Research Paper No. 2004/60

A Contract Perspective on the International Finance Facility

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September 2004

Abstract

The present paper is a first attempt to develop a theoretical model using a short-term vis-à-vis long-term contract framework within which donor countries' endorsement or rejection decision towards the recently proposed International Finance Facility (IFF) is rationalized. The current foreign aid system is portrayed as being similar to a series of short-term contracts, where donor countries are able to adjust the aid amount to reflect environmental change (broadly defined to take into account changes in public opinion, domestic situation, etc.). The benefit of this system is in its flexibility. Frontloading aid, on the other hand, as proposed by the IFF proposal, has the benefit of smoothing out the flows over time. In the model presented in this paper, donor countries balance these two contract schemes to determine the endorsement or rejection of the IFF proposal. By using historical aid data covering the period 1990-2003 for all DAC donor countries, our empirical analysis shows the payoffs and relative advantages of these two contract schemes.

Keywords: foreign aid, International Finance Facility, contract theory, uncertainty

JEL classification: F35, D81

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This study has been prepared within the UNU-WIDER project on New Directions in Development Economics, directed by Tony Addison.

UNU-WIDER acknowledges the financial contributions to the research programme by the governments of Denmark (Royal Ministry of Foreign Affairs), Finland (Ministry for Foreign Affairs), Norway (Royal Ministry of Foreign Affairs), Sweden (Swedish International Development Cooperation Agency—Sida) and the United Kingdom (Department for International Development).

ISSN 1810-2611 ISBN 92-9190-655-7 (internet version)

Acknowledgements

The authors would like to thank Yu Jin for assistance in developing the simulation codes. The views expressed in the paper are those of the authors' and should not be attributed to UNU-WIDER.

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Camera-ready typescript prepared by Liisa Roponen at UNU-WIDER
Printed at UNU-WIDER, Helsinki

1 Introduction

The final declaration of the Monterrey Conference in 2002 highlighted the dramatic decline in the resources required to meet the internationally agreed Millennium Development Goals (MDGs). The MDGs can be financed either through existing sources of development finance such as official development assistance (ODA) or, in view of the decline of aid in recent years, by seeking new sources of development finance. Bearing in mind that the total volume of aid flows in 2002 was US$58 billion, the existing amount of ODA and other private capital flows need to be doubled, or more domestic development finance mobilized, in order to achieve the MDGs.

Recent estimates from the OECD-DAC seem to suggest that, based on the assumption that donor countries will deliver what was agreed at Monterrey, ODA is expected to rise to around US$75 billion (at 2002 prices and exchange rates) by 2006. This, as a proportion of donor GNI, is equivalent to an increase of about 29 per cent from a low of 22 per cent in 2001. But it is still well below the comparable levels of the early 1990s (OECD 2003).[1] It needs to be stressed, however, that even if these commitments were met by 2006, the financing gap with regard to the MDGs will be about US$25 billion (based obviously on the 'working' assumption that the total extra cost for meeting the MDGs is US$50 billion).

It is now generally agreed in connection with ODA that:

i) while MDGs require more aid, accelerating aid finance and dramatically improving aid effectiveness at the same time, are crucial factors;

ii) improving our overall understanding of aid effectiveness is vital and should be a key priority for the achievement of the MDGs;

iii) there is now more consensus regarding the positive overall impact of aid on growth, but, at the same time, there is less consensus concerning the extent to which aid can in fact be effective in promoting growth in poor policy environments, and on what constitutes an effective, pro-poor growth policy, beyond the achievement of basic macroeconomic stability and the avoidance of gross price distortions;

iv) aid effectiveness issues are now becoming more central than ever: under the (plausible) assumption that it will not be an easy task to raise the additional aid flows needed for the MDGs, significantly improving the effectiveness of current (and future) aid is important. But even if a substantial amount of fresh aid is raised, *accelerating* finance is a crucial issue with regard to the MDGs (Mavrotas 2003a).

Against this background, recent years have witnessed an ongoing discussion of a number of innovative proposals for achieving the MDGs by 2015.[2] One such proposal

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[1] According to the OECD-DAC Report, the bulk of the absolute increases is expected from five donors, namely the United States, the UK, France, Italy and Germany.

[2] It is clearly beyond the scope of the present paper to discuss in detail the various proposals of development finance currently being debated in the international development community. For a

receiving much attention was launched in January 2003 jointly by HM Treasury and the UK Department for International Development (DFID) and is the International Finance Facility (IFF).

The IFF proposal aims to accelerate progress by 'frontloading' aid so that the Millennium Goals can be achieved. It concerns long-term, but conditional, funding guaranteed to the poorest countries by the donor countries. These long-term pledges of a flow of annual payments to the IFF would leverage additional money from the international capital markets (through a securitization process) in the magnitude of about US$50 billion over the period 2010-15, building up from 2006 and falling to zero by 2020.[3]

The above generic features of the IFF scheme seem to suggest that the proposal can be analysed either as a net addition to existing development aid, *or* as a comparison with 'straight' ODA of the same net present value, *or* as a comparison with ODA with the same time path. In the first case, we have all the benefits of increased flows, but also with some potential problems related to the absorptive capacity constraints of aid recipients. In the second case, we have the benefit of being able to bring forward disbursements. Finally, in the third case, we have the certainty of the flows, with the difference being much smaller (Mavrotas 2004).

At the time of writing, the scheme has the full support of the UK and France while other donors are still considering their position. Some developing countries which are not expected to be recipients of IFF, such as South Africa, India and China have also voiced their support for the proposal. The US, on the other hand, is considered unlikely to support the scheme in view of its focus on the Millennium Challenge Account, and Germany is legally banned from committing future governments to budgetary outlays (Mavrotas 2004).[4]

The overall need to increase aid flows substantially (via new sources such as the IFF, for example) so that MDGs can be achieved has recently triggered discussions on the potential effects of increased aid on aid-recipient countries. For instance, it has been argued that if donors were to meet the ODA target of 0.7 per cent of donor country GNP, aid flows would increase to about US$175 billion, or more than three times the current levels. This, obviously, would help with the achievement of the MDGs, but at the same time would pose challenges for aid-recipient countries at both the micro and macro level (Heller and Gupta 2002). Without implying an increase in aid flows equivalent to the targeted 0.7 per cent of donor country GNP, many of the current proposals to finance the MDGs (including the IFF) are associated with a sharp increase

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comprehensive, in-depth discussion of the various proposals, see Atkinson (2003, 2004). It is also important to stress that in some of the proposals, there is a distinct risk of crowding-out, i.e. new sources of development finance may crowd-out existing sources in certain cases (i.e. the additionality issue); furthermore, political economy issues are also of relevance to the new sources of development financing.

3 See Mavrotas (2003b, 2004) for a detailed discussion.

4 The IFF has been broadly discussed within the G8, the EU, the UN, the G20 group of leading and emerging economies, the Commonwealth, at international meetings of the IMF and the World Bank, with emerging market and developing countries and with international organizations such as the UNDP, the UN Commission for Africa, the WHO and NEPAD (HM Treasury 2004).

in aid over the next decade until 2015. This may pose crucial challenges for the potential recipients, such as the ability to absorb additional flows in a number of aid-dependent countries. More precisely, doubling aid, through the IFF for instance or other proposed schemes, might induce diminishing marginal rates of return. This could become a serious problem at very high aid-to-GDP ratios. In real terms, however, if aid were to double by 2005, it would bring only 14 countries, constituting a combined population of 109 million, above the 20 per cent aid-to-GNI (Mavrotas 2003b). At the same time, there is no evidence to support that countries receiving high levels of aid have performed poorly. Some of recent success stories in Africa clearly show that large amounts of aid can yield substantial rates of return. Although Uganda's aid in the early 1990s exceeded 20 per cent of GDP, it registered growth rates of over 7 per cent, effectively reducing poverty at the same time by 20 per cent (mainly through the Poverty Eradication Action Fund). Mozambique, with a 50 per cent aid-to-GDP ratio in the 1990s, achieved high growth rates, reaching 12 per cent in 1998. Recipients with aid levels over 20 per cent of income, most of which were in Africa, increased on average their per capita GDP by 1.3 per cent per year over the period 1995-2000 (World Bank 2003).[5] Needless to say, these trends cannot support a robust conclusion regarding the impact of aid on growth (or the overall effectiveness of development aid) in developing countries since the aid-growth relationship can be affected by factors not captured by simple empirical analysis.

The greatest controversy about the IFF arises from the trade-offs between the benefits of stable and predictable aid flows (as proposed in the IFF), i.e., frontloading future aid flows to meet today's challenge (e.g., the MDGs), and the benefits of a flexible aid flow policy as the status quo. The binding commitment of the IFF scheme constrains the amount of aid from donor countries in future years. This may limit flexibility to respond to changes in the domestic or international environment, which theoretically speaking, could be equivalent to a welfare loss for donor countries. The main advantage of the proposal, however, lies in the fact that by resolving the uncertainties of aid flows inherent in the current status quo system, it is able to finance and upload future flows to smooth out the aid consumption over time. This is welfare improving.

Along these lines, to the best of our knowledge, the present paper is the first attempt to formalize the above mentioned scheme within a short-term vis-à-vis long-term contract framework. We model the current development aid system as a series of short-term contracts, which allows donor countries to adjust the aid amount (i.e., consumption of foreign production) to respond to environmental changes (broadly defined to include public opinion, domestic situation, etc.). IFF, on the other hand, is modelled as a long-term contract which makes the frontloading of future aid possible. The advantage of smoothing out consumption in the latter case stems from the difference between time discount rates and interest rates. The greater the differences between the two, the more willing donors are to frontload future consumption. Another reason for smoothing out the consumption is foreseeable future consumption growth (resulting from total GDP growth), that is, the aid donor who foresees future consumption growth, and is impatient, will frontload future consumption. We assume that the donor countries balance these two contract schemes before deciding whether or not to join the IFF. We

5 This is lower than China and India but greater than the average for all low-income countries (World Bank 2003).

subsequently test our theoretical model by using historical aid data for the period 1960-2003 for all DAC donors, to show countries' payoffs and relative advantages of these two contract schemes.

The question we have formulated above with regard to contracts differs from many studies on contract length in the fact that no asymmetric information is assumed. Given symmetric information, the obvious advantage of a long-term contract is its ability to smooth consumption. This perspective is, however, often disregarded on empirical grounds by most of the previous studies. Funderburg and Holmstrom (1990) studied the long-term agency contract (e.g., many managerial contracts) vis-à-vis short-term contracts (e.g., wage contracts for many factory workers) in the labour market. They dismissed the consumption-smoothing consideration based on the empirical fact that low-income employees, who have less access to capital markets and who therefore would prefer long-term labour contracts for consumption smoothing, in reality receive mostly short-term labour contracts. This observation is certainly correct with regard to labour market contracts. However, we believe that for the foreign aid schemes herein, the consumption-smoothing perspective lies in the heart of a long-term contract proposal, such as the IFF.

The remainder of the paper is organized as follows. The next section presents the theoretical model under the two different scenarios, i.e., aid without and with the IFF and their implications for dealing with the growing economy assumption. Section 3 deals with empirical analysis, combining historical aid data and Monte Carlo simulation, to indicate the payoffs and relative advantages of these two contract schemes. The last section concludes the paper.

2 Current aid architecture vis-à-vis IFF: a theoretical model

2.1 The donor country

We examine a donor country's choice under the two alternative contract schemes: a series of short-term contracts or a long-term contract. Time is continuous starting from zero to T.

The donor has to decide which contract scheme to adopt to maximize its expected utility flow. The donor country derives utility U by allocating resources to domestic production Q and foreign aid A (foreign production) with $U_A \geq 0$ and $U_Q \geq 0$ (subscripts are partial derivatives). The form of its utility U is assumed as:

$$U = \alpha A^{\theta} Q^{1-\theta} \tag{1}$$

A Cobb-Douglas form of the utility function is chosen mainly for ease of calculation and the qualitative results derived hold for other forms of utility functions as well. To further simplify our calculation, we assume that the donor country seeks to maximize the log form of its utility function (1) subject to a budget constraint. The donor's budget is the total resources available (e.g., GNP) for allocation between foreign aid and domestic production. Furthermore, donors are also assumed to be risk neutral.

θ is a time varying coefficient capturing the relative importance of foreign aid to domestic production governed by exogenous shock. The optimal allocation of A and Q is therefore parameterized by the realization of θ. θ is assumed to be a random draw, with each period's realization as θ_t, to capture the time-varying environment under which the donor country makes resource allocation decisions. For example, there are times when rising unemployment forces greater resource allocation towards domestic needs. Or merely the swing of public opinion in the political arena can cause a change in the donor's preferences with regard to resource allocation. These changes in policy environment are captured by the change of donor country's utility function, which is parameterized by θ with greater realization of θ representing a greater preference towards foreign aid. The expectation and variance of θ are assumed to be:

$$E(\theta) = \overline{\theta} \tag{2}$$

$$Var(\theta) = \sigma^2 \tag{3}$$

The realized level of θ, reflecting the relative importance of foreign aid as compared to domestic production in terms of donor country's utility, may also depend on the effectiveness of development aid. More effective aid presumably yields higher utility to the donor, thus greater θ. Issues related to development aid effectiveness have attracted much attention recently, particularly since the publication of the *Assessing Aid* (1998) study by the World Bank, which has been very influential among donors, but also debatable among researchers, as well as the current debate on development finance sources and new proposals (including the IFF) for financing the MDGs. The focus of much of the recent aid effectiveness literature is on the aid policy-growth nexus. A key conclusion of the World Bank research on the above relationship is that 'aid works well in good policy environments'. The above conclusion has been at the heart of the research and policy debate in recent years.[6]

For our comparison of the status quo aid system and the IFF proposal, however, we will not consider the aid allocation implications of recent research on aid effectiveness. It is our assumption that the composition of recipient countries does not change under IFF compared to the status quo system. In other words, aid allocation is assumed to be the same with and without IFF. This is a plausible working assumption because the proposal, at least in its current form, leaves room for flexibility regarding the recipients of IFF-related aid flows. This assumption is essential in order to focus on the 'commitment' (with IFF) versus 'non-commitment' (status quo) perspectives of the two options.

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6 It is notable, however, that although *Assessing Aid* (World Bank 1998) is an important contribution to the literature, its policy guidelines should be treated with much scepticism, given that the Burnside-Dollar 'approach' (1997, 2000) to aid effectiveness is statistically delicate, it has been contradicted by other recent studies (e.g. Hansen and Tarp 2000, 2001; Guillaumont and Chauvet 2001), and cannot be seen as robust (Tarp and Hjerholm 2000; Lensink and White 2000; Beynon 2002; Chauvet and Guillaumont 2002; Dalgaard, Hansen and Tarp 2004; Roodman 2003; Easterly, Levine and Roomand 2003). See Mavrotas (2000); Beynon (2002, 2003); McGillivray (2003); Langhammer (2004), and Collier and Dollar (2004) for recent assessments of aid effectiveness.

2.2 Scenario 1: aid without IFF

Let us first consider the case where the donor country chooses a series of short-term contracts. This is similar to the status quo scenario, except that status quo probably adds more rigidity in the system. Foreign aid budgets may, for example, be approved only by parliament at certain time intervals rather than at any time, as assumed in the model. Furthermore, contracts are both verifiable and enforceable.

The advantage of having a series of short-term contracts is mainly the flexibility these offer to respond to changing circumstances (i.e., the θ parameter) over time. The disadvantage, however, lies in the uncertainty regarding aid flows that makes frontloading future aid very costly, if not impossible. In our case we assume that no financing possibility exists under the flexible short-term contracts.

The donor country's problem therefore becomes:

$$\underset{A_t, Q_t}{Max} \int_{t=0}^{T} \ln(\alpha A_t^{\theta_t} Q_t^{1-\theta_t}) e^{-\rho t} dt \tag{4}$$

s.t. $A_t + Q_t = k$

where ρ is time discount rate, k is the total resource available for distribution by the donor country (i.e., GNP) and is assumed to be constant over time, an assumption we relax later on.

Solving the above optimization problem yields the optimal resource allocation scheme of domestic production and foreign aid:

$$A_t^* = \theta_t k$$
$$Q_t^* = (1-\theta_t)k \tag{5}$$

According to Equation (5), the optimal resource allocated to foreign aid is the θ_t portion of total resource at any particular time. With the fluctuation of θ over time, the optimal amount of foreign aid fluctuates accordingly to maximize utility. Presumably the greater the fluctuation of θ, the greater benefit of such a flexible scheme (discussed in subsection 2.5).

2.3 Scenario 2: aid under IFF

The IFF is essentially a pre-negotiated long-term contract in which each donor country makes a commitment regarding its foreign aid contribution during the life of the Facility. As with the short-term scheme, we are not concerned here with the verification and enforcement issues of the contract.

Compared to short-term contracts discussed above, a long-term contract does not have the flexibility of adjusting foreign aid flows to respond to environmental changes. The advantage of a long-term contract, however, is that it enhances the predictability of future aid flows, which can be used to frontload future aid through financial engineering (e.g., the securitization process discussed in the IFF proposal) at certain financing cost to smooth the consumption of foreign aid (foreign production) over time.

The above financing possibility essentially relaxes the temporal budget constraints by making inter-temporal borrowing possible, the benefit of the long-term contract. However it is worth noting that under the IFF, the amount financed is restricted to aid use only, so in our model the long-term contract specifies that the donor country pre-commits to a fixed amount of financial aid A_0 at the outset, and the total amount of foreign aid A_t is merely the sum of pre-committed amount A_0 and the amount financed at that period B_t. This rules out the possibility that the donor country will strategically channel the amount targeted for foreign aid to domestic production by setting the first years' commitment low as an amount of aid.

The donor country's problem, therefore, now becomes:

$$\underset{A_0, B_t}{Max} \int_{t=0}^{T} \ln(\alpha A_t^{\theta_t} Q_t^{1-\theta_t}) e^{-\rho t} dt \tag{6}$$

$$\text{s.t. } A_0 + Q_t = k$$

$$A_0 + B_t = A_t$$

$$\int_{t=0}^{T} B_t e^{-rt} dt = 0$$

$$A_t \geq 0$$

The objective function shows that the donor country maximizes its utility by setting the two contract terms: aid flow A_0 and financing amount $B_0 \ldots B_T$. The first condition is the budget constraint. The total resource to be allocated is a fixed amount k and we shall relax this assumption of no economic growth in subsection 2.5. This now consists of two components: the pre-committed amount of aid flow A_0 and the domestic production Q_t. The second condition shows that the aid flow at time t is the sum of the pre-committed amount of aid flow A_0 and the financed amount B_t. B_t could be either positive or negative depending on t in borrowing period (positive B_t) or returning period (negative B_t). The third condition ensures that the entire financed amount of aid is paid back with a fixed interest r, which is presumably smaller than time discount rate ρ. The last condition is to ensure that total aid amount in each period (the commitment amount and the financed amount) is non-negative (for realistic reasons and for guaranteeing a meaningful utility function). The last condition imposes an upper limit on the borrowed/repayment amount, which from now on will be referred to as 'no overdraft condition'.

Solving equation (6), we can derive the following optimal levels for the committed aid amount A_0 and the financed aid amount B_t (see Appendix A for derivation details).

$$A_0{*} = \bar{\theta} k$$

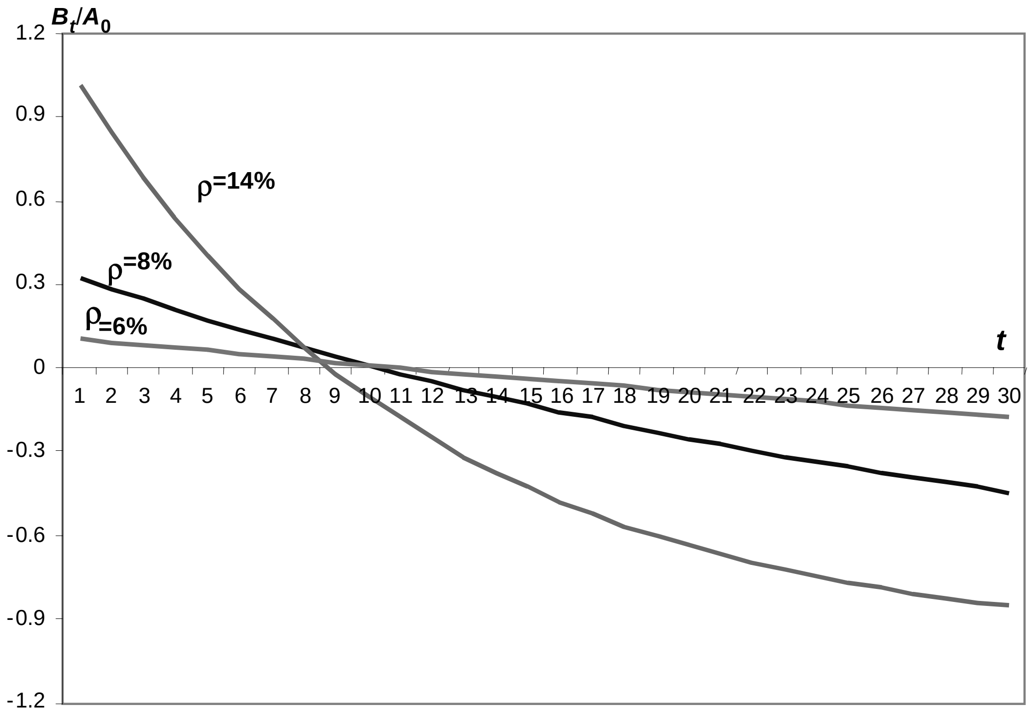
$$B_t{*} = \bar{\theta} k (w\text{-}1) \quad \text{where } w \equiv \frac{\rho(1-e^{-rT})}{r(1-e^{-\rho T})} \frac{1}{e^{(\rho-r)t}} \geq 0 \tag{7}$$

Equation (7) shows that under the long-term contract, the optimal policy is to pre-set the commitment aid amount to $\overline{\theta}$ (expected value of θ) portion of GDP. At the optimum, in each period the financed aid amount B_t is w-1 per cent of A_0, where w is a coefficient jointly determined by the total contract length T, time point t, time discount rate ρ, and interest rate r. When w is greater than 1, the country borrows money for aid and when w is smaller than 1, the country pays back the money. The fact that $\partial w/\partial t < 0$ (thus $\partial B_t^*/\partial t < 0$) suggests a non-linear financing scheme with the borrowed/repayment amount declining over time. The borrowing period starts from time 0 till the term in brackets equals zero ($w = 1$), and then the repayment periods follow. For example, given $T = 30$, $\rho = 6\%$ and $r = 5\%$, the repayment period starts on $t = 11.05$.

Figure 1 shows the simulations of different scenarios for ρ and r. Figure 1 indicates that the greater difference between ρ and r is the greater incentive the donor country has to frontload aid by borrowing. This also implies a faster start to the repayment period.

One point to note is that the 'no overdraft condition' is equivalent to imposing a restriction on w: $w \geq 0$, meaning that at any repayment period ($w < 1$) the maximum amount to be repaid cannot exceed the amount of the commitment (therefore $A_t \geq 0$). This condition is satisfied in all parameter values (T, t, ρ and r). (See Appendix A for details.)

Figure 1
Optimal financing amount of foreign aid when T = 30 and r = 5%



Note: T = 30 and r = 5% are in fact in line with the relevant assumptions adopted in the current IFF proposal.

2.4 Welfare comparison between the two contracts

In the preceding subsections, we derived the solution for optimal policies with regard to the two different contract schemes. The short-term contract has the flexibility of adjusting aid flows to reflect environmental changes but, given this uncertainty, no inter-temporal financing is possible. The long-term contract is exactly the opposite. In this subsection, we let the donor country optimize its choice of contracts contingent on two parameters. The first parameter is the volatility of θ over time. The greater the variation of θ over time, the more volatile the environment, a fact which underlines the magnitude of the benefit derived from flexible and short-term contract schemes. The second parameter is the difference between time discount rate ρ and interest rate r. In the current setting, the advantage of the long-term contract scheme is its ability to frontload financial aid. The greater the gap between time discount rate ρ and interest rate r, the more desirable it will be to frontload aid flows.

We denote the donor country's optimal welfare under the two contractual schemes (short-term and long-term), U^S and U^L, respectively. Substituting donor country's optimal aid policies under the two contract schemes back to its utility function, we take:

$$U^s = \int_{t=0}^{T} \left[\ln\alpha + \theta_t \ln k\theta_t + (1-\theta_t)\ln(1-\theta_t)k \right] e^{-\rho t} dt \tag{8}$$

$$U^L = \int_{t=0}^{T} \left[\ln\alpha + \theta_t \ln w\bar{\theta}k + (1-\theta_t)\ln(1-\bar{\theta})k \right] e^{-\rho t} dt \tag{9}$$

$$D \equiv U^s - U^L = \int_{t=0}^{T} \left[\theta_t \ln\theta_t + (1-\theta_t)\ln(1-\theta_t) - \theta_t \ln w\bar{\theta} - (1-\theta_t)\ln(1-\bar{\theta}) \right] e^{-\rho t} dt \tag{10}$$

$$where \qquad w \equiv \frac{\rho(1-e^{-rT})}{r(1-e^{-\rho T})} \frac{1}{e^{(\rho-r)t}} \tag{11}$$

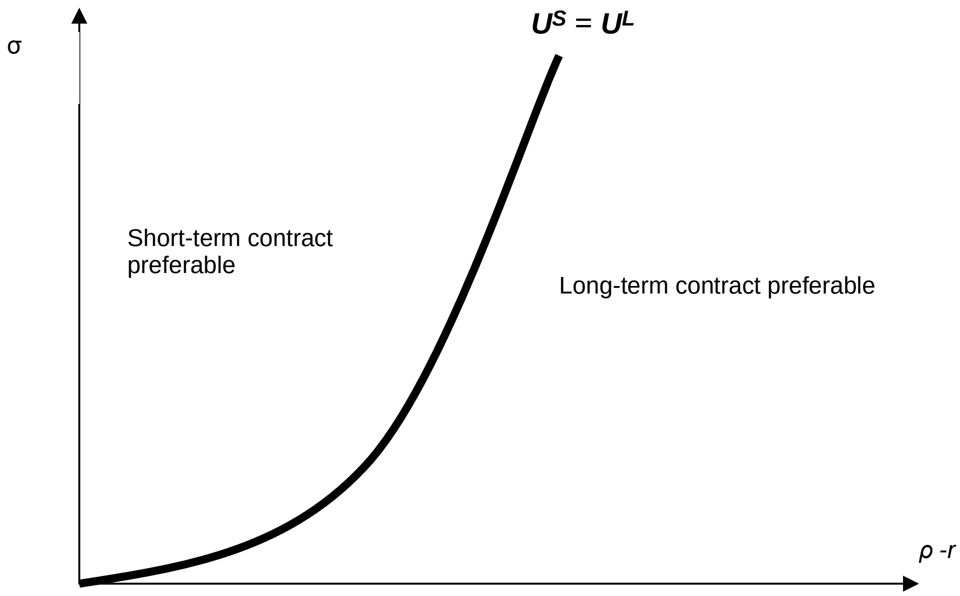
Equations (8) and (9) show the total (time discounted) utility under the two contract schemes. Their difference is represented by D in Equation (10), while w in Equation (11) captures how the difference between time discount rate ρ and interest rate r enters the utility balance between the two contracts. As already shown in the previous section, under the long-term contract, the greater difference between the time discount rate ρ and interest rate r, the more incentive the country has to frontload aid. We can interpret Equations (10) and (11) as follows. First, when the interest rate coincides with time discount rate ($\rho = r$), w equals to 1 and D is a non-negative number with $D = 0$ *iff* $\theta_t = \bar{\theta}$ $\forall t$. This means that when there is no benefit to frontloading aid (i.e., $\rho = r$), the short-term contract is always preferable. Moreover, we show that the more variation there is in θ_t, the more attractive the short-term contract becomes. Second, if there is no variation in θ_t (i.e., $\theta_t = \bar{\theta}$ $\forall t$ or $\sigma = 0$), the greater the time discount rate ρ compared to the interest rate r, the smaller D becomes, hence making long-term contract more appropriate. This is intuitive because the long-term contract allows the financing possibility of when the interest rate falls below time discount rate (i.e., $\rho > r$), frontloading future consumption is profitable. We can summarize the above in the following proposition (the proof is provided in Appendix B).

Proposition 1

$\partial D/\partial\sigma \geq 0$ and $\partial D/\partial(\rho\text{-}r) \leq 0$; there exists $\{\sigma^*,(\rho\text{-}r)^*\}$ so that $U^S = U^L$, i.e., $D = 0$.

Proposition 1 argues that the difference between U^S and U^L is monotonically increasing with σ and decreasing with (*ρ*-*r*). In the space of σ and (*ρ*-*r*), we are able to draw a upward sloping curve that equates U^S and U^L, as illustrated in Figure 2. It is notable that the short-term contract out-performs the long-term contract when (*ρ*-*r*) approaches zero. On the other hand, the long-term contract is preferable when the volatility σ is close to zero. When there is neither variation (σ = 0) nor financing benefit (*ρ*-*r*), there is no difference between the two contracts.

Figure 2
Choice between short-term and long-term contract without economic growth



2.5 Comparison of the two different schemes (contracts) under the growing economy assumption

In the previous subsections, the donor country's economy size was assumed to be fixed. The only incentive for the donor country to frontload aid is to take advantage of the difference between the time discount rate *ρ* and interest rate *r*. The greater difference between the two makes long-term contract more desirable. However, another important incentive source for frontloading aid should not be neglected: if the size of the economy increases over time, the donor country will anticipate the future growth of aid and will seek to smooth out the aid flow. This consumption-smoothing aspect is potentially important in the IFF proposal. In this section we examine this by relaxing the original assumption of fixed economy size, letting the economy instead grow at a constant rate *g*.

The optimization problem for the donor country under the short-term contract scheme remains the same except that the budget constraint is now different:

$$\underset{A_t, Q_t}{Max} \int_{t=0}^{T} \ln(\alpha A_t^{\theta_t} Q_t^{1-\theta_t}) e^{-\rho t} dt \tag{12}$$

$$\text{s.t. } A_t + Q_t = ke^{gt}$$

Solving the above optimization problem with the new budget constraint under the short-term contract scheme yields:

$$A_t{}^* = \theta_t ke^{gt}$$

$$Q_t{}^* = (1-\theta_t)ke^{gt} \tag{13}$$

The conclusion remains qualitatively the same as in subsection 2.3 and shows that under the short-term and flexible contract scheme, the donor country will allocate θ_t portion of total resources at that particular time to aid and the rest to domestic production.

A long-term contract in this scenario involves tying committed aid flows to the donor country's economic growth. Compared to the no-growth scenario where aid commitment is a fixed amount, this scenario in the contract term specifies the share of total GNP a donor country commits to foreign aid. A long-term contract allows the donor country to pre-specify its annual aid commitment A_0e^{gt}, which grows at the same pace as the economy. The optimization problem now becomes:

$$\underset{A_0, B_t}{Max} \int_{t=0}^{T} \ln(\alpha A_t^{\theta_t} Q_t^{1-\theta_t}) e^{-\rho t} dt \tag{14}$$

$$\text{s.t. } A_0 e^{gt} + Q_t = ke^{gt}$$

$$A_0 e^{gt} + B_t = A_t$$

$$\int_{t=0}^{T} B_t e^{-rt} dt = 0$$

$$A_t \geq 0$$

Solving the above optimization problem yields Equation (15) (the full mathematical solution given in Appendix C):

$$A_0{}^* = \bar{\theta} k$$

$$B_t{}^* = \bar{\theta} k (w' - e^{gt}) \quad \text{where } w' \equiv \frac{\rho(e^{(g-r)T} - 1)}{(r-g)e^{(\rho-r)t}(e^{-\rho T} - 1)} \geq 0 \tag{15}$$

Equation (15) is very similar to Equation (7), with Equation (7) being a special case of $g = 0$. The absolute amount of B_t^* is greater under the economic growth scenario ($g > 0$) than in the no economic growth scenario ($g = 0$). In view of Equation (15), by anticipating the future growth of resources to be allocated to aid, the donor country will frontload future aid by borrowing more in the first few periods, as illustrated in Figure 3.

Figure 3
Optimal financing amount of foreign aid when T = 30, r = 5%, and ρ = 6%

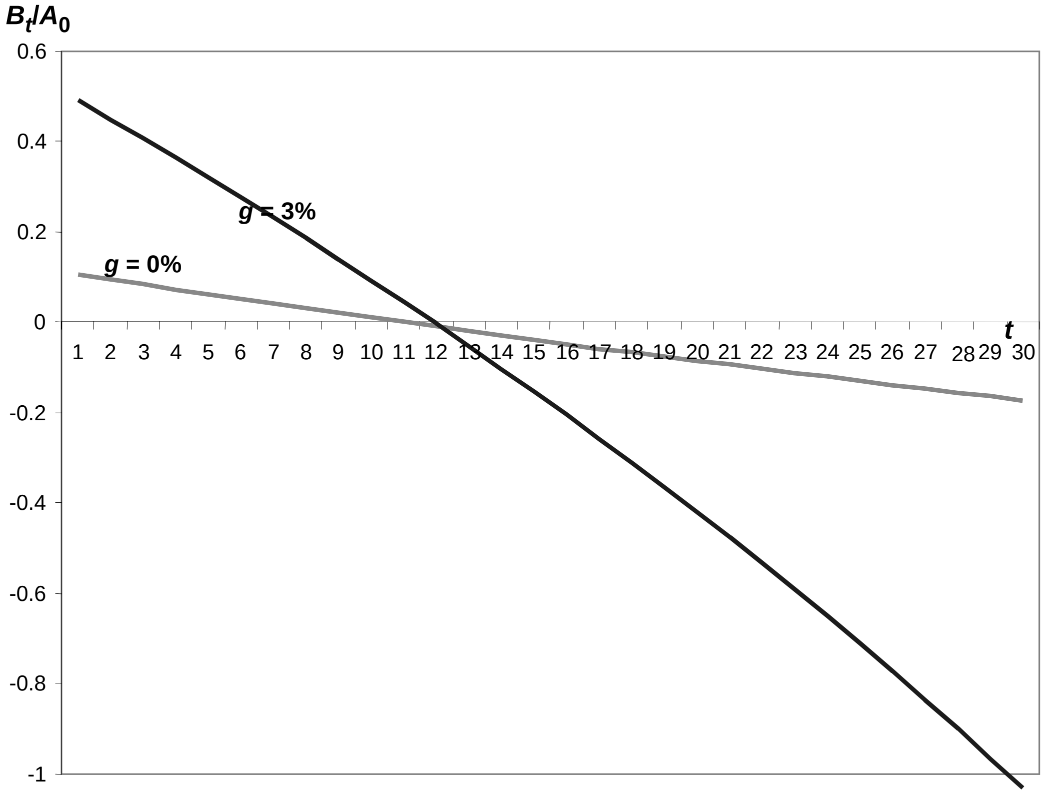
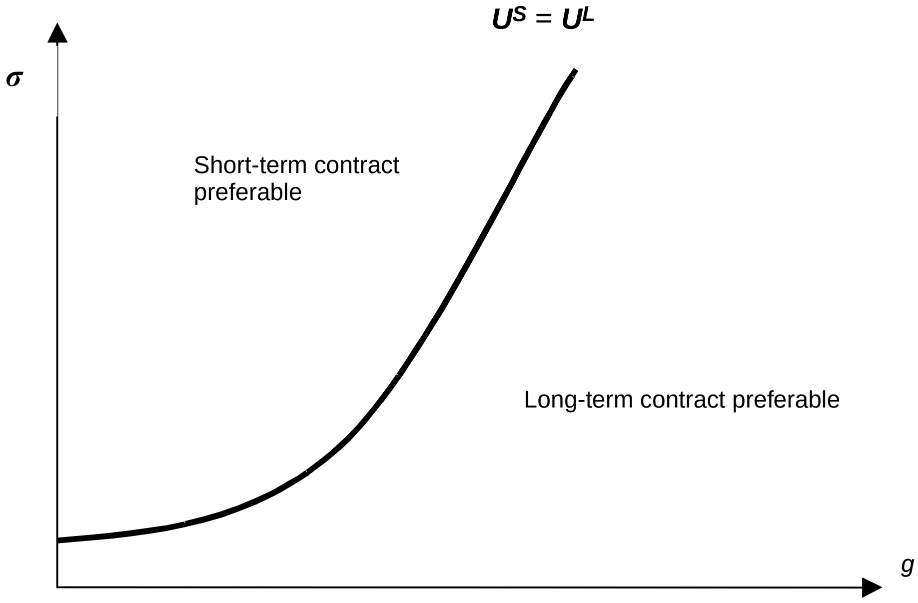


Figure 4
Choice between short-term and long-term contract with economic growth



Proposition 2

$\partial D / \partial g \leq 0$.

Proposition 2 means that the greater economic growth rate of the donor country, the more desirable IFF becomes relative to the status quo aid system. See Appendix C for a proof. The intuition behind this is that a faster growing economy makes aid smoothing more attractive. Long-term contract becomes more desirable as it enables financing/ frontloading future aid. This is illustrated in Figure 4.

3 Empirical illustration and Monte Carlo simulation

In the previous section, we established that the comparative advantages to the donor country of the status quo aid system and IFF depend on the magnitude of its economic growth (g), volatility in aid preference (σ), international capital market interest rate (r), and time discount rate (ρ). One of our main conclusions is that a long-term contract, like the IFF, is appreciated more when the economic growth rate is high and the aid preference volatility is low.

In this section, by examining the historical aid data[7] of the major donor countries from 1990 to 2003, we can provide empirical estimates of σ as well as the economic growth rate g. Given the prevailing interest rate r (5%) in the international capital market and the assumption of time discount rate ρ, we are able to gauge which donor countries will be more willing to choose the IFF option vis-à-vis the current short-term contract.

As shown earlier, Equations (5) and (13) in particular, θ, under the status quo system, is merely the ratio between foreign aid and total resources available. Therefore we take the aid-to-GNI ratio as an estimate of θ. Using the aid/GNI data from 1990 to 2003, we can estimate $\bar{\theta}$ and σ for each donor as follows:

$$\hat{\theta}_t = \frac{Aid_t}{GNI_t} \tag{16}$$

$$\hat{\hat{\theta}} = \frac{1}{m} \sum_{i=1}^{m} \hat{\theta}_{t-i}$$

$$\hat{\sigma} = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (\hat{\theta}_{t-i} - \hat{\hat{\theta}})^2}$$

The results are reported in Table 1. Columns 1 and 2 show the estimated average aid/GNI ratio and its standard deviation. The last column shows the average growth rate of the donor-country economy during the same period.

We run Monte Carlo simulations for each donor country to generate a random series of θ_t for 30 years ($T = 30$, as proposed in IFF) from a normal distribution $N(\hat{\hat{\theta}}, \hat{\sigma}^2)$.

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[7] Aid-to-GNI data are obtained from the OECD-DAC database (available online).

Given parameter assumptions of r and ρ, we are able to calculate the utility difference D between the short-term contract and the long-term one.[8] By running the simulation 1,000 times, we have 1,000 observations of D for each donor country. Based on the distribution of D observations, we are able to draw inference and test the hypothesis that $D > 0$ or $D < 0$. Table 2 shows the first moments of the simulation results and the tests for $D > 0$ (for positive means) and $D <0$ (for negative means) all return significant at < 0.01% level except Austria in the $\rho = 8\%$ case where the D is insignificant from zero at 10% significance level.

As we can see from Table 2, with 1 per cent difference between the time discount rate and the interest rate ($\rho = 6\%$ and $r = 5\%$), about half of the donor countries (including the United States) would prefer the short-term contract while the other half would favour the long-term contract. Countries with higher growth rates such as Ireland, Australia, and Norway generally prefer the long-term contract as predicted by Proposition 2. Countries with rather stable aid preferences such as New Zealand, Spain, and the United Kingdom generally have a stronger preference towards the long-term contract as predicted in Proposition 1.

Table 1
Donor countries' aid/GNI (mean and standard deviation) and economic growth rates, 1990-2003
(in per cent)

Country	$\hat{\bar{\theta}}$	$\hat{\sigma}$	$\hat{g}$
Australia	0.30	0.0483	3.15
Austria	0.21	0.0669	2.17
Belgium	0.39	0.0785	1.88
Canada	0.35	0.0858	2.58
Denmark	0.99	0.0565	2.03
Finland	0.41	0.1621	1.73
France	0.49	0.1210	1.76
Germany	0.31	0.0524	1.28
Greece	0.10	0.0912	2.58
Ireland	0.28	0.0791	6.39
Italy	0.21	0.0777	1.47
Japan	0.26	0.0401	1.62
Luxembourg	0.52	0.2051	4.30
Netherlands	0.82	0.0399	2.30
New Zealand	0.24	0.0183	2.68
Norway	0.94	0.1370	2.98
Portugal	0.27	0.0415	2.27
Spain	0.25	0.0275	2.66
Sweden	0.84	0.1070	1.77
Switzerland	0.35	0.0336	0.94
United Kingdom	0.30	0.0290	2.19
United States	0.14	0.0413	2.86

8 Matlab codes are available from the corresponding author upon request.

Table 2
Welfare comparison between the two contract schemes

Country	E(*D*) ρ = 6% r = 5%	E(*D*) ρ = 8% r = 5%	E(*D*) ρ = 14% r = 5%
Australia	-0.9810	-2.4861	-5.8083
Austria	1.1875	0.0059	-2.6956
Belgium	0.4730	-1.2161	-5.4658
Canada	0.4499	-1.3851	-5.5489
Denmark	-1.8297	-5.6217	-15.7015
Finland	3.9132	1.6182	-3.8823
France	1.2755	-0.9326	-6.3778
Germany	0.3236	-0.7710	-3.8429
Greece	3.7387	2.5098	0.1605
Ireland	-4.2547	-6.2348	-9.3381
Italy	2.0573	0.8903	-1.8598
Japan	0.0829	-0.8677	-3.5793
Luxembourg	1.3520	-2.5966	-10.2459
Netherlands	-2.0932	-5.4311	-13.8589
New Zealand	-0.7803	-1.8417	-4.3752
Norway	-2.8259	-7.4356	-17.6893
Portugal	-0.3031	-1.4782	-4.3510
Spain	-0.6546	-1.7923	-4.4531
Sweden	-0.3119	-3.5059	-12.1455
Switzerland	0.0503	-0.9283	-4.2013
United Kingdom	-0.5276	-1.7589	-4.8908
United States	0.2099	-0.5104	-2.2810

Note: E(*D*) is the mean value of the donor country's utility difference between the short-term contract (status quo) and the long-term contract system (IFF). A negative value indicates that the short-term contract is preferable and vice versa. All values are significant at 0.01% level except Austria in ρ = 8% case (insignificant from zero).

As the difference between the time discount rates and the interest rates increases to a moderate level (ρ = 8% and r = 5%), all countries would prefer a long-term contract such as the IFF. Four countries—Austria, Finland, Greece and Italy—are the exception, as none of these have either a very high growth rate or a stable aid preference. Austria is indifferent between the two contracts/options. Finland would have a strong preference towards the short-term contract mainly because of its second most volatile aid preference among all donors. Greece would favour short-term contracts mainly because of its relatively volatile aid preference in conjunction with only-medium growth rate. Italy would support short-term contracts mainly due to its combination of low growth rate and medium aid preference volatility. However, when the difference between the time discount rates and the interest rates increases further (ρ = 14% and r = 5%), all donor countries, with the exception of Greece, would prefer long-term contracts.

4 Concluding remarks

In this paper we have developed an analytical framework within which donor countries' endorsement or rejection decision of the recently proposed IFF is rationalized.

The current foreign aid system is portrayed as being similar to a series of short-term contracts, where donor countries are able to adjust the aid amount (i.e., consumption of foreign production) to respond to environmental changes (broadly defined to include public opinion, domestic situation, etc.). The advantage of this system is its flexibility.

Frontloading international aid, as proposed by the IFF proposal, has the benefit of smoothing out aid flows over time. The advantage of smoothing out consumption, theoretically speaking, comes from the difference between time discount rates and interest rates. The greater the difference, the more willing the donor is to frontload future aid. Another incentive for smoothing out aid is the future projections of aid growth, resulting from the donor country's economic growth. Anticipating future economic growth (therefore aid growth), the impatient donor will favour frontloading future aid.

In the model presented in this paper, donor countries evaluate these two contract schemes to determine whether or not to endorse the IFF proposal. Our empirical analysis, combining historical aid data and Monte Carlo simulations, shows the payoffs and relative advantages of these two contract schemes. It appears that with 1 per cent difference between the time discount rates and the interest rates (ρ = 6% and r = 5%), that about half of the donors would favour short-term contracts while the other half would favour the long-term contract. Countries with higher growth rates such as Ireland, Australia, and Norway, or countries with stable aid preference such as New Zealand, Spain and United Kingdom would generally side with the long-term contract. As the difference between the time discount rates and the interest rates increases, most countries would appear to prefer a long-term type contract, such as the IFF.

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Appendix A
Solution of the long-term contract problem

The donor country's problem is to maximize Equation (4), subject to three constraints.

$$\underset{A_0, B_t}{Max} \int_{t=0}^{T} \ln(\alpha A_t^{\theta_t} Q_t^{1-\theta_t}) e^{-\rho t} dt \qquad \text{(A-1)}$$

s.t. $A_0 + Q_t = k$; $A_0 + B_t = A_t$; $\int_{t=0}^{T} B_t e^{-rt} dt = 0$; $A_t \geq 0$

Since θ_t is unknown and the best estimate at contracting is its expectation $\overline{\theta}$,

First assuming the last condition satisfied, write the Lagrangian as follows:

$$L = \int_{t=0}^{T} [\ln\alpha + \overline{\theta}\ln(A_0 + B_t) + (1-\overline{\theta})\ln(k - A_0)] e^{-\rho t} dt - \lambda \int_{t=0}^{T} B_t e^{-rt} dt$$

FOC yields the following:

$$\int_{t=0}^{T} [\frac{\overline{\theta}}{A_0 + B_t} - \frac{(1-\overline{\theta})}{k - A_0}] e^{-\rho t} dt = 0 \qquad \text{(A-2)}$$

$$[\frac{\overline{\theta}}{A_0 + B_t}] e^{-\rho t} - \lambda e^{-rt} = 0 \quad \forall B_t \qquad \text{(A-3)}$$

$$\int_{t=0}^{T} B_t e^{-rt} dt = 0 \qquad \text{(A-4)}$$

From (A-3) we have: $\frac{\theta}{A_0 + B_t} = \lambda e^{(\rho - r)t}$ (A-5)

Substitute (A-5) into (A-2):

$$\int_{t=0}^{T} [\lambda e^{-rt} - \frac{(1-\overline{\theta})}{k - A_0} e^{-\rho t}] dt = 0$$

$$\lambda \frac{(1-e^{-rT})}{\text{r}} = \frac{(1-\overline{\theta})}{k - A_0} \frac{(1-e^{-\rho T})}{\rho} \qquad \text{(A-6)}$$

(A-5) and (A-6) yields:

$$B_t = \frac{\overline{\theta}}{\lambda e^{(\rho-r)t}} \text{-} A_0 = \frac{\overline{\theta}}{e^{(\rho-r)t}} \frac{(k - A_0)(1-e^{-rT})\rho}{(1-\overline{\theta})(1-e^{-\rho T})r} \text{-} A_0 \qquad \text{(A-7)}$$

Substitute (A-7) into (A-4):

$$\int_{t=0}^{T} [\frac{\overline{\theta}}{e^{(\rho-r)t}} \frac{(k - A_0)(1-e^{-rT})\rho}{(1-\overline{\theta})(1-e^{-\rho T})r} \text{-} A_0] e^{-rt} dt = 0$$

$$\frac{\overline{\theta}(k - A_0)(1-e^{-rT})\rho}{(1-\overline{\theta})(1-e^{-\rho T})r} \int_{t=0}^{T} [\frac{1}{e^{(\rho-r)t}}] e^{-rt} dt - \int_{t=0}^{T} A_0 e^{-rt} dt = 0$$

$$\frac{\overline{\theta}(k - A_0)(1-e^{-rT})}{(1-\overline{\theta})(1-e^{-\rho T})r} (1-e^{-\rho T}) - A_0 \frac{(1-e^{-rT})}{r} = 0 \qquad \text{(A-8)}$$

$$A_0{}^* = \frac{\overline{\theta}(k - A_0)}{(1-\overline{\theta})} = \overline{\theta} k \qquad \text{(A-9)}$$

$$B_t{}^* = \frac{\overline{\theta}}{\lambda e^{(\rho-r)t}} \text{-} A_0 = \frac{\overline{\theta}}{e^{(\rho-r)t}} \frac{(k - \overline{\theta}k)(1-e^{-rT})\rho}{(1-\overline{\theta})(1-e^{-\rho T})r} \text{-} \overline{\theta}k = \overline{\theta}k[\frac{(1-e^{-rT})\rho}{e^{(\rho-r)t}(1-e^{-\rho T})r} \text{-} 1] \qquad \text{(A-10)}$$

It is easy to verify $A_t = A_0{}^* + B_t{}^* \geq 0$

Appendix B
Welfare comparison between the short-term contract and the long-term contract

Donor's utilities under the two contract schemes and their difference D are defined in Equations (8)-(11).

$$D \equiv U^S - U^L = \int_{t=0}^{T} \left[\theta_t \ln \theta_t + (1-\theta_t)\ln(1-\theta_t) - \theta_t \ln w\bar{\theta} - (1-\theta_t)\ln(1-\bar{\theta}) \right] e^{-\rho t} dt \tag{B-1}$$

First we show that D is non-decreasing with σ. (B-1) can be rewritten as:

$$D = \int_{t=0}^{T} \left[[*] - \theta_t \ln w \right] e^{-\rho t} dt \quad \text{where } [*] \equiv \theta_t \ln \theta_t + (1-\theta_t)\ln(1-\theta_t) - \theta_t \ln \bar{\theta} - (1-\theta_t)\ln(1-\bar{\theta}) \tag{B-2}$$

As a special case when $w(t) = 1$ (i.e., $\rho = r$), (B-2) is further simplified as:

$$D = \int_{t=0}^{T} [*] e^{-\rho t} dt \tag{B-3}$$

It is easy to see that [*] function, therefore D, is non-decreasing with $|\theta_t - \bar{\theta}|$, or σ, and takes minimal value zero when $\theta_t = \bar{\theta}$. This also means that when $w = 1$, $D \geq 0$.

We then verify that D is non-increasing with $(\rho - r)$. For simplicity we do so by looking at the no variation case (σ = 0, [*] = 0, or $\theta_t = \bar{\theta}\ \forall\ t$). (B-2) is therefore simplified as:

$$D = \bar{\theta} \int_{t=0}^{T} \left[-\ln w \right] e^{-\rho t} dt = -\frac{\bar{\theta}}{\rho} [\ln \frac{\rho(1-e^{-rT})}{r(1-e^{-\rho T})}(1-e^{-\rho T}) + (r-\rho)(1-e^{-\rho T} - Te^{-\rho T})] \tag{B-4}$$

(B-4) is negative when $\rho > r$ and zero when $\rho = r$. Furthermore, given $\rho > r$, it is not difficult to verify from (B-4) $\partial D/\partial r > 0$ and $\partial D/\partial \rho < 0$.

Finally, we prove the existence of σ^* *and* $(\rho - r)^*$ such that $D = 0$. From above we see U^S is non-decreasing with σ while U^L non-decreasing with $(\rho - r)$ term, so there must exist a crossing point $\{\sigma^*, (\rho - r)^*\}$ where $U^S = U^L$. *Q.E.D.*

Appendix C
Optimal policies under the economic growth assumption

In this Appendix, we intend to show the optimal policy under IFF when the donor country's initial aid commitment is allowed to be tied to economic growth. We also prove that the utility difference between the short-term contract and IFF is further decreased with growing *g*.

$$\underset{A_0,B_t}{Max}\int_{t=0}^{T}\ln(\alpha A_t^{\theta_t}Q_t^{1-\theta_t})e^{-\rho t}dt \qquad \text{(C-1)}$$

s.t. $A_0e^{gt}+Q_t=ke^{gt}$; $\quad A_0e^{gt}+B_t=A_t$; $\quad \int_{t=0}^{T}B_te^{-rt}dt=0$; $\quad A_t\geq 0$

Since θ_t is unknown and the best estimate at contracting is its expectation $\overline{\theta}$,

Assuming the last condition satisfied, we write the Lagrangian as follows:

$$L=\int_{t=0}^{T}[\ln\alpha+\overline{\theta}\ln(A_0e^{gt}+B_t)+(1-\overline{\theta})\ln(ke^{gt}-A_0e^{gt})]e^{-\rho t}dt-\lambda\int_{t=0}^{T}B_te^{-rt}dt$$

FOC yields the following:

$$\int_{t=0}^{T}[\frac{\overline{\theta}e^{gt}}{A_0e^{gt}+B_t}-\frac{(1-\overline{\theta})}{k-A_0}]e^{-\rho t}dt=0 \qquad \text{(C-2)}$$

$$[\frac{\overline{\theta}}{A_0e^{gt}+B_t}]e^{-\rho t}-\lambda e^{-rt}=0 \quad \forall B_t \qquad \text{(C-3)}$$

$$\int_{t=0}^{T}B_te^{-rt}dt=0 \qquad \text{(C-4)}$$

From (C-3) we have:

$$\frac{\overline{\theta}}{A_0e^{gt}+B_t}=\lambda e^{(\rho-r)t} \qquad \text{(C-5)}$$

Substitute (C-5) into (C-2):

$$\int_{t=0}^{T}[\lambda e^{(g-r)t}-\frac{(1-\overline{\theta})}{k-A_0}e^{-\rho t}]dt=0$$

$$\lambda\frac{(1-e^{(g-r)T})}{\text{r-g}}=\frac{(1-\overline{\theta})}{k-A_0}\frac{(1-e^{-\rho T})}{\rho} \qquad \text{(C-6)}$$

(C-5) and (C-6) yields:

$$B_t=\frac{\overline{\theta}}{\lambda e^{(\rho-r)t}}\text{-}A_0e^{gt}=\frac{\overline{\theta}}{e^{(\rho-r)t}}\frac{(k-A_0)(1-e^{(g-r)T})\rho}{(1-\overline{\theta})(1-e^{-\rho T})(r-g)}\text{-}A_0e^{gt} \qquad \text{(C-7)}$$

Substitute (C-7) into (C-4):

$$\int_{t=0}^{T}[\frac{\overline{\theta}}{e^{(\rho-r)t}}\frac{(k-A_0)(1-e^{(g-r)T})\rho}{(1-\overline{\theta})(1-e^{-\rho T})(r-g)}\text{-}A_0e^{gt}]e^{-rt}dt=0$$

$$A_0{*}=\overline{\theta}k \qquad \text{(C-8)}$$

$$B_t{*}=\overline{\theta}k(w'-e^{gt}) \quad \text{where } w'\equiv\frac{(1-e^{(g-r)T})\rho}{e^{(\rho-r)t}(1-e^{-\rho T})(r-g)}\geq 0 \text{ with } \rho\geq \text{r} \qquad \text{(C-9)}$$

Verify $A_t^* = A_0^* e^{gt} + B_t^* = \bar{\theta} k e^{gt} + \bar{\theta} k(w' - e^{gt}) = \theta k w' \geq 0$

subsitute (C-8) and (C-9) back to utility difference function:

$$D = \int_{t=0}^{T} \left[\theta_t \ln \theta_t e^{gt} + (1-\theta_t)\ln(1-\theta_t)e^{gt} - \theta_t \ln w' \bar{\theta} - (1-\theta_t)\ln(1-\bar{\theta})e^{gt} \right] e^{-\rho t} dt$$

$$= \int_{t=0}^{T} \left[\theta_t \ln \theta_t + (1-\theta_t)\ln(1-\theta_t) + gt - \theta_t \ln w' \bar{\theta} - (1-\theta_t)\ln(1-\bar{\theta})e^{gt} \right] e^{-\rho t} dt \qquad \text{(C-10)}$$

when there is no fluctuation in θ_t (i.e. $\theta_t = \bar{\theta} \forall t$)

$$D = \int_{t=0}^{T} [gt - \bar{\theta} \ln w' - (1-\bar{\theta}) gt] = \int_{t=0}^{T} [-\bar{\theta} \ln \frac{(1-e^{(g-r)T})\rho}{e^{(g+\rho-r)t}(1-e^{-\rho T})(r-g)}] dt$$

g is similar to ρ in Appendix B $\therefore \partial D / \partial g \leq 0 \quad Q.E.D.$