in a given year. To calculate transmission investment costs [see Equation (3)], we use actual distances to calculate instead of reading the cost per megawatt directly from Figure A2. This is due to the significant variation in distances between nodes for country pairs in the >200-mile category. Almost half of the node pairs fell into this category and the distances within this group varied significantly from 200 miles to almost 1000 miles. As transmission line costs depend on the line length, it was not reasonable to use the same cost per megawatt for a 300-mile line as a 1000-mile line.

Appendix B: Approximation method for shared operating reserves

Operating reserves refer to unused generating capacity that is readily available for unexpected events such as surges in demand, plant failures, or transmission line outages. These reserve requirements, generally predetermined by a regulator or system operator, typically must be sufficient to cover the outage of the largest unit plus a fraction of demand. For this model, we calculate the operating reserve requirement as the size of the largest plant plus 5% of peak demand.

Sharing operating reserves allows countries to reduce their domestic reserve requirements. The combined reserve requirement for interconnected countries is equal to the largest unit of the *interconnected* system plus a fraction of demand. Figure B1 shows a simple numerical example for two countries, A and B. Country A has a peak demand of 500 MW and its largest plant is 100 MW. Country B has a peak demand of 1000 MW and its largest unit is 200 MW. As the example shows, when both countries operate in a self-sufficiency mode where they must supply their own operating reserves, the total reserve requirements equal 375 MW. However, if the countries had sufficient transmission capacity and shared their operating reserves, their combined reserve requirement would fall to 275 MW, a 27% reduction from the self-sufficiency mode.

Figure B1: Benefits of sharing operating reserves



Source: Authors' illustration.

As more countries are interconnected, the total savings in operating reserves increases but the distribution of savings among different countries will vary depending on how reserves are allocated among interconnected systems. For this model, we allocate operating reserve requirements in proportion to their reserve requirements in isolation. For the simple example in Figure B1, country A must supply 275 MW×125/(125+250) = 92 MW and country B must supply 183 MW×100/(50+100) = 138.3 MW. Both countries are still better off (lower reserve requirements) than in the self-sufficiency case.

B1 Reserve allocation with insufficient transmission capacity

The previous example assumes there is sufficient transmission capacity between the two systems. We define sufficient transmission capacity as the amount needed to transfer a country's entire operating reserve requirement. For Countries A and B, this is 125 and 250 MW, respectively. If there is insufficient transmission capacity to provide support during unexpected events, each country will need to supply a higher share of operating reserves domestically. Any additional transmission investments above the sufficient transmission capacity would not reduce the country's reserve requirement. To demonstrate this, we will use another simple example of country A, which has the ability to connect to neighbouring countries B, C, and D with the characteristics shown in Table B1.

Table B1: System characteristics for sample countries

Country	Largest unit (MW)	Demand (MW)	Operating reserve (MW)
A	100	500	125
B	150	1000	200
C	200	1000	250
D	300	1250	362.5
Total reserves			937.5

Source: Authors' compilation.

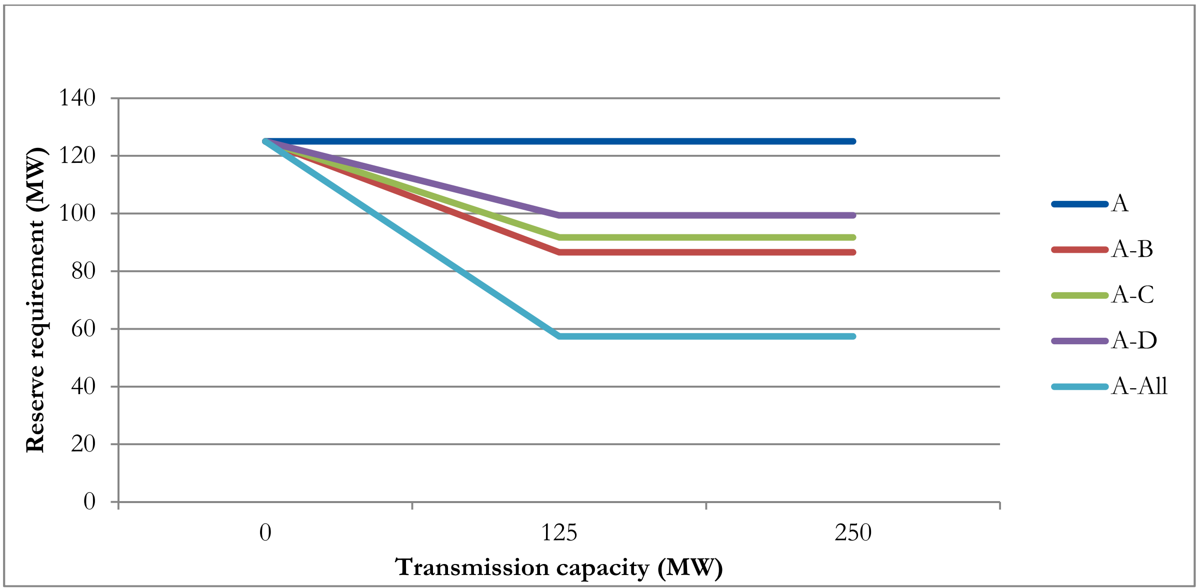
Using the method described above, we calculate the fraction of reserves that country A would need to supply if it is connected to any or all of its neighbours given sufficient transmission capacity. This serves as a minimum as any additional transmission capacity would not reduce the ability to share reserves between countries. The relationship between transmission capacity and domestic operating reserve requirements can then be represented as a piece-wise linear function (Figure B2). In this example, country A benefits the most if it shares reserves with all of its neighbours (curve 'A-All').

Using this relationship, we define a reserve coefficient α as the slope of the first section of this line. As shown in Figure B2, a steeper slope indicates a greater reduction in reserve requirements for a given country. Letting $\tau_{A\text{-}B}$ represent the transfer capacity between countries A and B, the operating reserve requirement for country A can be approximated as the initial self-sufficiency reserve requirement minus any shared portion of reserves from interconnected countries.

$$SharedOpRes_A \geq OpRes + \alpha_B\left(\tau_{A-B}\right) + \alpha_C\left(\tau_{A-C}\right) + \alpha_D\left(\tau_{A-D}\right) \quad 0 \leq \tau \leq OpRes_A$$

$$SharedOpRes_A \geq OpResMin_A \quad \tau \geq OpRes_A$$

Figure B2: Operating reserve requirements for country A for different interconnection scenarios and levels of transmission capacity



Source: Authors' interpretation based on study data.

Note that since the slope of the line is negative, α will have a negative value. In the model formulation, α is represented as *pResCoef*. The minimum operating reserve requirement for country A occurs when it is connected to all of its neighbours. This value is represented as $OpResMin_A$.