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Measuring Economic Insecurity

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Abstract

We provide an axiomatic treatment of the measurement of economic insecurity, assuming that individual insecurity depends on the current wealth level and its variations experienced in the past. The first component plays the role of a buffer stock to rely on in case of an adverse future event. The second component determines the confidence an individual has on his ability to overcome a loss in the future. Experiences in the recent past are given higher weight than experiences that occurred in the more distant past. Two classes of measures are characterized with sets of plausible and intuitive axioms.

Keywords: insecurity, wealth streams, economic index numbers
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1 Introduction

The notion of economic insecurity has received an increasing amount of public attention in the recent past and is appearing frequently in policy debates. In spite of its widespread use, a precise definition of the term ‘economic insecurity’ appears to have remained elusive and the phenomenon has not yet been analyzed from a thorough theoretical perspective. There have been several attempts to design measures of economic insecurity but they appear to have, by and large, been developed on an *ad hoc* basis. Examples for such attempts to capture the notion of insecurity include (i) an initiative of the France-German Ministerial Council in response to the report of the Commission on the Measurement of Economic Performance and Social Progress (Stiglitz, Sen and Fitoussi, 2009); (ii) the Rockefeller Foundation’s Economic Security Index; and (iii) a proposal by the International Labour Organization. The respective recommended measures can roughly be described as (i) the share of the population facing the risk of poverty; (ii) the population share who experience at least a 25% drop in disposable family income who lack an adequate financial safety net; and (iii) a weighted average of the ‘scores’ achieved in seven specific forms of insecurity. These proposals do not appear to have a sound theoretical foundation and employ indices that may be more suitable to address other issues, such as the assessment of poverty. The objective of our contribution is to formulate an axiomatic framework that we consider suited to the phenomenon to be measured and characterize classes of indices on the basis of our axioms.

Clearly, economic insecurity is a multi-faceted issue and a comprehensive formal definition that subsumes all possible aspects of it is likely to remain difficult to be agreed upon for some time to come. According to Osberg (1998, p.23), “[A] definition of ‘economic insecurity’ which reflects the common usage meaning of the term ‘insecure’ might be: “the anxiety produced by the lack of economic safety”.” The United Nations Department of Economic and Social Affairs (2008, p.vi) writes that “It is not easy to give a precise meaning to the term economic insecurity. Partly because it often draws on comparisons with past experiences and practices, which have a tendency to be viewed through rose-tinted lenses, and also because security has a large subjective or psychological component linked to feelings of anxiety and safety, which draw heavily on personal circumstances. Still in general terms economic insecurity arises from the exposure of individuals, communities and countries to adverse events, and from their inability to cope with and recover from the costly consequences of those events.” Jacobs (2007) suggests that “Economic insecurity is perhaps best understood as the intersection between “perceived” and “actual” downside risk." According to Stiglitz, Sen and Fitoussi (2009, p.198), "Economic insecurity may be defined as uncertainty about the material conditions that may prevail in the future. This insecurity may generate stress and anxiety in the people concerned, and make it harder for families to invest in education and housing."

A plausible summary and synthesis of the above considerations can be captured in the following phrase: economic insecurity is the anxiety produced by the possible exposure to adverse events and by the anticipation of the difficulty to recover from them. Past, present and future are all involved. We are insecure about a future event and the anticipation of some difficulty in recovering generate anxiety to an individual. The resources we have today are important: the wealthier we are, the bigger the buffer stock we can rely on in case of an adverse future event. Our past experiences play a role in shaping our self-confidence on how well we can do in case of an adverse event. We remember gains and losses in our resources over time. Of course, the more recent these variations are, the more vivid our memories. Evidence from psychology and economics supports this view. For instance, Knight (1921, p.199) states that "[A]ll reasoning rests on the principle of analogy. We know the absent from the present, the future from the now, by assuming that connections or associations among phenomena which have been valid will be so; we judge the future by the past."

The way we model insecurity is similar to the formation of adaptive expectations (Cagan, 1956). What we are interested in is the subjective forecast of how well someone can handle a loss in the future. Past gains and losses determine the confidence an individual has today. We focus on wealth variations in the past and the current wealth level as the basic determinants of insecurity. Thus, the measures of individual insecurity we propose have as their domain wealth streams of varying lengths. The length of these streams is not assumed to be fixed because individuals are of different ages in a given time period and, moreover, the availability of data may impose restrictions on how far back in the past we can go when assessing economic insecurity.

While there are, of course, many aspects of life that may play an important role in assessing the economic insecurity faced by an agent, it seems to us that an adequate (and, from an applied perspective, realistic) option is to use a comprehensive notion of wealth as the relevant variable. By doing so, we abstract from determinants of insecurity that cannot be captured by a monetary variable (see Stiglitz, Sen and Fitoussi, 2009, pp.53–54, for a discussion and examples). However, this simplification does not seems to pose much of a problem if the notion of wealth employed is indeed defined in a comprehensive manner—wealth is assumed to encompass everything that may help an individual in coping with adverse events. The wealth of an individual includes, for instance, claims on governments, family, friends etc. Sen (1976) refers to these claims as entitlements—consumption bundles available to an agent given her or his rights and opportunities; see also Sen (1984, p.497).

Although the design of social rather than individual measures of economic insecurity also is an issue of considerable interest, we focus on the individual problem in this paper. We justify this choice by appealing to the observation that economic insecurity is very much a sentiment experienced by each individual. To draw a parallel to other questions involving economic index numbers, consider the measurement of deprivation as a prominent example. As illustrated in Yitzhaki (1979) and much of the subsequent literature, it is natural to first obtain an individual value of deprivation for any income distribution and then, in a second stage, aggregate these individual deprivation values into a social deprivation index. This second stage is frequently performed by calculating the arithmetic mean of the individual deprivation values, and the more substantive problem is that of designing the individual index. The measurement of economic insecurity is similar in this respect: once an individual index of insecurity is established, a social index can easily be obtained by applying a (possibly but not necessarily arithmetic) mean to the individual insecurity values. Note that this contrasts with economic measures of inequality: there is no 'individual' inequality because the phenomenon in itself is defined in terms of the disparity present in a distribution.

We propose a set of properties that we think a measure of economic insecurity should possess and use them to characterize specific linear measures of insecurity. According to these indices, insecurity is given by the current wealth level multiplied by minus one plus weighted sums of the wealth gains (losses) experienced in the past. Two sequences of coefficients are employed—one applies to gains, the other to losses. The coefficients are such that recent experiences are given higher weight than experiences that have occurred in the more distant past. A subclass of these measures is obtained by giving higher weights to the absolute values of past losses than to those of past gains, thereby reflecting an attitude that we may label loss aversion in analogy to risk aversion in models of individual decision making under uncertainty.

Section 2 presents the formal framework and the properties we consider important for a wealth-based measure of economic insecurity. In section 3 we characterize a class of linear measures and an important subclass. Section 4 concludes with a discussion of some open questions as well as brief remarks on the links between our approach and criteria employed in inequality measurement and the analysis of decision making under risk.

2 Wealth streams and individual insecurity

For any $T \in \mathbb{N}_0$, let $\mathbb{R}^{(T)}$ be the $(T+1)$-dimensional Euclidean space with components labeled $(-T, \dots, 0)$. Zero is interpreted as the current period and T is the number of past periods taken into consideration. We allow T to vary because people alive in the current period may have been born (or have become economic agents) in different periods. A measure of individual insecurity is a sequence of functions $V = \langle V^T \rangle_{T \in \mathbb{N}_0}$ where, for each $T \in \mathbb{N}_0$, $V^T \colon \mathbb{R}^{(T)} \to \mathbb{R}$. This index assigns a degree of insecurity to each individual (net) wealth stream $w = (w_{-T}, \dots, w_0) \in \bigcup_{T \in \mathbb{N}_0} \mathbb{R}^{(T)}$. We employ a comprehensive notion of wealth and we allow net wealth to be negative. The wealth stream $w = (1, 3, 3, -1, 0, 2) \in \mathbb{R}^{(5)}$ is illustrated in Figure 1.

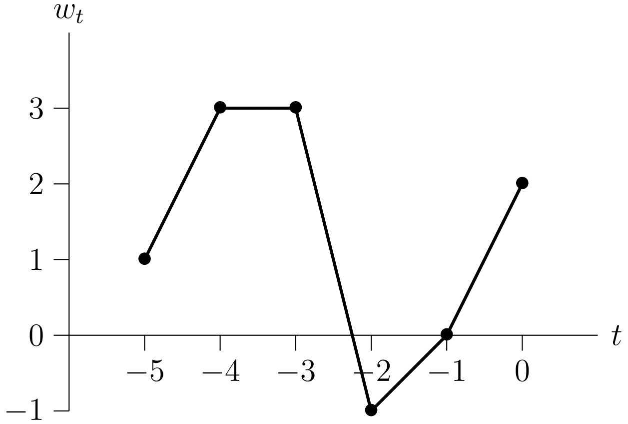

Figure 1: The wealth stream $w = (1, 3, 3, -1, 0, 2)$.

We want to design a class of measures the members of which capture the dependence of a sentiment of insecurity on past wealth movements in addition to today's wealth level. Loosely speaking, the basic hypothesis is that insecurity increases (decreases), *ceteris paribus,* with decreases (increases) of the wealth level experienced in the past with higher weight given to the recent past. Thus, our first property is the following difference monotonicity axiom.

Difference monotonicity. For all $T \in \mathbb{N}$, for all $w \in \mathbb{R}^{(T-1)}$ and for all $\gamma \in \mathbb{R}$,

$$V^T(w_{-(T-1)} + \gamma, w) \geq V^{T-1}(w) \;\Leftrightarrow\; \gamma \geq 0.$$

Difference monotonicity requires a decrease in insecurity as a consequence of the *ceteris paribus* addition of another period $-T$ which introduces a gain between periods $-T$ and

$-(T-1)$, thus allowing past gains to work against insecurity. Analogously, the measure of insecurity is assumed to increase if a period $-T$ is added in a way such that wealth decreases, *ceteris paribus,* when moving from $-T$ to $-(T-1)$. Finally, if the addition of period $-T$ involves a wealth level identical to that of period $-(T-1)$, insecurity is unchanged. This is a monotonicity requirement that appears to be essential in capturing the notion of increased (decreased, unchanged, respectively) insecurity as a response to additional losses (additional gains, no changes, respectively) in past wealth levels. Note that the axiom does not imply that gains and losses have to be treated symmetrically; it is possible, for instance, that adding a gain of a certain magnitude, *ceteris paribus,* decreases insecurity by less than a loss of the same magnitude increases insecurity. We will return to this issue in more detail at the end of this section. The axiom is illustrated in Figure 2. Starting from $w' = (3, 3, -1, 0, 2) \in \mathbb{R}^{(4)}$, an additional period -5 is added to arrive at the stream $w = (1, 3, 3, -1, 0, 2) \in \mathbb{R}^{(5)}$. The move from period -5 to period -4 involves a gain in net wealth and, thus, difference monotonicity demands that $V^5(w) < V^4(w')$.

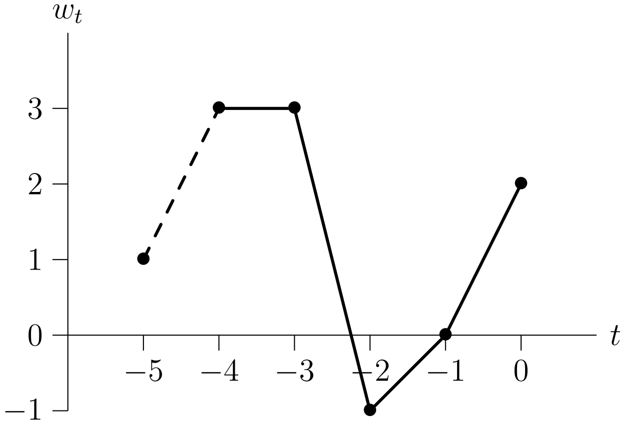

Figure 2: Difference monotonicity.

Next, we state a property that captures the observation that recent experiences carry a higher weight than experiences that occurred in the more distant past.

Proximity property. For all $T \in \mathbb{N} \setminus \{1\}$, for all $w \in \mathbb{R}^{(T)}$ and for all $\tau \in \{1, \ldots, T-1\}$,

$$\begin{aligned} & V^T(w_{-T}, \ldots, w_{-(\tau+1)}, w_{-(\tau+1)}, w_{-(\tau-1)}, \ldots, w_0) \geq \\ & V^T(w_{-T}, \ldots, w_{-(\tau+1)}, w_{-(\tau-1)}, w_{-(\tau-1)}, \ldots, w_0) \\ \Leftrightarrow \quad & w_{-(\tau+1)} \geq w_{-(\tau-1)}. \end{aligned}$$

The proximity property ensures that a gain (loss) of a given magnitude reduces (increases) insecurity, *ceteris paribus,* to a higher extent the closer to the present this gain (loss) occurs. That is, changes in wealth from one period to the next have a more severe impact the closer they are to the present period. Figure 3 illustrates the axiom. Comparing the streams $w = (1, 3, 3, -1, 0, 2) \in \mathbb{R}^{(5)}$ and $w' = (1, 3, -1, -1, 0, 2) \in \mathbb{R}^{(5)}$, we see that w' can be obtained from w by shifting the drop from 3 to -1 one period further into the past. According to the proximity property, the earlier loss affects the current sentiment of insecurity to a lesser extent than the original one and, thus, insecurity in w' is less that insecurity in w.

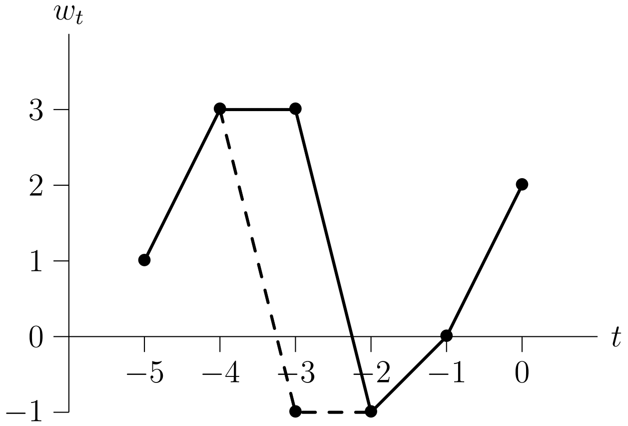

Figure 3: Proximity property.

We suggest to use the above three axioms as the fundamental properties of a measure of individual insecurity. This is parallel to the definition of an inequality measure as an S-convex function of income distributions. In addition to the defining properties, the following axioms are used to characterize the class of measures of individual insecurity.

A common property in the design of economic index numbers is homogeneity, an axiom that ensures that proportional changes in wealth are mirrored in the corresponding insecurity values. Thus, homogeneity requires insecurity to be measured by means of a ratio scale.

Homogeneity. For all $T \in \mathbb{N}_0$, for all $w \in \mathbb{R}^{(T)}$ and for all $\lambda \in \mathbb{R}_{++}$,

$$V^T(\lambda w) = \lambda V^T(w).$$

An analogous property applies to absolute instead of proportional changes. Formulated for insecurity measures, it is defined as follows. We use $\mathbf{1}_r$ to denote the vector consisting of $r \in \mathbb{N}$ ones.

Translatability. For all $T \in \mathbb{N}_0$, for all $w \in \mathbb{R}^{(T)}$ and for all $\delta \in \mathbb{R}$,

$$V^T(w + \delta \mathbf{1}_{T+1}) = V^T(w) - \delta.$$

Translatability differs from the usual translation scale property in that the value of δ is subtracted from the level of insecurity when δ is added to the wealth level in each period. This is a consequence of the inverse relationship between wealth and insecurity.

The conjunction of homogeneity and translatability implies that $V^0(w_0) = -w_0$ for all $w_0 \in \mathbb{R}$. Thus, V^0 is a decreasing linear function of w_0. We state this observation, to be used in the proof of our main theorem, in the following lemma.

Lemma 1. *If a measure of individual insecurity V satisfies homogeneity and translatability, then*

$$V^0(w_0) = -w_0 \quad \textit{for all } w_0 \in \mathbb{R}. \tag{1}$$

Proof. Setting $T = 0$ and $w_0 = 0$, homogeneity implies

$$V^0(0) = V^0(\lambda \cdot 0) = \lambda V^0(0) \quad \text{for all } \lambda \in \mathbb{R}_{++}$$

and, substituting any $\lambda \neq 1$, it follows that

$$V^0(0) = 0. \tag{2}$$

Setting $T = 0$ and $\delta = -w_0$ in the definition of translatability and using (2), we obtain

$$V^0(0) = V^0(w_0 + (-w_0)) = V^0(w_0) + w_0 = 0 \quad \text{for all } w_0 \in \mathbb{R}. \tag{3}$$

Clearly, the last equality in (3) is equivalent to (1). ■

Note that the full force of homogeneity and translatability is not needed for the above lemma; as is evident from the proof, it is sufficient to use the respective properties that are obtained by restricting the scopes of the axioms to the cases in which $T = 0$.

The next axiom combines a recursivity condition with the assumption that the role of past wealth enters through wealth differences only. Thus, in addition to the separability property encompassed by the axiom, this aggregation property expresses the assumption that past gains and losses are what matters to an agent; see also the discussion in the introduction.

Temporal aggregation property. For all $T \in \mathbb{N}\backslash\{1\}$, there exists a function $\Phi^T \colon \mathbb{R}^2 \to \mathbb{R}$ such that, for all $w \in \mathbb{R}^{(T)}$,

$$V^T(w) = \Phi^T \left(w_{-T} - w_{-(T-1)}, V^{T-1}(w_{-(T-1)}, \ldots, w_0) \right).$$

The temporal aggregation property is a separability condition that allows a measure of insecurity to be calculated by recursively moving back from the current period to the earliest relevant period where, in the step involving period $-t$, the part of insecurity that takes into consideration all periods from $-t$ to the current period is obtained as an aggregate of the insecurity resulting from considering periods $-(t-1)$ to period zero only and the change experienced in the wealth level between periods $-t$ and $-(t-1)$; see Blackorby, Primont and Russell (1978) for a detailed discussion of various recursivity properties.

3 Two-Sequences Gini Measures

We now characterize a specific class of measures that are inspired by the *single-series Gini* measures of inequality. The Gini index is one of the most established and well-known measures of income inequality. The generalized Ginis measures retain the linear structure of the Gini in rank ordered subspaces of the space of income distributions but allow for alternative degrees of inequality aversion by generalizing the coefficients to any rank ordered sequence of parameters. A subclass of the generalized Ginis is given by the single series Ginis, characterized in Bossert (1990). They are generalized Ginis such that the sequence of coefficients is the same for all population sizes. See, for instance, Donaldson and Weymark (1980), Weymark (1981) and Bossert (1990) for a discussion of the generalized and the single-series Ginis. Zank (2007) includes the generalized Ginis in his analysis of welfare functions with a reference income.

The class of *two-sequences Gini measures* involves two sequences of parameters—one the members of which are applied to past losses in wealth, one that is used for those period pairs in which there are gains. The sequences need not be the same but, within each sequence, some natural restrictions apply. Let $\alpha = \langle \alpha_{-t} \rangle_{t \in \mathbb{N}}$ and $\beta = \langle \beta_{-t} \rangle_{t \in \mathbb{N}}$ be two sequences of parameters such that

$$\left[\alpha_{-t} > \alpha_{-(t+1)} > 0 \quad \text{and} \quad \beta_{-t} > \beta_{-(t+1)} > 0 \right] \quad \text{for all } t \in \mathbb{N}. \tag{4}$$

The set of all sequences α such that $\alpha_{-t} > \alpha_{-(t+1)} > 0$ for all $t \in \mathbb{N}$ is denoted by $\mathcal{C}$. $\mathcal{C}^2$ is the Cartesian product of $\mathcal{C}$ with itself, that is, $\mathcal{C}^2$ is the set of all pairs of sequences satisfying (4). The *two-sequences Gini measure of insecurity corresponding to a pair of sequences* $(\alpha, \beta) \in \mathcal{C}^2$, $V_{(\alpha,\beta)} = \left\langle V^T_{(\alpha,\beta)} \right\rangle_{T \in \mathbb{N}_0}$, is defined by letting, for all $T \in \mathbb{N}_0$ and for all $w = (w_{-T}, \ldots, w_0) \in \mathbb{R}^{(T)}$,

$$V^T_{(\alpha,\beta)}(w) = \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)} \right) + \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)} \right) - w_0.$$

The measures defined above bear some formal resemblance to specific social evaluation functions that have appeared in the context of the ethical approach to income inequality measures (see Kolm, 1969, Atkinson, 1970, and Sen, 1973) and models of choice under risk (see, for instance, Rothschild and Stiglitz, 1970, 1971).

Our first result characterizes the class of measures based on two sequences of parameters as defined above.

Theorem 1. *A measure of individual insecurity* V *satisfies difference monotonicity, the proximity property, homogeneity, translatability and the temporal aggregation property if and only if there exists* $(\alpha, \beta) \in \mathcal{C}^2$ *such that* $V = V_{(\alpha,\beta)}$.

Proof. 'If.' Let $(\alpha, \beta) \in \mathcal{C}^2$. That $V_{(\alpha,\beta)}$ satisfies homogeneity and translatability is immediate. Difference monotonicity follows from the positivity of the coefficients α_{-t} and β_{-t}; see the definition of $\mathcal{C}$. The proximity property is satisfied because of the inequalities that apply to the sequences of parameters; see, again, the definition of $\mathcal{C}$. To see that the temporal aggregation property is satisfied, define, for all $T \in \mathbb{N} \setminus \{1\}$, the function $\Phi^T \colon \mathbb{R}^2 \to \mathbb{R}$ by letting, for all $(x, y) \in \mathbb{R}^2$,

$$\Phi^T(x, y) = \begin{cases} \alpha_{-T}\, x + y & \text{if } x > 0 \\ y & \text{if } x = 0 \\ \beta_{-T}\, x + y & \text{if } x < 0. \end{cases}$$

'Only if.' Suppose V satisfies the required axioms. We prove the relevant implication by inductively constructing a pair of sequences $(\alpha, \beta) \in \mathcal{C}^2$ such that

$$V^T(w) = \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)} \right) + \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)} \right) - w_0 \tag{5}$$

for all $T \in \mathbb{N}_0$ and for all $w \in \mathbb{R}^{(T)}$.

If $T = 0$, (5) is satisfied for all $w = (w_0) \in \mathbb{R}^{(0)}$ (trivially, for *any* pair $(\alpha, \beta) \in \mathcal{C}^2$ and, in particular, for the pair of sequences to be constructed below) because of (1).

Now let $T = 1$.

If $w \in \mathbb{R}^{(1)}$ is such that $w_{-1} = w_0$, difference monotonicity and (1) together imply

$$V^1(w) = V^0(w) = -w_0. \tag{6}$$

If w is such that $w_{-1} > w_0$, translatability with $\delta = -w_0$ implies

$$V^1(w_{-1} - w_0, 0) = V^1(w_{-1} - w_0, w_0 - w_0) = V^1(w_{-1}, w_0) + w_0 = V^1(w) + w_0$$

and, therefore,

$$V^1(w) = V^1(w_{-1} - w_0, 0) - w_0. \tag{7}$$

Applying homogeneity with $\lambda = w_{-1} - w_0 > 0$, it follows that

$$V^1(w_{-1} - w_0, 0) = V^1((w_{-1} - w_0) \cdot 1, (w_{-1} - w_0) \cdot 0) = (w_{-1} - w_0)V^1(1, 0)$$

and, together with (7),

$$V^1(w) = \alpha_{-1}(w_{-1} - w_0) - w_0 \tag{8}$$

where $\alpha_{-1} = V^1(1, 0)$. By difference monotonicity, $\alpha_{-1} > 0$.

If w is such that $w_{-1} < w_0$, a parallel argument yields

$$V^1(w) = \beta_{-1}(w_{-1} - w_0) - w_0 \tag{9}$$

where $\beta_{-1} = -V^1(-1, 0) > 0$.

Combining (6), (8) and (9), we obtain

$$V^1(w) = \sum_{\substack{t \in \{1\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0$$

for all $w \in \mathbb{R}^{(1)}$.

Now suppose that $T \in \mathbb{N} \setminus \{1\}$ and

$$V^{T-1}(w) = \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0 \tag{10}$$

for all $w \in \mathbb{R}^{(T-1)}$ where $(\alpha_{-(T-1)}, \ldots, \alpha_{-1})$ and $(\beta_{-(T-1)}, \ldots, \beta_{-1})$ are such that $\alpha_{-1} > \ldots > 0$ and $\beta_{-1} > \ldots > 0$. We have to show that there exists $(\alpha_{-T}, \beta_{-T})$ such that

$$\alpha_{-1} > \ldots > \alpha_{-T} > 0 \quad \text{and} \quad \beta_{-1} > \ldots > \beta_{-T} > 0 \tag{11}$$

and

$$V^T(w) = \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0 \tag{12}$$

for all $w \in \mathbb{R}^{(T)}$.

Together with (10), the temporal aggregation property implies the existence of a function $\Phi^T \colon \mathbb{R}^2 \to \mathbb{R}$ such that

$$\begin{aligned} V^T(w) &= \Phi^T\left(w_{-T} - w_{-(T-1)}, V^{T-1}(w_{-(T-1)}, \ldots, w_0)\right) \\ &= \Phi^T\Bigg(w_{-T} - w_{-(T-1)}, \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t}\left(w_{-t} - w_{-(t-1)}\right) \\ &\qquad + \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t}\left(w_{-t} - w_{-(t-1)}\right) - w_0\Bigg) \end{aligned} \tag{13}$$

for all $w \in \mathbb{R}^{(T)}$.

First, consider $w \in \mathbb{R}^{(T)}$ such that $w_{-T} = w_{-(T-1)}$. Difference monotonicity and (10) together imply

$$\begin{aligned} V^T(w) &= V^{T-1}(w_{-(T-1)}, \ldots, w_0) \\ &= \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t}\left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t}\left(w_{-t} - w_{-(t-1)}\right) - w_0 \end{aligned}$$

and it follows that

$$\Phi^T(0, y) = y \quad \text{for all } y \in \mathbb{R}^2. \tag{14}$$

Now consider the case in which w is such that $w_{-T} > w_{-(T-1)}$. Homogeneity implies that Φ^T satisfies

$$\Phi^T(\lambda x, \lambda y) = \lambda \Phi^T(x, y) \quad \text{for all } \lambda, x \in \mathbb{R}_{++} \text{ and for all } y \in \mathbb{R} \tag{15}$$

and translatability implies

$$\Phi^T(x, y - \delta) = \Phi^T(x, y) - \delta \quad \text{for all } x \in \mathbb{R}_{++} \text{ and for all } \delta, y \in \mathbb{R}. \tag{16}$$

Letting $\delta = y$, (16) implies $\Phi^T(x, 0) = \Phi^T(x, y) - y$ and, thus,

$$\Phi^T(x, y) = \Phi^T(x, 0) + y \quad \text{for all } x \in \mathbb{R}_{++} \text{ and for all } y \in \mathbb{R}. \tag{17}$$

Letting $\lambda = x > 0$, (15) implies

$$\Phi^T(x, 0) = \Phi^T(x \cdot 1, x \cdot 0) = x \Phi^T(1, 0) \quad \text{for all } x \in \mathbb{R}_{++}$$

and, together with (17), we obtain

$$\Phi^T(x, y) = \alpha_{-T}\, x + y \quad \text{for all } x \in \mathbb{R}_{++} \text{ and for all } y \in \mathbb{R} \tag{18}$$

with $\alpha_{-T} = \Phi^T(1, 0)$. By difference monotonicity, $\alpha_{-T} > 0$ and by the proximity property, $\alpha_{-T} < \alpha_{-(T-1)}$ and, thus,

$$\alpha_{-1} > \ldots > \alpha_{-T} > 0. \tag{19}$$

If w is such that $w_{-T} < w_{-(T-1)}$, an argument parallel to that used above to derive (18) can be employed to obtain

$$\Phi^T(x, y) = \beta_{-T}\, x + y \quad \text{for all } x \in \mathbb{R}_{--} \text{ and for all } y \in \mathbb{R} \tag{20}$$

with $\beta_{-T} = -\Phi^T(-1, 0)$. By difference monotonicity, $\beta_{-T} > 0$ and by the proximity property, $\beta_{-T} < \beta_{-(T-1)}$ and, thus,

$$\beta_{-1} > \ldots > \beta_{-T} > 0. \tag{21}$$

Combining (14), (18) and (20), it follows that

$$\Phi^T(x, y) = \begin{cases} \alpha_{-T}\, x + y & \text{if } x > 0 \\ y & \text{if } x = 0 \\ \beta_{-T}\, x + y & \text{if } x < 0 \end{cases}$$

for all $(x, y) \in \mathbb{R}^2$. Substituting back into (13), we obtain (12) for all $w \in \mathbb{R}^{(T)}$. Because $\alpha_{-1} > 0$ and $\beta_{-1} > 0$ and, moreover, (19) and (21) are satisfied for all $T \in \mathbb{N} \setminus \{1\}$, the pair of sequences (α, β) thus constructed satisfies (11) and therefore is in $\mathcal{C}^2$ as required. ■

The result of the previous theorem does not impose any restrictions on the relationship between the sequences α and β. A plausible assumption appears to be the requirement that *ceteris paribus* losses of a certain magnitude in a given period have at least as strong an impact on insecurity as *ceteris paribus* gains of the same magnitude in the same period. This assumption is captured in the weak loss priority axiom. If this property is added to those of theorem 1, the parameter values must be such that losses carry a weight that is at least as high as that for gains in each period, which leads to a subclass of the measures identified in the previous characterization. To define this loss averse class formally, let $\mathcal{D}$ be the set of all pairs of sequences $(\alpha, \beta) \in \mathcal{C}^2$ such that

$$\alpha_{-t} \geq \beta_{-t} \quad \text{for all } t \in \mathbb{N}.$$

Weak loss priority. For all $T \in \mathbb{N}$, for all $w \in \mathbb{R}^{(T-1)}$ and for all $\gamma \in \mathbb{R}_{++}$,

$$V^T(w_{-(T-1)} + \gamma, w) - V^T(w_{-(T-1)}, w) \;\geq\; V^T(w_{-(T-1)}, w) - V^T(w_{-(T-1)} - \gamma, w).$$

Weak loss priority can be interpreted as an insecurity analogue of weak risk aversion in the context of individual choice under uncertainty. Figure 4 provides an example of the application of this axiom. The three wealth streams indicated differ in the earliest period (period -5) only. The uppermost stream starts at a net wealth level of 5, the second at 3 and the third at 1. Thus, the absolute value of the difference between the first and the second stream in this period is the same as the absolute difference between the second and the third stream (this difference is given by 2). The distinguishing feature between these differences is that the first represents a loss while the second is a gain with respect to the middle stream. Weak loss priority requires that the loss has a larger impact on insecurity than the (equal-sized) gain so that

$$V^5(5,3,3,-1,0,2) - V^5(3,3,3,-1,0,2) \;\geq\; V^5(3,3,3,-1,0,2) - V^5(1,3,3,-1,0,2).$$

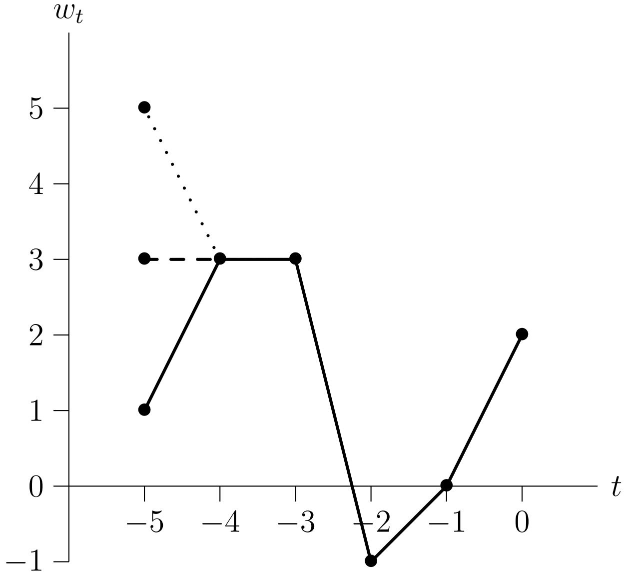

Figure 4: Weak loss priority.

Adding weak loss priority to the axioms of theorem 1 leads to a characterization of these loss averse measures. This is stated in the following theorem, the proof of which is straightforward and left to the reader.

Theorem 2. *A measure of individual insecurity V satisfies difference monotonicity, the proximity property, homogeneity, translatability, the temporal aggregation property and weak loss priority if and only if there exists $(\alpha, \beta) \in \mathcal{D}$ such that $V = V_{(\alpha,\beta)}$.*

Theorem 2 identifies the class of insecurity measures that we advocate in this paper. As an example, consider the measure obtained by choosing the sequences α and β so that

$$\alpha_{-t} = \frac{1}{2t-1} \quad \text{and} \quad \beta_{-t} = \frac{\alpha_{-t}}{2}$$

for all $t \in \mathbb{N}$. Clearly, $(\alpha, \beta) \in \mathcal{D}$. The coefficients according to the sequence α are the inverses of the coefficients corresponding to the Gini social evaluation function; see, for instance, Donaldson and Weymark (1980) and Weymark (1981).

4 Concluding remarks

In this paper, we propose classes of measures of economic insecurity analyzed from a thorough theoretical perspective. The measures of individual insecurity characterized in this paper share a linear structure with the generalized Gini social evaluation functions used in ethical approaches to inequality measurement. Furthermore, they resemble rank ordered decision criteria employed in theories of choice under uncertainty.

We provide a detailed treatment of individual insecurity based on wealth considerations in this paper. This leaves open the problem of aggregating individual insecurity values into a social index. Thus, an explicit study of economic insecurity for society as a whole is another task to be undertaken in future work. In addition, one may want to explore the possibility of including (non-monetary) variables other than wealth in order to arrive at a more comprehensive notion of insecurity.

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