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1. INTRODUCTION

In the measurement of poverty, Sen (1976) proposed the "normalization axiom" which has been used as the basis for deriving several alternative measures of poverty including that of Sen (Anand 1977, Kakwani 1980 and Clark, Hemming and Ulph 1981). In a recent note, Basu (1985) provided an axiomatic justification of the "normalization axiom" by means of three elementary axioms. He demonstrated that these three axioms are equivalent to Sen's normalization axiom. The purpose of the present note is to demonstrate that the theorem proved by Basu is incorrect, i.e., the three axioms proposed by him are insufficient to derive the normalization axiom.

The present note also comments on Sen's measure and proposes an alternative measure of poverty which, in some respect, is more desirable than that of Sen.

2. BASU'S DECOMPOSITION THEOREM

Suppose H is the head-count ratio which is the proportion of people whose income is below the poverty line and I is the income-gap ratio which measures the average income short-fall of the poor from the poverty line, then some function f(H,I) may be a reasonable measure of poverty provided all the poor have the same income. Three elementary axioms proposed by Basu (1985) are as follows:

Axiom 1: (a) $f(1,1) = 1$;

(b) $\lim_{H \to 0} f(H,I) = \lim_{I \to 0} f(H,I) = 0$

Axiom 2: $[H_1 - H_2 > H_3 - H_4] \to [f(H_1,I) - f(H_2,I) > f(H_3,I) - f(H_4,I)]$

Axiom 3: $[I_1 - I_2 > I_3 - I_4] \to [f(H,I_1) - f(H,I_2) > f(H,I_3) - f(H,I_4)]$

Basu proved that axioms 1, 2 and 3 lead to Sen's normalization axiom:

Axiom N: <u>If all the poor have the same income, then</u> $f(H,I) = HI$

One counter example is sufficient to disprove Basu's theorem. It is easy to show that

$$f(H,I) = \alpha H + \beta I + \gamma HI \qquad (1)$$

where $\alpha + \beta + \gamma = 1$, satisfies all three axioms 1 to 3. Obviously, Sen's normalization axiom is a special case of (1) when $\alpha = \beta = 0$.

Where did Basu go wrong? The answer is given below.

Axiom 3 implies that

$$f(H,I) = a(H) + b(H)I \qquad (2)$$

where a(H) and b(H) are same functions of H only. Note that as $I \to 0$, i.e., all poor have income equal to the poverty line, H must also approach zero.[1] Thus, equation (2) implies

$$\lim_{H \to 0} a(H) = 0$$

whereas Basu assumes a(H) is zero for all H which is clearly incorrect. Similarly, axiom 2 implies that

$$f(H,I) = c(I) + d(I)H, \qquad (3)$$

since as $H \to 0$, I will either be zero or negative. Since I measures the percentage mean income short-fall of the poor from the poverty line, it should never be negative. Therefore, as $H \to 0$, I must be zero which implies that

$$\lim_{I \to 0} c(I) = 0,$$

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1. The number of poor persons cannot be assumed to be fixed, when their incomes have been raised to the poverty line (as soon as a person's income has reached the poverty line, that person cannot be regarded as poor by definition).

whereas Basu assumes $c(I) = 0$ for all values of I.[2] It can easily be demonstrated that equations (2) and (3) in conjunction with axiom 1 lead to equation (1). Clearly then, some more axioms are needed to arrive at Sen's normalization axiom.

If axioms 2 and 3 are intuitively reasonable, axiom 4 proposed below could also be considered appealing.

Axiom 4: $[H_1 - H_2 > H_3 - H_4,\ I_1 - I_2 > I_3 - I_4] \rightarrow [f(H_1, I_1) - f(H_2, I_2) > f(H_3, I_3) - f(H_4, I_4)]$

It is obvious that Sen's normalization axiom does not satisfy axiom 4.

3. A COMMENT ON SEN'S POVERTY MEASURE

Sen's (1976) poverty measure makes use of three poverty indicators: the proportion of poor, the income-gap ratio, and the distribution of income among the poor. His measure is given

$$S = H\,[I + (1 - I)\,G]$$

where G is the Gini index of the income distribution among the poor. Differentiating S with respect to I and G, gives

$$\frac{\partial}{\partial G}\left(\frac{\partial S}{\partial I}\right) = -H < 0 \qquad (4)$$

which implies that the larger (smaller) the inequality of income among the poor the smaller (larger) the increase in poverty will be when I increases. Intuitively, this result should be other way round, i.e., the larger the income inequality, the greater the increase in poverty should be when the poverty-gap ratio increases. Thus, the derivatives in (4) should

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2. Even if I is allowed to be negative, Basu's theorem is still incorrect because in that case $c(I) = 0$ only for $I < 0$ which is not the same thing as assuming $c(I) = 0$ for all values of I.

be strictly positive instead of being negative. An alternative poverty measure which gives positive sign to these derivatives and at the same time satisfies all Sen's basic axioms may be written as

$$K = HI(1 + G)$$

which, of course, requires an axiomatic justification similar to that of Sen's measure.

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