UNU-WIDER | 30 YEARS OF RESEARCH FOR DEVELOPMENT

WIDER Working Paper 2015/002

Reference groups and the poverty line

An axiomatic approach with an empirical illustration

Satya R. Chakravarty,[1] Nachiketa Chattopadhyay,[1] Zoya Nissanov,[2] and Jacques Silber[3]

January 2015

Abstract: A recent trend in the study of poverty is to consider a relative poverty line, one that is responsive to the nature of the income distribution. We develop an axiomatic approach to the determination of an amalgam poverty line. Given a reference income (e.g. the mean or the median), the amalgam poverty line becomes a weighted average of the absolute poverty line and the reference income, where the weights depend on the policy maker's preferences for aggregating the two components. The paper ends with an empirical illustration comparing rural and urban and areas in the People's Republic of China and India.

Keywords: absolute poverty, amalgam threshold, India, People's Republic of China, poverty line, relative poverty
JEL classification: D31, D63, I32

Acknowledgements: For comments and suggestions, the authors first thank participants in the Asian Development Bank International Workshop on the 'Untold Side of the Asian Poverty Reduction Story', Southwest University, People's Republic of China, 19-20 November 2013, in particular B. Zheng for helpful discussions and comments. The authors wish also to thank Bhaskar Dutta and participants in the UNU-WIDER Development Conference on 'Inequality – Measurement, Trends, Impacts, and Policies', Helsinki, 5-6 September 2014. Finally, the authors are grateful to Iva Sebastian-Samaniego of the Asian Development Bank for helping them with the computations based on the Shorrocks-Wan algorithm.

[1]Indian Statistical Institute, Kolkata; [2]Ariel University; [3] Bar-Ilan University; corresponding author: jsilber_2000@yahoo.com

This paper has been presented at the UNU-WIDER 'Inequality—Measurement, Trends, Impacts, and Policies' conference, held 5–6 September 2014 in Helsinki, Finland.

ISSN 1798-7237 ISBN 978-92-9230-887-2 https://doi.org/10.35188/UNU-WIDER/2015/887-2

Typescript prepared by Sophie Richmond for UNU-WIDER.

UNU-WIDER gratefully acknowledges the financial contributions to the research programme from the governments of Denmark, Finland, Sweden, and the United Kingdom.

The World Institute for Development Economics Research (WIDER) was established by the United Nations University (UNU) as its first research and training centre and started work in Helsinki, Finland in 1985. The Institute undertakes applied research and policy analysis on structural changes affecting the developing and transitional economies, provides a forum for the advocacy of policies leading to robust, equitable and environmentally sustainable growth, and promotes capacity strengthening and training in the field of economic and social policy-making. Work is carried out by staff researchers and visiting scholars in Helsinki and through networks of collaborating scholars and institutions around the world.

UNU-WIDER, Katajanokanlaituri 6 B,00160 Helsinki, Finland, wider.unu.edu

1 Introduction

Even in the early years of the twenty-first century, removal of poverty remains one of the major goals of economic policy in many countries of the world. As a consequence, a wide variety of poverty indices have been proposed in the literature and the construction of such indices depends explicitly on the poverty line, which is often taken as given. The determination of such an income or consumption threshold on which the definition of poverty relies has been a debatable issue for quite some time (see among others, Citro and Michael 1995; Ravallion 1994; Ruggles 1990). The choice of a poverty line may involve a high degree of arbitrariness. Consequently, an index of poverty which relies on identifying who the poor are, and aggregation of their income shortfalls from the poverty line, may represent inaccurate information. Therefore, the choice of an appropriate poverty line has a high impact on anti-poverty policies.

Based on a report documented in Ravallion et al. (1991), the World Bank used a US$1 per day poverty line for the developing world. A proposal to update this international poverty line was attempted in Ravallion et al. (2009). A new international line of US$1.25 a day at 2005 purchasing power parity (PPP) for household consumption was suggested in that article. Deaton (2010) argued that many problems are involved in the calculation of a global poverty line and correction of international price differences using PPP exchange rates. He discussed difficulties of making inter-country comparisons where relative prices and consumption patterns are different. Some recommendations for improving calculation of the poverty count were also made by him.

In fact, quite a clear distinction is made in the literature between an 'absolute poverty line' and a 'relative poverty line'. While the former with a fixed real value over time is given exogenously, the latter is made responsive to the income distribution. For instance, a household with less than 60 per cent of the median income may be counted as poor in the relative sense.[1] The major distinction between the relative and absolute thresholds arises not from specification of their values but from how the values change under changes in the distribution.

Each society has its own way of setting the absolute or relative poverty line. For instance, the European Union (EU) standard set the poverty line as 60 per cent of the median. In contrast, the US official poverty line, which is largely indebted to Orshansky (1965), is based on family pre-tax income and an absolute poverty threshold. Currently, a new supplemental poverty measure, which uses more general definitions and adjustments for family size and

[1] Examples include 50 per cent of the median (Fuchs 1969) and 50 per cent of the mean (O'Higgins and Jenkins 1990). Atkinson and Bourguignon (2001) considered a relative poverty line equal to the mean income (or expenditure) multiplied by0.37. Chen and Ravallion (2001) preferred to use0.33 instead of0.37 as the multiplicative factor.

composition, has been introduced in 2011.[2] India uses separate absolute poverty lines for rural and urban sectors. (See Subramanian 2011, for a recent discussion.)

Attempts have been made to incorporate relativity in poverty measurement by adjusting poverty lines across demographic sub-groups. One way is to choose poverty lines for each sub-group separately. However, this may give rise to non-monotonicity of the poverty line with respect to the family size (Ruggles 1990: ch. 4). Further, it is implicitly assumed that the overall poverty indicator is decomposable across population sub-groups. The idea of equivalence scale has been used for determining a relative poverty line as well. Here the drive is to set the line for one reference sub-group and use a conversion rate representing the equivalence scale to derive the line for any other sub-group, so that differing needs of families of different sizes are taken into account.[3] Kakwani (2011) employed consumer theory to construct food and non-food poverty thresholds.

Given that the determination of the poverty line is still a disputable matter, we wish to propose an axiomatic approach to the calculation of a relative poverty line. It is assumed that the poverty line is relative in the income/consumption space. Our approach follows a long tradition of identifying welfare with utility. Utility depends on the absolute income and the relative income, that is, income relative to some reference standard. There is a growing economic literature that stresses the importance of incorporating relative position in decision-making analysis. In a seminal contribution, Duesenberry (1949), stipulated that social comparisons are in general asymmetric, which in the context of income implies that the happiness of a poor person is negatively affected by the income of anyone richer than him. Kahneman and Tversky (1991) argued that the average income of the reference group for a person can be taken as a natural reference standard for comparing his own income. According to Frank (1985, 1999), a description of human behaviour that does not take relative standing into account is implausible. Clark and Oswald (1996) provided an empirical support to demonstrate that a person's awareness about subjective well-being depends explicitly on relative position (see also Easterlin 2001; Falk and Knell 2004; Ferrer-i-Carbonell 2005). The focus on relative economic position in utility analysis has also been recognized in the theory of relative deprivation (Runciman 1966).[4]

Individual utility is assumed to be increasing, concave in absolute income but decreasing, convex in the reference standard (see Clark and Oswald 1998). Our analysis relies on a general reference income level, of which some proportions of mean or median income can be special cases. An additive form and a multiplicative form of the utility function are characterized using two different sets of intuitively reasonable axioms. Now, suppose a reference income is given. We then employ a utility-consistency condition suggested by Kakwani (2011) to determine the poverty line uniquely in terms of a reference income and a given poverty line. More precisely, given a reference income and a person with income equal to an arbitrarily set poverty line,

[2] Slesnick (1993) advocated the use of consumption expenditure instead of pre-tax income adjusted by a household-specific true cost of living index and a general household equivalence scale and a 'reference' poor household.

[3] See Blackorby and Donaldson (1994) and Foster (1998) for further discussion.

[4] See also, Yitzhaki (1979), Berrebi and Silber (1985), Chakravarty and Moyes (2003), Bossert and D'Ambrosio (2007) and Zheng (2007).

who is only just poor, we determine the level of the corresponding utility. We then consider an alternative setting where the person is again only just poor, that is with income at some alternative poverty line. However, in this situation, his utility is not affected by the reference income. Since the effect of the reference income on utility is captured through the absolute or relative divergence of a person's income from the reference income, the annulment of the effect is obtained by setting his own income to be his reference income. Utility consistency requires that the person is equally satisfied in both the positions. In other words, we equate the utility in this later state of affairs with the level of utility derived for the arbitrarily set poverty line and reference income situation to determine the arbitrary poverty line uniquely. This assumption of equal satisfaction is quite plausible because in each case the individual is at the existing poverty line income.

An innovative feature of our paper is that for either form of the utility function the new poverty line becomes an amalgam, a weighted average, of the given poverty line and the reference income. Therefore, our derivation allows the possibility of a change in the question 'absolute or relative?' to 'how relative?' A second novelty of our contribution is that Foster's (1998) suggestion for a 'hybrid' poverty threshold, a weighted geometric mean of a relative threshold and an absolute threshold, can be supported by our utility-consistency condition. Thus, our suggestion bears a close similarity to that of Foster (1998) and hence can be treated as a hybrid approach as well.

Another attractive feature of our framework is that some of the suggestions that exist in the literature (e.g. Atkinson and Bourguignon 2001, and the EU standard) for basing the poverty line directly on some location parameter, such as the mean or median, become particular cases of our formulation.

The paper is organized as follows. Section 2 develops the theoretical framework while section 3 gives a short empirical illustration based on separate data on rural and urban areas in the People's Republic of China (PRC) and India. Section 4 then briefly concludes.

2 Formal framework

The model relies on two assumptions about an individual's utility function. The first assumption specifies that utility depends in part on the individual's absolute income. According to the second assumption, utility also depends on the relative income, income relative to some reference standard. This latter condition is one way of ensuring that utility partly depends on his relative position (or 'status') in the society in terms of some attribute of well-being. Such assumptions about utility functions are quite common in the literature (see, for example, Clark and Oswald 1998). As Clark and Oswald (1998) suggested, relativity can be incorporated into the framework by having difference comparisons or ratio comparisons.

Let x and m respectively be the absolute income and reference income of an individual in the society. Both x and m are assumed to be drawn from the finite non-negative non-degenerate interval $[0,\infty)$, that is, $x, m \in [0,\infty)$.The reference income m can be treated as a positional good and it is assumed that x does not exceed the reference income.[5] Examples of

[5] For a somewhat different position, see Hopkins (2008).

m can be the mean and the median incomes in the population or some positive scalar transformations of them.

Let U denote the non-constant real valued utility function of the individual. The function $U(x,m)$ is assumed to be increasing, concave in x and decreasing, convex in m. Increasingness and concavity assumptions in absolute income are quite standard.[6] Suppose a person with a low income regards the income level m as his targeted income. He may be optimistic about receiving this income by working hard and/or receiving some subsidy. An increase in m might increase his difficulty to fulfil the objective of receiving the higher targeted income. This means that his additional utility from an increase in m will be negative, that is, U is decreasing in m. Convexity means that his dissatisfaction from an increase in m increases at a non-decreasing rate.

The difference form comparison demands that the utility function should be of the form $U(x, x-m)$. The argument $x-m$ can be thought of as capturing dis-utility from comparison. That is, in this case the determinant of relative status depends on difference $x-m$. Since $x, m \in [0,\infty)$ and $x \le m$, it is clear that $x-m \in (-\infty,0]$.

We now propose the following axioms for a utility function $U : [0,\infty)\times(-\infty,0] \to R$ involving difference form comparison, where R is the set of real numbers.

Linear Translatability (LIT): For any real c such that $x+c \in [0,\infty)$, $U(x+c,(x+c)-(m+c)) = U(x, x-m)+kc$, where $k > 0$ is some scalar.

Linear Homogeneity (LIH): For any $c \in (0,\infty)$, $U(cx, cx-cm) = cU(x, x-m)$.

Since under equal increase of the absolute and reference incomes the relative status $(x-m)$ remains unchanged but the absolute income increases, individual utility should increase. LIT is a simple way of specifying this increment. It demands that when the absolute and reference incomes are changed by a given amount, then utility changes by a constant times the given amount. In other words, it shows how utility changes when the absolute and reference incomes are diminished or augmented by the same amount. This axiom can be treated as an absolute counterpart to LIH, which says that an equi-proportionate change in the absolute and the reference incomes changes utility equi-proportionately. This postulate is weaker than the requirement that U is increasing in x.

In the literature on income inequality measurement, a social welfare function that satisfies linear homogeneity and linear translatability simultaneously is called a compromise welfare function. The Gini welfare function is an example of a welfare function of this type (see Blackorby and Donaldson 1980). Such welfare functions are helpful for measuring economic distance between income distributions, which quantifies well-being of one population relative to that of another (see Chakravarty 2014).

[6] We can definitely replace concavity by strict concavity and develop a similar analysis. See Remark 1 at the end of this section.

Proposition 1: The only utility function that satisfies LIT and LIH is of the form

$$U(x, x-m) = (k-a)x + am \tag{1}$$

where $k > 0$ is same as in LIT and $a < 0$ is a constant.

Proof: By LIT $U(x-x, x-x-m+x) = U(x, x-m) - kx$. We rewrite this equation as $U(x, x-m) = U(0, -m+x) + kx$. By LIH it follows that $U(0, -m+x)$ $= (m-x)U(0,-1) = a(m-x)$, where $a = U(0,-1)$. Hence $U(x, x-m) = (k-a)x + am$. Decreasingness of U in m requires that $a < 0$. This establishes the necessity part of the proposition. The sufficiency part can be checked easily. Δ

The weights $(k-a)$ and a in (1) provide a simple way of capturing the mixture of two effects. For $a = 0$ the preferences are private and self-interested. This becomes ensured under the mild condition that $k > 0$. The individual does not look at his position in terms of the reference income. He does not care about what other individuals are doing. It follows that U is concave in x and convex in m under the restrictions $(k-a) > 0$ and $a < 0$. The utility function in (1) is a particular form of the '*additive comparisons model*' suggested by Clark and Oswald (1998). However, no characterization has been developed by them.

Let us now consider a situation in which an individual does not compare his/her absolute income with the reference income because the reference income itself is identical to the absolute income. If we denote this absolute income by z_0, then from (1) we have, $U(z_0, 0) = kz_0$. This absolute income can be taken as the current poverty line. The utility corresponding to some arbitrary poverty line z_1 and the reference income m will then be given by $U(z_1, z_1 - m) = (k-a)z_1 + am$. Let us now find the income z_1 which would guarantee the individual a level of utility identical to the utility level $U(z_0, 0)$. That is, the level of happiness that the person had in the earlier scenario when he was enjoying the poverty line income remains the same in the present case, characterized by a new poverty line and a reference income. Equality of the two utility levels can be justified on the ground that in both circumstances the individual's income coincides with the poverty line income. Following Kakwani (2011) we refer to this as a utility-consistency condition. To understand this further, suppose for a given time point the absolute poverty line is well-defined at z_0. Suppose now the distribution changes. Given a reference income m, if we want to determine a poverty line z_1 that will keep the utility of the person at the old poverty line unchanged, we should readjust the poverty line. The readjustment is done by equating the utility levels.

Equating the two expressions $U(z_0, 0)$ and $U(z_1, z_1 - m)$, we get

$$z_1 = qz_0 + (1-q)m, \tag{2}$$

where $q = \dfrac{k}{(k-a)}$. Given that $a < 0$, we can say that the revised poverty line is a convex mixture, a weighted average, of the existing poverty line and the specified reference income. For a one unit increase in the living standard (m), $(1-q)$ represents the increase in the threshold z_1. Therefore, q may be interpreted as a policy parameter in the sense that it reflects the relative importance of the current poverty line in reaching its revised estimate. As the weight q increases from 0 to 1, more and more importance is assigned to the current poverty line in the averaging in Equation (2). For $q = 1$, z_1 coincides with the existing poverty line z_0, whereas for $q = 0$, z_1 becomes the reference income m. A compromise choice for q is $q = 0.5$. Since in general $m > z_0$, it follows that $z_1 > z_0$.

As Clark and Oswald (1998) argued, an alternative specification can be a '*ratio comparisons model*'. In this case the individual's utility depends directly on the absolute income x and also on the relative factor $\dfrac{x}{m}$. Thus, in this case the determinant of the status is the ratio $\dfrac{x}{m}$. We consider a general form of the utility function $U\left(x, f\left(\dfrac{x}{m}\right)\right)$, where f is a positive valued and increasing transformation of the ratio $\dfrac{x}{m}$. This is a fairly general version of a ratio comparisons model. As before, we maintain the assumptions that U is increasing, concave in x and decreasing, convex in m. By our formulation, U is increasing in $f\left(\dfrac{x}{m}\right)$.

In order to characterize a particular form of the utility function which we wish to use for determining a poverty line in the ratio comparisons framework, we consider the following axioms for $U : (0,\infty)\times(0,\infty) \rightarrow R_{++}$, where R_{++} is the strictly positive part of R.

Linear Homogeneity (LIH): For any $\left(x, f\left(\dfrac{x}{m}\right)\right) \in (0,\infty)\times(0,\infty)$,

$U\left(cx, f\left(\dfrac{cx}{cm}\right)\right) = cU\left(x, f\left(\dfrac{x}{m}\right)\right)$, where $c>0$ is arbitrary.

Since $f\left(\dfrac{x}{m}\right)$ remains unaltered under positive scale transformation of the absolute income x and the reference income m, LIH shows how utility should be adjusted under such transformation of the variables.

Normalization (NOM): If $x = 1$, then $U\left(x, f\left(\dfrac{x}{m}\right)\right) = f\left(\dfrac{1}{m}\right)$.

Continuity (CON): U is continuous in its arguments. NOM is a cardinality principle which says that if the individual's income is 1, then corresponding utility value is given simply by the transformed value $f\left(\frac{1}{m}\right)$ of the ratio $\frac{1}{m}$. Variants of this are certainly possible. But given that the income is fixed at 1, the utility should be dependent on the ratio $\frac{1}{m}$ in a negative monotonic way and NOM ensures this. Continuity assures that minor observational errors in incomes will not change utility abruptly.
Axioms LIH, NOM, and CON uniquely identify a specific functional form of the utility function.

Proposition 2: The only utility function $U:(0,\infty)\times(0,\infty)\to R_{++}$ that satisfies LIH, NOM, and CON is of the form

$$U\left(x, f\left(\frac{x}{m}\right)\right) = xf\left(\frac{x}{m}\right). \tag{3}$$

Proof: Let us denote the ratio $\frac{x}{m}$ by A. LIH implies that

$$U(cx, f(A)) = cU(x, f(A)), \tag{4}$$

where $c>0$.This equality holds for all $x>0$ and $c>0$. Consequently, for any $c>0$ it holds for $x=1$ also.

Now, given $x=1$, using NOM in (4), we get

$$U(c.1, f(A)) = cU(1, f(A)) = cf(A). \tag{5}$$

From (5) it follows that

$$c = \frac{U(c, f(A))}{f(A)}. \tag{6}$$

Plugging the value of c from (6) into (4) we get

$$U(cx, f(A)) = cU(x, f(A)) = \frac{U(c, f(A))}{f(A)} U(x, f(A)). \tag{7}$$

Let $$V(x, f(A)) = \frac{U(x, f(A))}{f(A)}. \tag{8}$$

From (7) and (8) it now follows that

$$V(cx, f(A)) = \frac{U(cx, f(A))}{f(A)}$$

$$= \frac{1}{f(A)}\left[\frac{U(c, f(A))}{f(A)} U(x, f(A))\right]$$

$$= \left[\frac{U(c, f(A))}{f(A)}\right]\left[\frac{U(x, f(A))}{f(A)}\right]$$

$$= V(c, f(A)) V(x, f(A)). \tag{9}$$

Now, define $g_{f(A)}(x) = V(x, f(A))$ so that we can rewrite (9) as

$$g_{f(A)}(cx) = g_{f(A)}(c) g_{f(A)}(x). \tag{10}$$

Given A, by non-constancy of U we rule out the trivial solutions $g_{f(A)}(t) = 0$ and $g_{f(A)}(t) = 1$ of the functional equation (10). Since U (hence g) is positive valued, we can take logarithmic transformation on both sides of (10) to get

$$\log(g_{f(A)}(cx)) = \log(g_{f(A)}(c)) + \log(g_{f(A)}(x)). \tag{11}$$

Substitution of $c = e^u$ and $x = e^v$ into (11) yields the functional equation

$$\log(g_{f(A)}(e^{u+v})) = \log(g_{f(A)}(e^u)) + \log(g_{f(A)}(e^v)). \tag{12}$$

Define $h_{f(A))}(t) = \log(g_{f(A)}(t))$, where $t \in R$. CON implies continuity of $h_{f(A))}$. Then the functional equation (12) reduces to

$$h_{f(A)}(u+v) = h_{f(A)}(u) + h_{f(A)}(v), \tag{13}$$

of which the only continuous solution is $h_{f(A)}(u) = \delta u$, where δ is a non-zero constant that depends on $f(A)$)(Aczel, 1966: 34). Using $h_{f(A)}(u) = \delta u$, in the definition of $h_{f(A))}(t)$, we get $\log(g_{f(A)}(u)) = \delta u$ and with $u = \log t$, it follows that $g_{f(A)}(t) = t^{\delta}$.

From the definition of $g_{f(A)}$ it then follows that

$$V(x, f(A)) = x^{\delta(f(A))}. \tag{14}$$

Using the definition of $V(x, f(A))$ in Equation (14) we get

$$U(x, f(A)) = x^{\delta(f(A))} f(A). \tag{15}$$

LIH ensures that $\delta f(A) = 1$, which in turn shows that

$$U(x, f(A)) = xf(A) = xf\left(\frac{x}{m}\right). \tag{16}$$

This completes the necessity part of the theorem. The sufficiency can be checked easily. Δ

Clark and Oswald (1998) specified, without characterization, a utility function which is additively separable in the absolute income x and the relative income $\frac{x}{m}$. However, the functional form we have characterized is of the product type in its arguments. The essential idea of dependence of the utility function on the relative as well as absolute statuses is well maintained in our characterized form also. Further, our form becomes additively separable under the logarithmic transformation.

An appropriate choice of $f\left(\frac{x}{m}\right)$ under which all the assumptions for U are retained is $f\left(\frac{x}{m}\right) = \beta - \frac{m}{x}$, where $\beta > 1$ is a constant such that $f\left(\frac{x}{m}\right) > 0$. For an observed income distribution we can take $\beta = \frac{u}{l} + 1$, where $l > 0$ and u are respectively the lower and upper bounds on income. The corresponding utility function turns out to be $x\left(\beta - \frac{m}{x}\right)$. As in the additive case, we now wish to determine the value of z_1 such that $U\left(z_0, f\left(\frac{z_0}{z_0}\right)\right) = U\left(z_1, \frac{z_1}{m}\right)$

For the chosen form of $f\left(\frac{x}{m}\right)$, this equality becomes, using (3), $z_1\left(\beta - \frac{m}{z_1}\right) = z_0(\beta - 1)$, from which we get

$$z_1 = wz_0 + (1 - w)m, \tag{17}$$

where $w = \frac{\beta - 1}{\beta}$. Since $\beta > 1$, it follows that $0 < w < 1$. Thus, as in (2), here also the revised poverty line becomes a compound of the existing poverty line and the reference income. The parameter w has the same policy interpretation as in (2). Thus, irrespective of the form of the utility function, we have the same procedure of generating a relative poverty line from an existing poverty line and a reference income.

The choice of the weight w is evidently related to that of the parameter β. Assuming, as before, that in (16) the function f may be written as $f(x/m) = \beta - (m/x)$, we can express Equation

(16) as $U = x\beta - m$, from which we derive that $dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial m}dm = \beta dx - dm$ so that for a given utility level, $\frac{dm}{dx} = \beta$.

There are very few papers in the literature on subjective welfare that have estimated the simultaneous impact on happiness, *ceteris paribus*, of an increase in one's own income and in that of the reference group's income. One of these papers is a very recent study by Clark et al. (2013). In Table 4 of their paper the authors report the results of a regression where the dependent variable refers to satisfaction with income. It then appears that the coefficient of own income is about three times as high as that of self-reported reference income, and of the opposite sign. This would imply that the value of β is around 3 and, as a consequence, the value of the weight w would be equal to 2/3. We now show that some of the existing suggestions for treating the poverty line as some fraction of the mean or median income can be accommodated in our framework. The EU standard set the poverty line at 60 per cent of the median, which is equivalent to choosing a particular weight for the reference income in our formulation. If we take $(1-w) = \frac{0.6m - z_0}{m - z_0}$ in (17), where m is the median, then we get the poverty line set by the EU. Likewise, for $(1-w) = \frac{0.37m - z_0}{m - z_0}$, where m now stands for the mean, we get the Atkinson and Bourguignon (2001) relative poverty line.

It will now be worthwhile to compare our proposal with Foster's (1998) recommendation for a hybrid threshold. If m represents the median, then the threshold αm, where $0 < \alpha < 1$, is a general relative cut-off (Citro and Michael 1995). If we denote αm by z_m, then Foster (1998) suggested the use of a weighted geometric mean of the absolute threshold z_0 and the relative threshold z_m, namely , $z_0^{\rho} z_m^{1-\rho}$, as a threshold limit, where $0 < \rho < 1$ is a constant. A 1 per cent increase in the living standard m increases the poverty line by $\rho\%$ (see also Fisher 1995). Now, assume that the individual utility function is of the form $U\left(x, \frac{x}{m}\right) = x^{\rho}\left(\frac{x}{m}\right)^{1-\rho}$, $0 < \rho < 1$ is a constant. This utility function is increasing, concave in absolute income but decreasing convex in the reference level. Then our utility-consistency condition reveals that $z_1 = z_0^{\rho} z_m^{1-\rho}$, the hybrid cut-off advocated by Foster (1998). Thus, the Foster proposition can be justified by our utility-consistency condition.

Remark 1: The two forms of U given by $U(x,m) = (k-a)x^{\delta} + am^{\delta}$ and $U(x,m) = x^{\delta}\left(\beta - \left(\frac{m}{x}\right)^{\delta}\right)$, where $a < 0$, $k > 0$, $0 < \delta < 1$ and $\beta - \left(\frac{m}{x}\right)^{\delta} > 0$, are increasing and strictly concave in absolute income but decreasing and strictly convex in reference income. For each of these two specifications of U, by the utility-consistency condition, we have $z_1 = \left(sz_0^{\delta} + (1-s)m^{\delta}\right)^{\frac{1}{\delta}}$, where $0 < s < 1$. For $\delta = 1$, z_1 coincides with (2), whereas as $\delta \to 0$, it becomes Foster's hybrid poverty line. This form of z_1 is known as a quasilinear mean. Such a form has been characterized by Chakravarty (2011) as a generalized human development index using several dimensions of human well-being. A similar characterization can be developed in the current context.

3 An empirical illustration

In this section we present several measures of the extent of poverty in rural and urban areas of the PRC and India, when an 'amalgam poverty line', a weighted average of an absolute poverty line and of the mean or median income, is introduced. As absolute poverty line we have used a monthly income of US$38 (at 2005 PPP), which corresponds to US$1.25 per day, as originally suggested by Ravallion et al. (2009). We assumed various possible weights. More precisely, we supposed that the weight w given to the absolute poverty line (the weight of the median or of the mean being then $(1-w)$), could be 1,0.9,0.66, and 0.5.

The database consisted of information on the income shares of ten deciles in the rural and urban areas of the two countries mentioned previously. Two computation methods were used. The first is based on an algorithm originally proposed by Kakwani and Podder (1973), allowing one to estimate the Lorenz curve for each country and year on the basis of these ten observations (income shares). On the basis of this Lorenz curve it was then easy to find out what percentage of the population had an income (or expenditure level) smaller than that corresponding to some poverty line. The second approach used an algorithm proposed by Shorrocks and Wan (2009), which allows one to 'ungroup' income distributions, that is, to derive, for example, the share of each centile when the only data available originally are the income shares of deciles. Since the Shorrocks and Wan approach relies on 'ungrouped' income distributions, it appears to be more refined than the Kakwani and Podder method, which uses only ten income shares.

In Table 1 we present the value of the headcount ratio (in percentage) in the rural and urban areas in the PRC and India, under several possible scenarios. We give two sets of results: those based on the Kakwani and Podder approach and those derived from the Shorrocks and Wan algorithm. As expected, for a given weight, the headcount ratio is higher when the weight $(1-w)$ refers to the mean rather than the median. Needless to say, the headcount ratio increases with the weight w. There seem to be significant differences between the results obtained on the basis of the two approaches, although most of the time, though not always, the Shorrocks and Wan approach leads to smaller headcount rates. In all cases, whether for urban or rural areas, the headcount ratio is smaller in the PRC than in India. Note, however, that whereas with the regular US$38 poverty line, there is no urban poverty in China, when some weight is given to the mean or median income when defining the poverty line, the headcount ratio becomes significant, being even higher than 30 per cent when the weight of the mean is equal to 50 per cent. The differences between the rural and urban sectors are much less striking in India, poverty being quite high in both areas.

We then combined the data on the headcounts given in Table 1 with data on the total population around 2010, to derive an estimate of the total number of poor in the rural and urban areas of each of the two countries examined. These results are given in Table 2. To simplify the presentation we give only results based on the Shorrocks and Wan algorithm. It is then easy to compare the number of poor under various scenarios with those obtained on the basis of a weight w equal to 1 (so that the 'amalgam poverty line' is also equal to US$38). Here also we observe a very important increase in the number of poor in urban areas in China, when the poverty line depends on the median or mean income.

Finally, Table 3 gives the income gap ratios in the rural and urban areas of the People's Republic of China (PRC) and India under the various scenarios, the results being again based on the Shorrocks and Wan algorithm. This index is an indicator of poverty depths of different individuals. Here also the income gap ratio increases with the weight given to the median or mean income, whether in India or in the PRC. The income gap ratio is much smaller in urban than in rural areas of the PRC but this is not true for India since, when the weight given to the mean or median income becomes higher, the income gap ratio becomes higher in urban than in rural areas.

Note finally that, when multiplied by the poverty line and the total number of poor, this summary measure has a direct policy interpretation in the sense that the multiplied formula determines the total amount of money required to put all the poor people at the poverty line. Now, for a given country and area, with a given poverty line and the reference income, we determine the amalgam poverty line using a specific weighting scheme. Given an amalgam poverty line, we can then directly estimate the amount of money necessary to place the poor people of a given area in a given country at its poverty line, using the country's area income gap ratio from Table 3 and the number of poor from Table 2.

Table 1: Headcount ratios under various scenarios

Weighting scheme (weight given to the absolute poverty line)	PRC, rural areas (2009) Kakwani and Podder approach	PRC, rural areas (2009) Shorrocks and Wan approach	PRC, urban areas (2009) Kakwani and Podder approach	PRC, urban areas (2009) Shorrocks and Wan approach	India, rural areas (2010) Kakwani and Podder approach	India, rural areas (2010) Shorrocks and Wan approach	India, urban areas (2010) Kakwani and Podder approach	India, urban areas (2010) Shorrocks and Wan approach
Absolute poverty line: US$38, weighted with the median								
100%	0.29	0.21	0.00	0.00	0.36	0.34	0.34	0.29
90%	0.31	0.24	0.00	0.02	0.37	0.36	0.36	0.31
66%	0.37	0.31	0.16	0.11	0.40	0.40	0.40	0.37
50%	0.40	0.36	0.26	0.21	0.42	0.42	0.43	0.40
Absolute poverty line: US$38, weighted with the mean								
90%	0.33	0.27	0.00	0.02	0.38	0.38	0.38	0.34
66%	0.42	0.41	0.22	0.18	0.43	0.47	0.45	0.46
50%	0.47	0.49	0.33	0.31	0.47	0.52	0.49	0.52

Note: The complete income distributions were derived on the basis of data on the shares of the deciles in total income. Two estimation methods were used. The first applied the Kakwani and Podder (1973) approach to the parametrization of the Lorenz curve. The second implemented the Shorrocks and Wan (2009) proposal for 'ungrouping income distributions'.

The first column gives the weight (in percentage) given to the absolute poverty line (US$38), the complement (in percentage) giving the weight given to the median or the mean of the income distributions.

Source: Authors' calculations based on data provided by the ADB (various years).

Table 2: Number of poor (millions) in rural and urban areas of the PRC and India, depending on the weighting scheme

Country and area	US$38;median;100%	US$38;median;90%	US$38;median;66%	US$38;median;50%	US$38;mean;90%	US$38;mean;66%	US$38;mean;50%
PRC, rural areas (2009)	142.84	165.95	217.82	251.45	187.37	284.32	342.89
PRC, urban areas (2009)	2.05	9.69	71.79	131.31	13.97	116.01	194.79
India, rural areas (2009)	285.87	299.92	332.82	353.72	318.04	390.67	434.58
India, urban areas (2009)	108.67	116.49	136.55	149.60	126.85	169.65	194.15

Note: The heading of each column shows that the poverty line is assumed to be equal to US$38 and it also indicates which other indicator is weighted (median or mean) and which weight is given to the absolute poverty line. The computations were based on the Shorrocks and Wan (2009) approach. Results based on the approach of Kakwani and Podder (1973) are available on request from the authors.

Source: Authors' calculations based on data provided by the ADB (various years).

Table 3: Poverty gap ratios in the rural and urban areas of the PRC and of India, depending on the weighting scheme

Country and area	US$38;median;100%	US$38;median;90%	US$38;median;66%	US$38;median;50%	US$38;mean;90%	US$38;mean;66%	US$38;mean;50%
PRC, rural areas (2009)	0.05	0.06	0.09	0.11	0.07	0.13	0.17
PRC, urban areas (2009)	0.00	0.00	0.02	0.05	0.00	0.04	0.09
India, rural areas (2009)	0.08	0.08	0.09	0.10	0.09	0.12	0.14
India, urban areas (2009)	0.07	0.08	0.11	0.12	0.09	0.15	0.18

Source: Authors' calculations based on data provided by the ADB (various years).

4 Conclusions

We have followed Clark and Oswald's (1998) suggestion that an individual cares about his absolute position (his own income) and his relative position (his own income in comparison with a reference income, such as the mean or the median). Two different forms of the utility function that depend on a person's absolute and relative statuses have been characterized. These two utility functions have been employed to determine a relative poverty line endogenous to the income distribution. It turns out that, in either case, the relative poverty line becomes a combination, a weighted mean, of a given poverty line and a reference income, where the weights add up to one. This is similar in spirit to Foster's (1998) hybrid poverty threshold, a weighted geometric mean of a relative and an absolute cut-off point. This weight enables a policy maker to express his preference for absolute or relative poverty. Interestingly enough, some of the existing suggestions for the choice of the relative poverty line drop out as special cases of our general approach.

The empirical illustration has shown that, no matter how we define the 'amalgam poverty line', the extent of poverty is generally smaller in the PRC than in India. In an interview given to *Prospect Magazine*, Sen (2013) attempted to explain such differences:

> Over the decades China has presented examples of good and smart as well as weak and confused authoritarian rule. The gigantic famines of 1958-61 resulted from terrible policy choices that could not be changed for three years despite tens of millions dying each year – no political party could criticize the terrible policies, and newspapers could not even cover the bad news. But after that, despite many other problems, China did remarkably fast progress in education and healthcare for all – an example of good authoritarianism. But they were irrationally prejudiced about the use of markets, which they shunned until the reforms of 1979. With the reforms there were some smart moves (with marketization China did brilliantly in manufacturing and agriculture) but also a big mistake when they marketized health insurance, so you had to buy health insurance rather than being insured by the state or the commune; the Chinese were not alert to the terrible consequences of marketizing everything. The percentage of health coverage went down after 1979 from 100 per cent to 10 or 12 per cent, with downward effects on the high pace of China's progress in life expectancy. Again, it took them many years to recognize that they had made a mistake, and went about un-doing the harm, a correction that became full speed only in 2004 – a quarter century after the error of marketizing health insurance in 1979. Now they have nearly 100 per cent coverage – and with much better quality health care thanks to China's economic prosperity.
>
> An authoritarian system, if it is intelligently and humanely led (but there is no guarantee of that), can get its way quickly. A democratic system is somewhat slower, because you have to convince everyone. In the case of India, the question is which issues get dramatized and politicized. Famine was instantly politicized, because it is so central to the Indian view of the British Raj. The Raj began with a famine [in 1769] and ended with a famine [in 1943]. The elimination of famines was an immediate success of democratic India. There have been other successes, particularly when there have been crises – with HIV, for example – when there has been a real sense of urgency, which the media discussion and democratic pressure reflected. Five or ten years ago, people were saying that India was going to have more cases of HIV than anywhere else in the world – not only as an absolute number, but as a proportion. But it hasn't happened. That challenge was met and

things were done to reduce the vulnerability of the population. These challenges received public attention and advocacy, and a democratic success followed.

Unfortunately, the challenge has not been seized in the case of general healthcare, not even general immunization. And nor has it happened with general education. So I think there is no guarantee that democracy will get there immediately, but it depends on the people to make it happen. And of course the foundational reason for wanting democracy isn't that. The basic reason for wanting democracy is that it gives people dignity, political freedom, and voice – democracy has its own value. If that is compatible with doing good things, and if what happened with famine and HIV crisis could be translated to general healthcare and chronic undernourishment, then that would be a wonderful combination. There is no reason at all why we – and here I speak as an Indian citizen – cannot make that happen.

Appendix A: On Shorrocks and Wan's (2009) 'ungrouping income distributions'

Assume a Lorenz curve with $(m+1)$ co-ordinates (p_k^*, L_k^*) where p_k^* and L_k^* $(k = 1\ m)$ refer respectively to the cumulative shares in the total population and in total income of income classes 1 to *k*, while $p_0^* = L_0^* = 0$. These Lorenz co-ordinates can, for example, refer to decile shares published on a given country. Since often the corresponding average income is not available, it will be assumed to be equal to 1 so that the mean income μ_k^* of class *k* will be expressed as

$$\mu_k^* = \frac{L_k^* - L_{k-1}^*}{p_k^* - p_{k-1}^*}\ k = 1 \text{ to } m \tag{A1}$$

The goal is to obtain a synthetic sample of *n* equally weighted observations whose mean value is 1 and which conform to the original data. These *n* observations are therefore partitioned into *m* non-overlapping and ordered groups having each $m_k = n(p_k^* - p_{k-1}^*)$ observations. Call x_{ki} the i^{th} observation in class *k*, the sample mean of this class being μ_k.

The algorithm proposed by Shorrocks and Wan (2009) includes two stages.

The first step consists of building an initial sample with a unit mean which is generated from a parametric form fitted to the grouped data (see, for example, Ryu and Slottje (1999) for a survey of various parametrizations of the Lorenz curve).[7]

In the second stage the algorithm adjusts the observations generated in the initial sample to the true values available from the grouped data. More precisely, the initial sample value x_j , assumed to belong to class *k*, is transformed into an intermediate value $\widehat{x_j}$ via the following rule:

$$\frac{\widehat{x_j} - \mu_k^*}{\mu_{k+1}^* - \mu_k^*} = \frac{x_j - \mu_k}{\mu_{k+1} - \mu_k} \tag{A2}$$

For the first class we will write that

$$\frac{\widehat{x_j}}{\mu_1^*} = \frac{x_j}{\mu_1} \text{ for } x_j \leq \mu_1 \tag{A3}$$

while for the last class we have

$$\frac{\widehat{x_j}}{\mu_m^*} = \frac{x_j}{\mu_m} \text{ for } x_j \geq \mu_m \tag{A4}$$

Obviously, in the next iteration the intermediate values, $\widehat{x_j}$ are themselves transformed into new values until the algorithm produces an ordered sample which exactly replicates the properties of the original grouped data. Convergence is in fact very quickly obtained.

Appendix B: The Kakwani and Podder (1973) approach

Let L refer to the height of the Lorenz curve (income share) and z to the corresponding abscissa (population share). Kakwani and Podder (1973) proposed then the following equation for the

[7] Shorrocks and Wan chose to generate the initial sample on the basis of a lognormal distribution. For more detail, see Shorrocks and Wan (2009).

Lorenz curve (and showed that such a formulation satisfies all the desired properties of a Lorenz curve):

$$lnL = lnz + hz - h \tag{B1}$$

It is hence possible to derive the value of the parameter h by regressing lnL on lnz and z.

From (1) we also derive that

$$L = e^{lnz+h(z-1)} = e^{lnz}e^{h(z-1)} = ze^{h(z-1)} \tag{B2}$$

Remembering that the slope along the Lorenz curve is equal to the ratio of the income corresponding to this point of the Lorenz curve to the mean income, we can apply (2) to the poverty line and write that

$$\frac{\partial L}{\partial z} = \left(\frac{poverty\ line}{mean}\right) = e^{h(z-1)} + zhe^{h(z-1)}$$

$$= e^{h(z-1)}(1 + zh) \tag{B3}$$

We are therefore looking for the population share z for which the equation below holds

$$ln\left(\frac{poverty\ line}{mean}\right) = h(z-1) + \ln(1 + zh) \tag{B4}$$

that is,

$$\ln(poverty\ line) = \ln(mean) + \ h(z-1) + \ln(1 + zh) \tag{B5}$$

Given the poverty line selected, the mean income and the parameter h determined previously, it is easy to derive the value of z for which (B5) holds, that is the headcount ratio corresponding to the chosen poverty line.

References

ADB (various years). 'Data'. Available at: www.adb.org

Aczel, J. (1966). *Lectures on Functional Equations and Their Applications*. New York: Academic Press.

Atkinson, A. B., and F. Bourguignon (2001). 'Poverty and Inclusion from a World Perspective'. In J.E. Stiglitz and P.A. Muet (eds), *Governance, Equity and Global Markets*. Oxford: Oxford University Press.

Berrebi, Z.M., and J. Silber (1985). 'Income Inequality Indices and Deprivation: A Generalisation'. *Quarterly Journal of Economics*, 100: 807-81.

Blackorby, C., and D. Donaldson (1980). 'A Theoretical Treatment of Indices of Absolute Inequality'. *International Economic Review*, 21: 107-36.

Blackorby, C., and D. Donaldson (1994). 'Measuring the Cost of Children: A Theoretical Framework'. In R. Blundell, I. Preston, and I. Walker (eds), *The Measurement of Household Welfare*. Cambridge: Cambridge University Press.

Bossert, W., and C. D'Ambrosio (2007). 'Dynamic Measures of Individual Deprivation'. *Social Choice and Welfare*, 28: 77-88.

Chakravarty, S.R. (2011). 'A Reconsideration of the Trade-offs in the New Human Development Index'. *Journal of Economic Inequality*, 9: 471-4.

Chakravarty, S.R. (2014). *Inequality, Polarization and Conflict: An Analytical Study*. New York: Springer.

Chakravarty, S.R., and P. Moyes (2003). 'Individual Welfare, Social Deprivation and Income Taxation'. *Economic Theory*, 21: 843-69.

Chen, S., and M. Ravallion (2001). 'How Did the World's Poorest Fare in the 1990s?' *Review of Income and Wealth*, 47: 283-300.

Citro, C.F., and R.T. Michael (1995). *Measuring Poverty: A New Approach*. Washington, DC: National Academy Press.

Clark, A.E., and A.J. Oswald (1996). 'Satisfaction and Comparison Income'. *Journal of Public Economics*, 61: 359-81.

Clark, A.E., and A.J. Oswald (1998). 'Comparison-concave Utility and Following Behaviour in Social and Economic Settings'. *Journal of Public Economics*, 70: 133-55.

Clark, A., C. Senik, and K. Yamada (2013). 'The Joneses in Japan: Income Comparisons and Financial Satisfaction'. Discussion Paper 866. Osaka: Institute of Social and Economic Research, Osaka University.

Deaton, A. (2010). 'Price Indexes, Inequality, and the Measurement of World Poverty'. *American Economic Review*, 100: 5-34.

Duesenberry, J.S. (1949). *Income, Saving and the Theory of Consumer Behaviour*. Cambridge, MA: Harvard University Press.

Easterlin, R.A. (2001). 'Income and Happiness: Towards a Unified Theory'. *Economic Journal*, 111: 465-84.

Falk, A., and M. Knell (2004). 'Choosing the Joneses: Endogenous Goals and Reference Standards'. *Scandinavian Journal of Economics*, 106: 417-35.

Ferrer-i-Carbonell, A. (2005). 'Income and Well-being: An Empirical Analysis of the Comparison Income Effect'. *Journal of Public Economics*, 89: 997-1019.

Fisher, G.M. (1995). 'Is There Such a Thing as an Absolute Poverty Line Over Time?' Mimeo. Washington, DC: US Department of Health and Human Services.

Foster, J.E. (1998). 'Absolute versus Relative Poverty'. *American Economic Review*, 88: 335-41.

Frank, R.H. (1985). *Choosing the Right Pond.* Oxford: Oxford University Press.

Frank, R.H. (1999). *Luxury Fever*. New York: The Free Press.

Fuchs, V. (1969). 'Comment on Measuring the Size of the Low-income Population'. In L. Soltow (ed.), *Six Papers on the Size Distribution of Wealth and Income*. New York: National Bureau of Economic Research, 198-202.

Hopkins, E. (2008). 'Inequality, Happiness and Relative Concerns: What Actually Is Their Relationship?' *Journal of Economic Inequality*, 6: 351-72.

Kahneman, D., and A. Tversky (1991). 'Loss Aversion in Riskless Choice: A Reference-dependent Model'. *Quarterly Journal of Economics*, 106: 1039-61.

Kakwani, N.C. (2011). 'A New Model for Constructing Poverty Thresholds'. In J. Deutsch and J. Silber (eds), *The Measurement of Individual Well-Being and Group Inequalities: Essays in Memory of Z.M. Berrebi.* London: Routledge.

Kakwani, N.C., and N. Podder (1973). 'On the Estimation of Lorenz Curve from Grouped Observations'. *International Economic Review*, 14: 278-92.

O'Higgins, M., and S. Jenkins (1990). 'Poverty in the EC: Estimates for 1975, 1980 and 1985'. In R. Teekens and B.M.S. van Praag (eds), *Analysing Poverty in the European Community: Policy Issues, Research Options, and Data Sources*. Luxembourg: Office of Official Publications of the European Communities.

Orshansky, M. (1965). 'Counting the Poor: Another Look at the Poverty Profile'. *Social Security Bulletin*, 28: 3-29.

Ravallion, M. (1994). *Poverty Comparisons*. Chur: Harwood.

Ravallion, M., G. Datt, and D. van de Walle (1991). 'Quantifying Absolute Poverty in the Developing World'. *Review of Income and Wealth*, 37: 345-61.

Ravallion, M., S. Chen, and P. Sangraula (2009). 'Dollar a Day Revisited'. *The World Bank Economic Review*, 23: 163-84.

Ruggles, P. (1990). *Drawing the Line*. Washington, DC: Urban Institute Press.

Runciman, W.G. (1966). *Relative Deprivation and Social Justice*. London: Routledge.

Ryu, H.K., and D.S. Slottje (1999). 'Parametric Approximations to the Lorenz Curve'. In J. Silber (ed.), *Handbook on Income Inequality Measurement*. Dordrecht: Kluwer Academic.

Sen, A. (2013). 'Interview'. *Prospect Magazine*, August.

Shorrocks, A., and G. Wan (2009). 'Ungrouping Income Distributions. Synthesizing Samples for Inequality and Poverty Analysis'. In K. Basu and R. Kanbur (eds), *Arguments for a Better World. Essays in Honor of Amartya Sen*, Vpol. 1, *Ethics, Welfare and Measurement.* Oxford: Oxford University Press.

Slesnick, D.T. (1993). 'Gaining Ground: Poverty in the Post-war United States'. *Journal of Political Economy*, 101: 1-38.

Subramanian, S. (2011). 'The Poverty Line: Getting It Wrong Again'. *Economic and Political Weekly*, 46: 37-42.

Yitzhaki, S. (1979). 'Relative Deprivation and the Gini Coefficient'. *Quarterly Journal of Economics*, 93: 321-4.

Zheng, B. (2007). 'Utility-gap Dominances and Inequality Orderings'. *Social Choice and Welfare*, 28: 255-80.