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The social profitability of rural roads in a small open economy

Do urban agglomeration economies matter?

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Abstract: In the presence of agglomeration economies, the effects of a rural roads programme depend not only on the reduction in transportation costs, but also on the form of labour mobility. When financed by a poll tax on rural households, the wage will rise, accompanied by some return migration, provided both cross-price effects in production and consumption and agglomeration economies are sufficiently small. With empirically plausible elasticities of agglomeration economies, urban households may be worse off. A tax on exports provides a countervailing distortion, yielding them some relief, yet with rather small adverse effects on rural households. If mobility takes the form of rural–urban commuting, cheaper fares will promote the exploitation of agglomeration economies. An export tax may then improve urban welfare. Using the change in the value, at producer prices, of the rural sector's net supply vector as the measure of the programme's social profitability can yield serious errors.

Key words: rural roads, social profitability, transportation costs, agglomeration economies

JEL classification: H54, O18, O22, R13

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1 Introduction

Providing villages with all-weather roads promises to improve rural welfare. Farmers should enjoy better net terms of trade and all villagers, as consumers, should pay less for urban goods. There are also potential benefits in the form of better schooling and a faster trip to a clinic or hospital. A prime example of a nationwide scheme is India's *Pradhan Mantri Gram Sadak Yojana* (PMGSY). Launched in December 2000 to cover some 170,000 habitations, this programme is almost complete at the time of writing. Its ultimate cost was earlier put at in excess of US$50 billion (World Bank 2010).

While estimating these diverse, direct benefits is a central and demanding task,[1] the consequences of extending the whole network of rural roads for the wider economy have been rather neglected. Greatly expanded movements of goods and people will normally affect activities—and hence prices and wages—in towns, with associated effects on both rural and urban welfare. Neglecting these effects may lead to serious errors when evaluating the social profitability of large-scale programmes. In particular, by making rural life more attractive, such programmes may well stem rural–urban migration, thus slowing urban output and so reducing the efficiency with which it is produced when there are agglomeration economies. The authors of *Reshaping Economic Geography* (World Bank 2009), for example, do not address this possibility directly, although their strictures on the folly of limiting rural–urban migration (World Bank 2009: 140–42) rather lead one to infer that investing in rural roads, in whose financing the World Bank has been heavily involved, has at least one serious drawback.[2]

The object of this paper is to analyse the effects of such programmes on welfare in both town and country when there are agglomeration economies, paying particular attention to the mobility of rural labour and how the programme is financed.[3] This calls for a general equilibrium treatment, in which the reallocation of resources induced by improved rural roads depends on, *inter alia*, the extent to which goods are internationally tradeable at parametric world prices; for with such market opportunities, there is no lack of demand for the goods in question.

Consider, for example, a small open economy, in which all goods are thus tradeable, labour is intersectorally immobile, and unit transport costs between the ports, border crossings, and towns are constant. Then prices in towns will be independent of the condition of the rural road network when improvements therein are financed by lump-sum taxes on rural households. If socially profitable, a better network will improve rural welfare, but leave urban activity and welfare unchanged.

In practice, transport costs—and other barriers to trade—are so high that a whole variety of goods are neither exported nor imported, nor are they likely to be under any conceivable constellation of domestic productivity levels and tastes. Domestic demand is then more fully in play. The structure adopted here involves three goods, all freely traded internally, with 'iceberg' transport costs. Villagers produce a single, internationally tradeable good by means of labour, the two urban goods, and land. Urban firms

[1] Studies of the effects of rural roads programmes on rural output, incomes, and poverty in various developing countries include those by Fan et al. (2000), Jacoby (2000), Escobal and Ponce (2002), Jacoby and Minten (2009), Khandker et al. (2009), and Warr (2010). Stifel et al. (2016) estimate willingness to pay in Ethiopia. Bell and van Dillen (2014, 2018) investigate PMGSY's effects on transport costs, schooling, and morbidity in a sub-region of the Deccan Plateau. Asher and Novosad (2020) execute an empirically arduous and wide-ranging study of PMGSY's effects in six states. Using a regression discontinuity approach on a sample of some 11,400 villages, they arrive at the finding that the programme generated minor changes in agricultural outcomes, incomes, and assets, but has induced a substantial shift of workers out of agriculture.

[2] The authors of *Infrastructure for Development* (World Bank 1994) are largely concerned with improving efficiency in the provision of infrastructure. Rural roads receive little attention, and rural–urban migration is hardly mentioned in any connection.

[3] The financing of programmes is not considered at all in Bell (2018), and the formal treatment of the small open economy with a non-tradeable good is confined to the special case of immobile labour.

are concentrated in a single port-city. They produce a second such tradeable and a good that is traded only domestically, labelled the (internationally) non-tradeable. The former is produced by means of labour, the rural good, the non-tradeable, and capital; the non-tradeable, by unassisted labour. The production of both goods is subject to Marshallian external economies.[4] Rural households are extended families, whose working members are mobile, thus augmenting the supply of labour to urban firms. Urban households are engaged only in urban production.

The roads programme reduces transportation costs between the rural hinterland and the port-city. In order to keep things tractable, certain effects are ruled out. Schooling is unaffected, as is health, whatever the levels of air and water pollution, as well as those personal contacts that further the propagation of diseases.[5] Congestion is treated as an external diseconomy, and agglomeration economies are represented as net of the latter. Trade and transport are competitively organized.[6]

The lack of all-weather rural roads does not prevent rural workers from migrating to seek urban employment: they simply take up residence in the towns, though not necessarily permanently. The provision of such roads may, however, make daily commuting an attractive proposition. This variant is treated separately.[7]

Underpinned by theoretical results, the sizes of diverse effects are explored using numerical examples with Cobb–Douglas technologies and preferences. Employing a constellation of arguably plausible parameter values, one can compute the equivalent variation with and without agglomeration economies, with poll and export taxes as alternative means of finance. In this iso-elastic world, improving the network of rural roads will generate substantially smaller aggregate benefits when the elasticity of agglomeration efficiencies is at the upper end of empirical estimates and the programme is financed by poll taxes. An export tax performs much better. The sectoral distribution of the aggregate is rather sensitive to both agglomeration economies and the form of taxation. Estimating the aggregate as the change in the value, at producer prices, of the rural sector's net supply vector may also yield serious errors.

The plan of the paper is as follows. Section 2 sets out the basic structure, treating the rural and urban sectors in detail. Poll taxes constitute the benchmark. Section 3 deals with prices and the wage in equilibrium, laying the basis for the analysis of the programme's effects on social welfare in Section 4. The two main alternatives—export taxes and commuting—are treated in Section 5. Illustrative numerical examples of all variants follow in Section 6. The robustness of the findings to the assumptions about substitution in consumption, family structure, congestion, and to the values of key parameters is examined in Section 7. The main conclusions are drawn together in Section 8.

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[4] In keeping with the present paper's central concern with public finance, a simple specification is chosen. For extensive surveys of the theoretical and empirical literature on agglomeration economies, see Behrens and Robert-Nicoud (2016) and Combes and Gobillon (2016), respectively. Cottineau et al. (2016) demonstrate, using French data, that the size of agglomeration economies depends on the definition of what is 'urban'.

[5] In a broad-ranging survey of urbanization that dwells on the distinct possibility that there are too many mega-cities, Henderson (2002) emphasizes the costs of congestion and pollution.

[6] Casaburi et al. (2013) analyse various market structures with reference to rural Senegal. In their partial equilibrium framework, rural output is assumed to be fixed, as are the only urban variables—namely, the urban prices of rural goods.

[7] It is motivated by Asher and Novosad's (2020) finding that PMGSY has induced a substantial shift of workers out of agriculture, whereby it should be noted that the said finding involves a local average treatment effect.

2 The model

A small open economy comprises a port-city and its rural hinterland. Villagers produce good 1, whose FOB (free on board) world price, p_1^*, is parametrically given. What they do not consume themselves, they sell to agents in the city, where goods 2 and 3 are produced. The marketed surplus of good 1 can be consumed by urban households, used as an input in the production of good 2, or exported. The world CIF (cost, insurance, and freight) price of good 2, p_2^*, is also parametrically given. Like land in the rural sector, let there be a specific urban fixed factor, capital, which is used in the production of good 2, so that both goods will be produced domestically. Initially, neither good is taxed.

Good 3 is tradeable domestically, but not internationally. The port-city is, in principle, independent of its hinterland; for goods 1 and 2 can be traded internationally, and good 3 can be produced there to satisfy any urban demand. If the city does not trade with its hinterland, it must produce and export good 2 in order to meet its needs for good 1. The hinterland, in contrast, is not independent of the city; for although villagers can export good 1 in exchange for imports of good 2 (both through the port-city), they must trade with the city in order to obtain good 3. In equilibrium, therefore, some of the city's demand for good 1 must be met from domestic production. Given that the economy is still rather agrarian in nature, let the rural sector's endowment of land be so large that, in equilibrium, good 1 is exported and good 2 imported, an assumption that does not necessarily rule out some domestic production of good 2.

The two locations—rural hinterland and port-city—are denoted by the index $k = 1,2$, respectively, and the price of good i in location k by p_{ik}. With domestic prices tethered to world prices by arbitrage and domestic transport costs, the farm-gate price of good 1 is $p_{11} = (1-\tau_1)p_1^*$, where τ_1 is the fractional (iceberg) cost of transporting good 1 to the port-city. Villagers pay $p_{21} = (1+\tau_2)p_2^*$ and $p_{31} = (1+\tau_3)p_3$ for goods 2 and 3, respectively. Households and firms in the city face the price vector $\mathbf{p}_2 = (p_1^*, p_2^*, p_3)$

All households supply their labour endowments completely inelastically, but whereas rural labour is mobile, urban households' endowments of capital are assumed to be such that it is never attractive for their members to take up rural employment. It is also assumed that in all allocations, some workers engaged in urban production are members of rural households.

Various possibilities—and complications—arise from rural–urban migration. If villagers commute to urban jobs, they pay fares and lose time in travelling; and if they buy goods in the towns, part of their families' total expenditure is made at urban prices. If, instead, they move to towns, they may lose their claims on the imputed rents arising from the family's fixed endowments (principally land); and in that event, a new urban household is formed, but without claims on the incomes derived from the urban fixed factor. Then again, the rural household may remain an extended family unit, pooling all income, but making some expenditures at urban prices.

There is no space to go through all such variations; the following will serve as a benchmark. Suppose migrants remain members of the extended rural family,[8] and that all rural family expenditures are made at village prices—a simplification that is defensible if urban and rural prices do not differ too strongly and migrants make up a sufficiently small fraction of the population belonging to rural households. The latter decide how to allocate their labour between the family farm and urban jobs.

[8] An alternative family structure is discussed in Section 7.2.

Daily commuting is ruled out. It involves the complication of expenditure of time and cash on transport, but the assumption that all rural family expenditures are made at village prices is then innocuous. This variant is treated in detail in Section 5.2.

The government now undertakes a rural roads programme, which reduces transport costs between the port-city and its hinterland. This improvement in the network is produced by unassisted rural workers, and in keeping with the above assumptions on labour mobility, they are paid the going urban wage w. Let this programme be financed by a poll tax on rural households.

2.1 The rural economy

Rural households, which are assumed to be identical, choose inputs of goods 2 and 3 and labour in rural production so as to maximize their net revenues, taking prices as parametrically given. In aggregate,

$$R_1 = p_{11}Y_1 + p_{21}Y_{21} + p_{31}Y_{31} - wL_1, \tag{1}$$

where Y_1 denotes the aggregate output of good 1 and, with the usual convention that inputs have a negative sign, $Y_{i1}\,(<0, i=2,3)$ denotes the aggregate output of good i in sector 1. Aggregate rural income is $M_1 = R_1 + w\overline{L}_1 - T_1$, where $\overline{L}_1$ is the aggregate rural labour endowment and $T_1\ (= wL_{1p})$ is the sum paid in poll taxes to finance the programme's requirement of L_{1p} units of labour. Those individuals who are resident in the city supply $\overline{L}_1 - L_1 - L_{1p} \geq 0$ units of labour there.

Since rural households are assumed to be identical, the individual household's decision problem may be written in the form

$$\max_{(\mathbf{X}_1, L_1, Y_{12}, Y_{13})} U_1(\mathbf{X}_1) \quad \text{s.t. } M_1 \geq \mathbf{p}_1\mathbf{X}_1,\ (\mathbf{X}_1, L_1, -Y_{12}, -Y_{13}) \geq \mathbf{0},\ (1), \tag{2}$$

where $\mathbf{X}_1 = (X_{11}, X_{21}, X_{13})$ denotes the aggregate rural consumption bundle. By assumption, the rural sector is a net supplier of good 1: $Y_1 > X_{11}$. Problem (2) is separable in the spheres of production and consumption. Applying the envelope theorem to the former (maximizing M_1 is equivalent to maximizing R_1), we obtain

$$dM_1^0 = (Y_1, Y_{21}, Y_{31}) \cdot \mathbf{dp}_1 + (\overline{L}_1 - L_1^0)dw - T_1, \tag{3}$$

where the second-order term $L_{1p}dw$ can be neglected.

Let the programme yield a small reduction $\boldsymbol{d\tau}\,(\ll \mathbf{0})$[9] in $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$. Then the resulting change in the rural price vector is $\mathbf{dp}_1 = (-p_1^* d\tau_1, p_2^* d\tau_2,\ p_3 d\tau_3 + (1+\tau_3)dp_3)$. The programme brings about an increase in the output price and a fall in the input price of good 2. The sign of the change in p_{31} is ambiguous, despite the reduction in τ_3; for p_{31} also depends on w, which may rise or fall. On balance, an increase in rural output and income is the likely outcome, but that remains to be established.

It follows from the envelope theorem and $Y_1 > X_{11}$ that, *ceteris paribus*, the programme will make villagers better off if it satisfies two conditions. First, that the poll tax $T_1 = wL_{1p}$ be not so large as to reduce M_1. Second, that the programme not lead to a sufficiently sharp fall in w whenever some villagers already have urban jobs. Such a fall would, however, almost surely induce a sharp fall in p_{31}. Hence, the effects of changes in the urban wage and producer price of good 3 require detailed examination.

Let $V_1(M_1, \mathbf{p_1}, w)$ denote the rural sector's indirect utility function. Then

$$dV_1 = (\nabla M_1 \cdot (\mathbf{dp}_1, dw)) \cdot \frac{\partial V_1}{\partial M_1} + \nabla V_1 \cdot \mathbf{dp}_1.$$

[9] All elements of $\boldsymbol{d\tau}$ are negative.

Using Hotelling's lemma and Roy's identity, and collecting terms, we obtain

$$dV_1 = [(Y_1 - X_{11})dp_{11} + (Y_{21} - X_{21})dp_{21} + (Y_{31} - X_{31})dp_{31} + (\overline{L}_1 - L_1 - L_{1p})dw - T_1] \cdot \frac{\partial V_1}{\partial M_1}.$$

The rural economy supplies the urban economy with $\overline{L}_1 - L_1 - L_{1p}$ units of labour. The vector of its *net* supplies of goods to the urban economy is $\mathbf{Z}_1 = (Y_1 - X_{11}, Y_{21} - X_{21}, Y_{31} - X_{31})$. The change in V_1 induced by $d\boldsymbol{\tau}$ may be written

$$\begin{aligned} dV_1 &= [\mathrm{diag}(-p_1^*, p_2^*, p_3)\mathbf{Z}_1 \cdot d\boldsymbol{\tau} + (1+\tau_3)Z_{31}dp_3 + (\overline{L}_1 - L_1)dw - T_1] \cdot \frac{\partial V_1}{\partial M_1} \\ &\equiv [B_1^\tau - T_1 + (1+\tau_3)Z_{31}dp_3 + (\overline{L}_1 - L_1)dw] \cdot \partial V_1/\partial M_1. \end{aligned} \quad (4)$$

The sum of the direct effects of the reduction in $\boldsymbol{\tau}$, evaluated at the urban prices $\mathbf{p}_2$ and denoted by B_1^τ, is positive, for each of its three terms is positive. If $B_1^\tau > T_1$, the programme would increase rural income at those prices. Its effect on V_1 through the induced changes in the urban wage and producer price of good 3 is ambiguous if, as is to be expected, these prices move together. Ignoring second-order terms and holding quantities constant, $(1+\tau_3)Z_{31}dp_3 + (\overline{L}_1 - L_1)dw$ is the change in migrant workers' earnings minus the change in rural households' expenditures on good 3.

2.2 The urban economy

Sector 2 comprises n, numerous and identical firms, which produce good 2 by means of labour, capital, and inputs of goods 1 and 3. Firms choose inputs so as to maximize profits, taking prices as parametrically given. Labour and capital are necessary and substitutable in the production of good 2; but the associated unit input requirements for goods 1 and 3 are fixed, at a_{12} and a_{32}, respectively. Production is also subject to Marshallian external economies: the higher is the level of total employment in that sector, the higher is the level of efficiency of each firm's factor inputs.[10] If firm j, say, chooses the factor bundle (l_j, k_j), let its level of output—given an efficient input bundle of goods 1 and 3—be given by $A_2\varphi(L_2)f(l_j, k_j)$, where $L_2 = l_j + L_2(-j)$ is the level of total employment in sector 2, φ and f are increasing and differentiable in their arguments, f is homogeneous of degree one, and A_2 is a constant. The firm's revenue, net of the costs of intermediate inputs and wages, is $R_j = \hat{p}_2 A_2 \varphi(L_2) f(l_j, k_j) - wl_j$, where $\hat{p}_2 = p_2^* - p_1^* a_{12} - p_3 a_{32}$ is the value added per unit of gross output. The net revenue is returned to the households that supply k_j.

When choosing a production plan, let each firm make Nash conjectures concerning the plans of the rest. Then, ignoring the negligible influence on φ of its contribution to L_2, firm j will choose l_j so as to equate marginal (private) revenue with marginal (private) cost: $\hat{p}_2 A_2 \varphi(L_2) \cdot \partial f(l_j, k_j)/\partial l_j = w$. We confine our attention to symmetric equilibria. Each firm then chooses the same input of capital, k_2, so as to exhaust the aggregate endowment thereof, and the same level of employment, l_2^0, where the latter satisfies the foregoing marginal condition and, in aggregate, $L_2 = nl_2^0$:

$$\hat{p}_2 A_2 \varphi(nl_2^0) \cdot (\partial f(l_j, k_2)/\partial l_j)|_{l_j = l_2^0} = w, \ \forall j. \quad (5)$$

Good 3 is produced by means of labour alone, and may also be subject to Marshallian external economies. The unit input requirement of labour is given by $l_3 \cdot g(L_2, L_3)$, where g is non-increasing and differentiable in both arguments and l_3 is a constant. The sector is competitively organized, so that price is equal to unit cost,

$$p_3 = l_3 \cdot g(L_2, L_3) \cdot w, \quad (6)$$

and there are no net revenues to return to urban households.

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[10] Capital is supplied inelastically, so that total output will move with total employment.

The relation between employment and the wage is complicated by the presence of external economies. Differentiating Equation (5) totally, noting Equation (6), and rearranging, we obtain

$$\begin{aligned}[(1+p_3a_{32}/\hat{p}_2]dw &= \left[\hat{p}_2A_2(\varphi(nl_2^0)f_{ll}+n\varphi'\cdot f_l)-a_{32}wl_3\cdot(w/\hat{p}_2)\cdot g_l\right]dl_2^0\\ &- [p_3a_{32}A_2\varphi(nl_2^0)f_l\cdot(g_{L_3}/g)]dL_3\equiv\xi_2\cdot dl_2^0-\xi_3\cdot dL_3, \end{aligned}\tag{7}$$

where $f_l=\partial f(l_2^0,k_2)/\partial l_2^0$, $g_l=\partial g(nl_2^0,L_3)/\partial l_2^0$ and $g_{L_3}=\partial g(nl_2^0,L_3)/\partial L_3$. If g is decreasing in L_3, w and L_3 will move in the same direction for any given L_2. The sign of ξ_2 is ambiguous. In the absence of agglomeration economies, the remaining term, $\hat{p}_2A_2(\varphi(nl_2^0)f_{ll}$, is negative, reflecting the strict concavity of f in labour alone. In that event, w and l_2^0 will move in opposite directions. The other two terms, which arise from agglomeration economies, are both positive.

Using Equations (5) and (6) once more, some manipulation yields ξ_2 as a function of a weighted sum of certain elasticities:

$$\xi_2=\frac{w}{l_2^0}\left[\frac{l_2^0f_{ll}}{f_l}+\frac{nl_2^0\cdot\varphi'}{\varphi}-\frac{p_3a_{32}}{\hat{p}_2}\cdot\frac{l_2^0\cdot g_l}{g}\right]\equiv w\zeta/l_2^0.\tag{8}$$

The partial derivative f_l is proportional to the marginal product of labour. The expression $l_2^0f_{ll}/f_l$ is the elasticity thereof with respect to labour, evaluated at l_2^0. The term $nl_2^0\cdot\varphi'/\varphi$ is the elasticity of the function representing the effects of external economies on the efficiency of production of good 2. The third term is the product of the ratio of the cost of inputs of good 3 to the price of value added and the *partial* elasticity of the function representing the effects of external economies on the efficiency of production of good 3, arising from total employment in sector 2. This establishes:

Lemma 1. *If agglomeration economies affect efficiency in the production of the non-tradeable good only through employment in sector 2 and rural–urban migration is governed by the mechanism described in Section 2.1, then employment in sector 2 and the wage rate will move in the opposite or the same direction according to* $\zeta\lesseqgtr 0$.

In practice, ζ is almost surely negative; for empirical estimates of the elasticities associated with agglomeration effects are quite close to zero (see Section 6), whereas the own-elasticity of the marginal product of labour is not. The ratio of the cost of inputs of good 3 to the price of value added is at most 1 if, and only if, $p_2^*-p_1^*a_{12}-p_3a_{32}\geq p_3a_{32}$. Since the left-hand side is the sum of payments to labour and the fixed factor per unit of gross output, this condition, too, almost surely holds empirically.

Urban households are identical, each endowed with labour and capital. Their aggregate income is therefore

$$M_2=n[\hat{p}_2A_2\varphi(nl_2^0)\cdot f(l_2^0,k_2)-wl_2^0]+w\overline{L}_2,\tag{9}$$

where $\overline{L}_2$ is their total endowment of labour. As consumers, their decision problem is

$$\max_{(\mathbf{X}_2|\mathbf{p}_2,w)} U_2(\mathbf{X}_2)\quad \text{s.t. } M_2\geq\mathbf{p}_2\mathbf{X}_2,\ \mathbf{X}_2\geq\mathbf{0},\ (9).$$

Let the corresponding indirect utility function be denoted by $V_2(M_2,\mathbf{p}_2)$. Proceeding as in Section 2.1 and noting that $[Y_3+(1+\tau_3)Z_{31}]$ is the total absorption of good 3 within the urban economy itself, Roy's identity and some manipulation yield

$$\begin{aligned} dV_2 &= \{[\overline{L}_2-nl_2^0-l_3\cdot g(L_2,L_3)(Y_3+(1+\tau_3)Z_{31})]dw+\frac{\hat{p}_2Y_2}{nl_2^0}\cdot\frac{nl_2^0\cdot\varphi'}{\varphi}ndl_2^0\\ &- wl_3[Y_3+(1+\tau_3)Z_{31}]dg\}\frac{\partial V_2}{\partial M_2}, \end{aligned}\tag{10}$$

where $\hat{p}_2Y_2/nl_2^0$ is value added per worker in sector 2, $nl_2^0\cdot\varphi'/\varphi$ is the elasticity of the function yielding enhanced factor productivity through agglomeration economies in that sector, and $dg=g_l\cdot ndl_2^0+$

$g_{L_3}dL_3$. The level of aggregate urban employment is $nl_2^0 + l_3 \cdot g(L_2, L_3)Y_3 > \overline{L}_2$, since some workers from rural households are engaged in urban production. In equilibrium, the urban economy's excess demand for labour over its own endowment is exactly met by migrants' supply. The term $-l_3 g \cdot (1+\tau_3)Z_{31}$ is the labour needed to meet rural demand for good 3. In view of Lemma 1, this establishes:

Lemma 2. *If agglomeration economies affect efficiency in the production of the non-tradeable only through employment in sector 2, then urban welfare is decreasing in the wage rate if the migrants' labour supply is greater than the labour needed to meet rural demand for good 3.*

Remark. Agglomeration economies strengthen this effect through the terms involving φ' and g_l. Thus, the lemma will also hold if the migrants' labour supply is smaller than the labour needed to meet rural demand for good 3 by a sufficiently small amount.

3 The wage rate and prices in equilibrium

Markets clear through a flexible wage rate and price of the non-tradeable, which respond mutually and simultaneously to the reductions in rural–urban transport costs.

The net supply of good 1 at the farm gate is $Z_{11} = Y_1(\mathbf{p}_1, w) - X_{11}(M_1, \mathbf{p}_1)$, which becomes $(1-\tau_1)Z_{11}$ at the port-city. The programme will yield an increase in the latter directly by reducing τ_1. It will also affect supply and demand through the changes it induces in prices and incomes:

$$d[(1-\tau_1)Z_{11}] = -Z_{11}d\tau_1 + (1-\tau_1)[\nabla Y_1 \cdot (\mathbf{dp}_1, dw) - \nabla X_{11} \cdot (dM_1, \mathbf{dp}_1)]. \quad (11)$$

Lemma 3. *Suppose all goods are normal and sufficiently good substitutes in consumption such that $\nabla X_{11} \cdot (0, \mathbf{dp}_1) \leq 0$. If, at a constant wage and in the absence of agglomeration economies, $(B^\tau - T_1 + p_1^* Z_{11} d\tau_1)$ would not be too large, then the programme would generate an increase in the amount of good 1 delivered to the port-city.*

Proof. See Appendix.

Remark. In practice, the marginal propensity to spend on good 1 is much less than 1, and the component $-p_1^* Z_{11} d\tau_1$ accounts for the lion's share of the direct net benefits $B^\tau - T_1$, so that $B^\tau - T_1 + p_1^* Z_{11} d\tau_1 < 0$ is certainly possible. Thus, the auxiliary conditions in Lemma 3 are not especially restrictive.

If the whole of such an increase is not absorbed in the port-city, exports of good 1, E_1, will necessarily increase. By Walras' Law, the value of the economy's net exports at world prices must be zero in equilibrium: $p_1^* E_1 + p_2^* E_2 = 0$. Together with Lemma 3, this identity underpins the argument yielding:

Proposition 1. *The programme will almost always induce an increase in the wage, even if good 2 is highly substitutable in rural production and consumption.*

Proof. See Appendix.

Lemma 1 and the continuity of g then yield:

Corollary 1. *If g is sufficiently weakly decreasing in L_2 and L_3, and $\zeta < 0$, then Proposition 1 will hold in the presence of agglomeration economies.*

Lemma 2 yields:

Corollary 2. *The programme will improve urban welfare only if the labour supplied by migrants is smaller than the labour needed to meet rural demand for good 3.*

An analysis of the condition for the labour market to clear yields further insights. This condition can be written in the form

$$L^r + L^u \equiv [L_1 + gl_3(1+\tau_3)(-Y_{31}+X_{31})] + [L_2 + gl_3(a_{32}Y_2 + X_{32})] = \overline{L}_1 + \overline{L}_2 - L_{1p}, \tag{12}$$

where the first expression in brackets is the total employment of labour engaged in producing good 1 and satisfying rural demand for good 3, the second is the employment of labour to meet the corresponding requirements of urban production and consumption, and L_{1p} is zero in the absence of the programme. When the latter is undertaken, the sum of such employments must fall by the programme's own requirement of labour:

$$dL^r + dL^u \equiv d[L_1 + gl_3(1+\tau_3)(-Y_{31}+X_{31})] + d[L_2 + gl_3(a_{32}Y_2 + X_{32})] = -L_{1p}. \tag{13}$$

A consideration of cross-price effects points to the following result:

Proposition 2. *Let the technology for producing good 1 and the preferences of rural households be Cobb–Douglas, and let urban households' preferences be such that their expenditure share for good 3 varies sufficiently weakly with the wage.*[11] *Then, in the absence of urban agglomeration economies, the programme will induce an increase in the wage if it does not reduce rural income net of the poll tax, T_1, needed to finance it.*

Proof. See Appendix.

The assumptions about substitutability are stronger than those in Proposition 1, and yield a sharper result. They can clearly be weakened. Corollary 1 also holds.

4 Changes in welfare

Let the social welfare function, W, have as arguments V_1 and V_2, and be increasing and differentiable in both. Hence, from Equations (4) and (10), and noting that $dp_3 = l_3(gdw + wdg)$, the change in welfare induced by the programme is

$$\begin{aligned} dW &= \beta_1[B^r + (1+\tau_3)Z_{31}dp_3 + (\overline{L}_1 - L_1 - L_{1p})dw - T_1] \\ &+ \beta_2[(\overline{L}_2 - nl_2^0)dw - (Y_3 + (1+\tau_3)Z_{31})dp_3 + \hat{p}_2 Y_2 \cdot (\varphi'/\varphi)ndl_2^0], \end{aligned} \tag{14}$$

where $\beta_k = W_k \cdot \partial V_k / \partial M_k$ is the social value of a small increase in income accruing to households in location k.

It is instructive to examine the special case in which distributional considerations are put aside ($\beta_1 = \beta_2 = 1$). The sum of the two expressions in brackets is then the change in aggregate, net (money) benefits. Since $L_3 = \overline{L}_1 + \overline{L}_2 - L_1 - L_{1p} - nl_2^0$,

$$B = (B_1^\tau - T_1) + L_3 dw - Y_3 dp_3 + \hat{p}_2 Y_2 \cdot (\varphi'/\varphi)ndl_2^0. \tag{15}$$

The term $B_1^\tau - T_1$ is the net benefit accruing to rural households that arises directly from the reduction in transport costs, which their labour has produced and financed, with the sector's net supplies valued at *urban* prices. This is a consequence of the assumptions that world prices are parametrically given and

[11] This need hold only locally. The limiting general case is Cobb–Douglas.

that good 3 is produced exclusively in the port-city. The economy's producer price vector is therefore $\mathbf{p}_2$, and B_1^τ is the change in the value of the rural sector's net supply vector at $\mathbf{p}_2$.

The term $L_3 dw - Y_3 dp_3$ arises from the programme's effects on the wage and the price of good 3, which are related by Equation (6). From the latter, we have

$$L_3 dw - Y_3 dp_3 = -wl_3 g(L_2, L_3) \cdot Y_3 \cdot (dg/g) = -wL_3 \cdot (dg/g) = -p_3 Y_3 \cdot (dg/g), \tag{16}$$

and hence, from Equation (15):

Proposition 3. *In the absence of any external economies, the programme will generate the aggregate net benefit $B_1^\tau - T_1$.*

The intuition for this familiar result is that facing a parametric world price for good 2, firms in that sector are never constrained by domestic demand. The production of good 3, in contrast, is wholly driven by it, and if there are no agglomeration economies, the programme's gross benefits are just B_1^τ, provided the change in τ is sufficiently small.

Allocations will be Pareto-efficient in the setting of Proposition 3. That will not hold in the presence of agglomeration economies; for even if $g_{L_3} = 0$, $dg = g_l \cdot dl_2^0$, which is positive in virtue of Corollary 1. If $g_{L_3} = 0$, then Equation (16) yields

$$L_3 dw - Y_3 dp_3 + \hat{p}_2 Y_2 \cdot (\varphi'/\varphi) n dl_2^0 = [-wL_3 g_l/g + \hat{p}_2 Y_2 \cdot (n\varphi'/\varphi)] dl_2^0 < 0.$$

By inducing an increase in p_3, an increase in w reduces domestic demand for good 3, and hence the level of efficiency in its production. From Equations (7) and (8), the absolute size of the ensuing loss is

$$\Lambda = \frac{p_2^* - p_1^* a_{12}}{\hat{p}_2 w \zeta} \left(p_3 Y_3 \cdot \frac{l_2^0 g_l}{g} - \hat{p}_2 Y_2 \cdot \frac{n l_2^0 \cdot \varphi'}{\varphi} \right) dw, \tag{17}$$

wherein it is seen that the components of urban value added are weighted by their respective elasticities with respect to agglomeration economies. To summarize:

Proposition 4. *If the production of goods 2 and 3 is subject to external economies, but the latter only through activity in sector 2, and the programme is financed by a poll tax, then its net benefit is smaller than $B^\tau - T_1$ in the amount Λ.*

5 Two variations: export taxes and commuting

How robust are the foregoing findings to certain, arguably key, assumptions? First, there is the financing of the programme by means of a rural poll tax, which is analytically clear, but almost surely a non-starter in practice. Second, rural labour is fully mobile in the sense that migration is unhindered, but daily commuting to urban jobs is ruled out. A discussion of robustness to other assumptions is deferred to Section 7.

5.1 An export tax

A tax on exports to finance a rural roads programme would be in keeping with the aim of having the principal beneficiaries pay for it. Although distortionary in itself, such a tax would tend to reduce the programme's contractionary effects on urban activity; for it would directly counteract the programme's effect on the price of good 1 at the farm gate, thus making return migration less attractive, and also lower the price of good 1 in the port-city.

The financing requirement is now $t_1 p_1^* E_1 = wL_{1p}$, where t_1 denotes the ad valorem rate on exports; it is endogenously determined. The farm-gate price is $p_{11} = (1-\tau_1-t_1)p_1^*$; so that $\mathbf{dp_1} = (-p_1^*(d\tau_1 + t_1), p_2^* d\tau_2, p_3 d\tau_3 + (1+\tau_3)dp_3)$. Proceeding as in Section 2.1, T_1 no longer enters Equation (3), being levied indirectly. Thus, Equation (4) becomes

$$dV_1 = [B_1^\tau - t_1 p_1^* Z_{11} + (1+\tau_3)Z_{31}dp_3 + (\overline{L}_1 - L_1)dw] \cdot \partial V_1/\partial M_1,$$

which differs from Equation (4) only in that $t_1 p_1^* Z_{11}$ replaces T_1 $(= wL_{1p})$.

Urban firms and households enjoy the more favourable price $p_{12} = (1-t_1)p_1^*$, so that the expression in braces on the right-hand side of Equation (10) is augmented by $t_1 p_1^*(a_{12}Y_2 + X_{12})$:

$$\begin{aligned} dV_2 &= \{[\overline{L}_2 - nl_2^0 - l_3 \cdot g(L_2, L_3)(Y_3 + (1+\tau_3)Z_{31})]dw + \hat{p}_2 Y_2 \cdot (\varphi'/\varphi)nl_2^0 \\ &+ t_1 p_1^*(a_{12}Y_2 + X_{12}) - wl_3[Y_3 + (1+\tau_3)Z_{31}]dg\} \frac{\partial V_2}{\partial M_2}. \end{aligned}$$

Proceeding as in Section 4, we obtain

$$B = B_1^\tau - t_1 p_1^*(Z_{11} - (a_{12}Y_2 + X_{12})) + L_3 dw - Y_3 dp_3 + \hat{p}_2 Y_2 \cdot (\varphi'/\varphi)nl_2^0. \tag{18}$$

Since $E_1 = (1-\tau_1)Z_{11} - (a_{12}Y_2 + X_{12})$, it follows that $t_1 p_1^*(Z_{11} - (a_{12}Y_2 + X_{12}))$ exceeds the required revenue $t_1 p_1^* E_1$. This yields the counterpart of Proposition 3:

Proposition 5. *If, in the absence of any external economies, the programme is financed by an export tax on good 1, then the size of the deadweight loss is* $t_1 p_1^* \cdot (\tau_1 Z_{11})$.

The export tax has the direct effect of lowering input costs for the firms producing good 2, thus promoting an increase in employment and output, with further effects in the presence of agglomeration economies. Equation (7) becomes

$$[(1 + p_3 a_{32}/\hat{p}_2]dw = \xi_2 \cdot dl_2^0 - \xi_3 \cdot dL_3 + t_1 p_1^* a_{12} w/\hat{p}_2,$$

where $\xi_3 = 0$ if $g_{L_3} = 0$. The change in the wage, dw, will be smaller than that with a rural poll tax; for by lowering the price of good 1 at the farm gate, an export tax reduces the level of return migration. The counterpart of Λ in Equation (17) is

$$\Lambda(t_1) = \frac{p_2^* - p_1^* a_{12}(1 + t_1 w/dw)}{\hat{p}_2 w \zeta} \left(p_3 Y_3 \cdot \frac{l_2^0 g_l}{g} - \hat{p}_2 Y_2 \cdot \frac{nl_2^0 \cdot \varphi'}{\varphi} \right) dw + t_1 p_1^* \cdot (\tau_1 Z_{11}),$$

where the term $t_1 w/dw$ is the ratio of the tax rate to the proportional change in the wage. Whether the combined effect of this reduction in costs and the smaller change in the wage will more than offset the loss $t_1 p_1^* \cdot (\tau_1 Z_{11})$ is unclear. This will be closely examined in the numerical examples in Section 6.

5.2 Rural–urban commuting

Suppose commuting wholly replaces rural–urban migration. The round-trip claims both time and the fare. Valuing the worker's time at the wage, let the combined cost be a fixed fraction τ_ℓ of the wage. In effect, there are iceberg costs, so that commuters receive the net wage $w_1 \equiv (1-\tau_\ell)w$, which is also the opportunity cost of farm labour. The time spent in commuting is a charge on rural households' endowments of labour: this is represented by $\tau_\ell(\overline{L} - L_1)$ and belongs in the labour-market clearing condition.

If the programme is financed by a rural poll tax, aggregate rural income is $M_1 = R_1(w_1) + w_1(\overline{L} - L_1) - T_1$, where T_1 $(= wL_{1p})$ now finances a reduction in τ_ℓ as well as in the vector $\boldsymbol{\tau}$ for goods. Thus, Equation (4) becomes

$$dV_1 = [B_1^\tau - w(\overline{L}_1 - L_1)d\tau_\ell - T_1 + (1+\tau_3)Z_{31}dp_3 + (\overline{L}_1 - L_1)dw] \cdot \partial V_1/\partial M_1.$$

The additional, direct benefit is $|w(\overline{L}_1 - L_1)d\tau_\ell|$, $(\overline{L}_1 - L_1)$ being the net supply of labour—at the village—to urban employment.

The immediate, direct effect of the reduction in τ_ℓ is to increase w_1 and so make commuting more attractive and cultivation less so. The resulting movement of rural labourers will put downward pressure on the urban wage; but if commuters earlier supplied only a small fraction of the aggregate employment in urban production, the net wage w_1 is sure to rise. Agglomeration economies will mitigate the adverse effects on urban households, who would welcome an export tax on good 1 in addition.

6 Numerical examples

Drawing on the results of Section 3, let the production technology in the rural sector be Cobb–Douglas. The solution of decision problem (2) yields the supply function:

$$Y_1(\mathbf{p}_1, w) = \left[A_1 \left(\frac{\alpha_{21} p_{11}}{p_{21}}\right)^{\alpha_{21}} \left(\frac{\alpha_{31} p_{11}}{p_{31}}\right)^{\alpha_{31}} \left(\frac{\alpha_{\ell 1} p_{11}}{w}\right)^{\alpha_{\ell 1}}\right]^{(1-\alpha_{21}-\alpha_{21}-\alpha_{\ell 1})^{-1}}. \tag{19}$$

The cost shares of urban goods—artificial fertilizers, other chemicals, fuel, and certain urban services—are small in peasant agriculture: let $\alpha_{21} = \alpha_{31} = 0.05$. Let labour claim two-thirds of value added—that is, $\alpha_{\ell 1} = 0.6$; the residual $(1 - \alpha_{21} - \alpha_{31} - \alpha_{\ell 1})$ accrues to the rural sector's specific factor. The value of the TFP-scalar A_1 must be sufficiently large to yield a surplus for export.

Likewise, let the value-added function f in sector 2 be Cobb–Douglas, where $\alpha_{\ell 2} = 1/2$; and let the input–output coefficients a_{12} and a_{32} for goods 1 and 3 be fixed, at 0.1. The TFP-scalar in the absence of agglomeration economies is $A_2 = 3/2$. In their presence, let the function $\varphi(L_2)$ be iso-elastic with parameter ε_2 and normalized such that $\varphi(L_2) = 1$ when aggregate employment in sector 2 has the level, $L_2(0)$, that rules in the absence of such economies: $\varphi(L_2) = [L_2/L_2(0)]^{\varepsilon_2}$. This normalization facilitates comparisons of the programme's effects with and without agglomeration economies.

Recalling that good 3 is produced by means of labour alone, the input–output coefficient, $l_3 \cdot g(L_2, L_3)$, is normalized in the same way, with $l_3 = 1$ and $g = [(L_2 + L_3)/(L_2(0) + L_3(0)]^{-\varepsilon_3}$. Where the values of ε_2 and ε_3 are concerned, Henderson (2002) reports localization elasticities for various industries in the range 0.05–0.08, to which must be added the contribution of general economies of urbanization. In the light of the heavy aggregation involving just two urban goods in the present structure, $\varepsilon_i = 0.2$ $(i = 2, 3)$ would represent extremely strong economies of agglomeration.

Let households' preferences in town and country be Cobb–Douglas. The taste parameter for households' consumption of good i in town and country, respectively, is denoted by b_{ik} $(i = 1, 2, 3; k = 1, 2)$. Rural households consume a substantial fraction of their own output. Let the expenditure shares be $b_{11} = 0.4$, $b_{21} = 0.3$, $b_{31} = 0.3$. Urban households' tastes for good 1 are arguably a bit weaker, being rather influenced by the port and its trade. Let $b_{12} = 0.3$, $b_{22} = 0.4$, $b_{32} = 0.3$.

Let the transport cost parameters before the roads programme is undertaken be a uniform 10 per cent ($\tau_i = 0.1$), and let the programme halve them. It should be remarked that this reduction in $\boldsymbol{\tau}$, while not strictly marginal, is arguably small enough for the results in Sections 4 and 5 to hold.

Turning to prices, the world prices of goods 1 and 2 may be normalized to unity: $p_1^* = p_2^* = 1$, either serving as numéraire. The rural and urban price vectors before the programme are therefore $\mathbf{p}_1 = (0.9, 1.1, 1.1p_3)$ and $\mathbf{p}_2 = (1, 1, p_3)$, respectively, whereby $p_3 = wl_3g(L_2, L_3)$ is endogenous. With the programme, the rural price vector becomes $\mathbf{p}_1 = (0.95, 1.05, 1.05p_3)$, whose accomplishment is assumed to require 0.025 units of rural labour—that is, 1 per cent of the rural endowment thereof.

Hence, $T_1 = 0.025w$. Rural households' barter terms of trade improve by 10.6 per cent, which is not so small.

To complete the constellation of technologies, preferences, transport costs, world prices, and their associated parameter values, there are rural and urban households' factor endowments. Land is subsumed under the TFP value $A_1 = 2.9$. Their respective labour endowments are $\overline{L}_1 = 2.5207$ and $\overline{L}_2 = 2.0$. Urban households own, in aggregate, 1.5 units of the specific factor (K_2) employed in producing good 2. These endowments, along with all of the foregoing, yield $w = 1$ in equilibrium in the absence of the programme. This endogenously derived value will be convenient when making comparisons of allocations in equilibrium. For ease of reference, the complete set of parameter values is set out in Table 1.

Table 1: Constellation of parameter values

Parameter	Value	Description
Rural		
A_1	2.9	TFP parameter
$\boldsymbol{\alpha}_1$	(0.05,0.05,0.6)	Elasticity of output with respect to goods 2 and 3 and labour
$\mathbf{b}_1$	(0.4, 0.3, 0.3)	Taste parameters
$\overline{L}_1$	2.5207	Labour endowment
$\boldsymbol{\tau}$	(0.1,0.1,0.1)	Transport cost factors without programme
$\boldsymbol{\tau}$	(0.05,0.05,0.05)	Transport cost factors with programme
L_{1p}	0.025	Programme input requirement
Urban		
A_2	1.5	TFP parameter
$\mathbf{a}_2$	(0.1, 0.1)	Input–output coefficients for goods 1 and 3 in sector 2
$\alpha_{\ell 2}$	0.5	Elasticity of value added with respect to labour in sector 2
ε_i	(0,0); (0.2, 0.2)	Elasticity of agglomeration function in sector $i = 2,3$
$\mathbf{b}_2$	(0.3, 0.4, 0.3)	Taste parameters
$\overline{L}_2$	2.0	Labour endowment
K_2	1.5	Specific factor endowment
$\mathbf{p}^*$	(1,1)	World prices of goods 1 and 2

Source: author's construction.

The benchmark case is that wherein there is neither a roads programme ($\tau_i = 0.1, i = 1,2,3$) nor agglomeration economies ($\varphi = 1$, $\varepsilon = 0$). The full details of the allocation in equilibrium are set out in column 1 of Table 2.[12] Wages account for 72 and 79 per cent, respectively, of rural and urban households' total incomes, so that movements in the wage rate induced by the roads programme have a powerful influence on welfare. It is also seen that the labour supplied by rural households to urban production is far smaller than the labour needed to meet their demand for good 3, thus satisfying Corollary 2.

6.1 Poll taxes: no agglomeration economies

The programme—in the form of a halving of unit transport costs—exerts a decisive influence on the whole allocation in equilibrium through its effects on the urban wage, whose increase from 1 to 1.054 is not especially small,[13] and return migration (L_1 increases by 3 per cent). There is a strong rise in deliveries of good 1 to the port-city, generated by a 2.8 per cent increase in output at the farm gate, a reduction of 5 percentage points in (iceberg) transportation costs, and virtually no change in rural consumption (the income effect on the latter being slightly more than offset by the increase in p_{11} and the reduction in p_{21}, with almost no net change in p_{31}). Although the increases in rural production and rural purchasing power result in more demand for good 2 at the farm gate, the halving of τ_2 results in a

[12] Recall that the normalizations $\varphi(L_2) = [L_2/L_2(0)]^{\varepsilon_2}$ and $l_3 = 1, g = [(L_2+L_3)/(L_2(0)+L_3(0)]^{-\varepsilon_3}$ yield identical allocations in the absence of the programme for all values of ε.

[13] In this setting, there is an explicit equation for the wage; see the Appendix.

much smaller net effect. In consequence, the output of good 2 must fall, which can be induced only by an increase in the wage, reinforced by the increase in the cost factor $p_3 a_{32}$.

To place the programme's effects in relation to the economy's macroeconomic magnitudes in its absence, which would be observable at the time of evaluating the programme *ex ante*, its labour requirement is 1 per cent of the rural labour force. This corresponds to 0.72 per cent of rural income and 0.42 per cent of gross domestic product (GDP) ($= M_1 + M_2$). Incomes in town and country increase by 2.89 per cent and 5.49 per cent, respectively, where the latter is *net* of the poll tax that finances the programme ($T_1 = 0.025w = 0.0264$); but prices also change, adversely for all households as consumers. An exact measure of the programme's welfare effects is needed.

Table 2: Allocations in equilibrium: a rural poll tax, an export tax, and commuting

Transport	$\tau_i = 0.1$	$\tau_i = 0.05$		$\tau_i = 0.05$		$\tau_\ell = 0.2$	$\tau_i = 0.05,\ \tau_\ell = 0.1$	
tax	none[a]	poll	poll	export	export	none[a]	poll	poll
Agglomeration elasticity	$\varepsilon = 0$	$\varepsilon = 0$	$\varepsilon = 0.2$	$\varepsilon = 0$	$\varepsilon = 0.2$	$\varepsilon = 0$	$\varepsilon = 0$	$\varepsilon = 0.2$
Rural	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
p_{11}	0.90	0.95	0.95	0.9231	0.9244	0.90	0.95	0.95
p_{21}	1.10	1.05	1.05	1.05	1.05	1.10	1.05	1.05
p_{31}	1.10	1.1067	1.1099	1.0812	1.0837	1.2832	1.1889	1.1858
Y_1	3.4945	3.5928	3.6354	3.5338	3.5603	3.9106	3.7978	3.7682
$-Y_{21}$	0.1430	0.1625	0.1645	0.1553	0.1567	0.1600	0.1718	0.1705
$-Y_{31}$	0.1430	0.1542	0.1556	0.1509	0.1518	0.1371	0.1517	0.1510
L_1	1.8871	1.9430	1.9871	1.9007	1.9219	2.2627	2.1243	2.1017
M_1	3.4642	3.6544	3.6505	3.5742	3.5769	3.4564	3.6782	3.6797
X_{11}	1.5397	1.5387	1.5370	1.5488	1.5479	1.5362	1.5487	1.5493
X_{21}	0.9448	1.0441	1.0430	1.0212	1.0220	0.9427	1.0509	1.0593
X_{31}	0.9448	0.9906	0.9867	0.9917	0.9902	0.8081	0.9281	0.9310
E_1	0.8625	1.0402	1.1038	0.9582	0.9968	1.1905	1.2016	1.1632
$(EV_1/M_1) \cdot 100$		4.49	4.29	4.11	4.06		8.05	8.20
Urban								
p_{12}	1.00	1.00	1.00	0.9731	0.9744	1.00	1.00	1.00
p_{22}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
p_{32}	1.00	1.0540	1.0570	1.0297	1.0321	1.1666	1.1323	1.1293
w	1.00	1.0540	1.0476	1.0297	1.0274	1.1666	1.1323	1.1355
Y_2	1.35	1.2722	1.1913	1.3106	1.2651	1.1332	1.1726	1.2201
$-Y_{12}$	0.135	0.1272	0.1191	0.1311	0.1265	0.1133	0.1173	0.1220
$-Y_{32}$	0.135	0.1272	0.1191	0.1311	0.1265	0.1133	0.1173	0.1220
L_2	0.54	0.4796	0.4516	0.5089	0.4921	0.3805	0.4074	0.4229
Y_3	2.0936	2.0732	2.0475	2.0860	2.0722	2.0390	1.9733	1.9891
L_3	2.0936	2.0732	2.0660	2.0860	2.0817	2.0390	1.9733	1.9782
M_2	2.54	2.6134	2.5682	2.5835	2.5604	2.7770	2.7258	2.7513
X_{12}	0.762	0.7840	0.7705	0.7965	0.7884	0.8331	0.8178	0.8254
X_{22}	1.016	1.0454	1.0273	1.0334	1.0242	1.1108	1.0903	1.1005
X_{32}	0.762	0.7439	0.7289	0.7527	0.7442	0.7141	0.7222	0.7309
E_2	–0.8625	–1.0402	–1.1038	–0.9582	–0.9967	–1.1905	–1.2016	–1.1632
$(EV_2/M_2) \cdot 100$		1.28	–0.56	1.65	0.63		–0.96	0.04

Note: world prices at the port-city: $p_1^* = p_2^* = 1$. $(EV_k/M_k) \cdot 100$ relative to the base case $\tau_i = 0.10$. [a] The functions φ and g representing agglomeration economies are normalized so as to yield, in the absence of the roads programme, the same allocation as that in which there are no agglomeration economies.

Source: author's calculations.

The natural candidate is the equivalent variation (EV)—namely, the lump-sum transfer such that households would be indifferent between having that sum with the initial transport costs and enjoying the programme to reduce τ_i to $0.05\,(i = 1, 2, 3)$. All households supply their labour perfectly inelastically, so that their money-metric welfare is inversely proportional to the level of the true cost-of-living index,

$\kappa(\mathbf{p}_k)$. Since the utility functions are Cobb–Douglas, $\kappa(\mathbf{p}_k) = p_{1k}^{b_{1k}} p_{2k}^{b_{2k}} p_{3k}^{b_{3k}}$.[14] In the absence of the programme, $\kappa(\mathbf{p}_1) = 1.01516$; with the programme, $\kappa(\mathbf{p}_1) = 1.02484$. The corresponding values for urban households are 1 and 1.01590, respectively. Applying these to M_k, we obtain proportional improvements in rural and urban money-metric utility of 4.49 and 1.28 per cent, respectively, with an aggregate improvement of 3.13 per cent.

In the light of Proposition 3, it is also instructive to examine the performance of the measure $B^\tau - T_1$. From Equation (4), we have $B^\tau = 0.2065$, evaluated at the (observable) pre-programme quantities, so that the net aggregate benefit is 0.1801. The sum $EV_1 + EV_2$ is 4.4 per cent higher, at 0.1881. As a further alternative, a simple partial equilibrium approach to evaluation might well stop at measuring the increase in rural incomes, namely, 0.1902. Although this estimate is almost spot on in aggregate, about one-sixth actually accrues to urban households. The failure of B^τ and the partial equilibrium measure to capture distributional effects is a definite weakness.

6.2 Agglomeration economies

In the presence of the very strong agglomeration economies represented by $\varepsilon = 0.2$, the programme yields the allocation in column 3. It conforms to intuition. More profitable opportunities in rural production lead to rural migrant workers withdrawing from urban production, thus reducing urban factor productivity. The contraction in urban production is all the stronger, with a slightly larger increase in the price of good 3, despite a smaller increase in the wage rate, to 1.0476, as opposed to 1.0540 in the absence of agglomeration economies. The programme's effects on welfare are, in certain respects, startlingly different. The EV for rural households, at 4.29 per cent of rural income, is almost as large as in the absence of agglomeration economies. For urban households, however, there is now an incidental loss of 0.56 per cent, as opposed to an incidental gain of 1.28 per cent—both without any contribution in taxes. In aggregate, the programme yields an EV of 0.1345, or 2.24 per cent of GDP, almost 30 per cent smaller than in the absence of agglomeration economies. Here, the measure $B^\tau - T_1 = 0.1815$ exhibits a large error, implying that Λ, the loss arising from weaker external effects as urban employment contracts is large.

6.3 Export taxes and commuting

Export taxes

In the absence of agglomeration economies, the required rate is 2.69 per cent, thereby offsetting just over one-half of the reduction in τ_1 (see p_{11} in column 4). Again, the results conform to intuition. Rural output, income, and the marketed surplus all respond much more modestly to the programme than under a rural poll tax. With the blunting of the incentive to work on the farm, fewer migrants return home. The increase in the wage rate is correspondingly smaller, so that the output of good 2 falls less. The programme now yields an EV_1 that is 8.5 per cent smaller, whereas EV_2 is 29 per cent larger. In aggregate, the improvement is 3.07 per cent of GDP, instead of 3.13 per cent with a rural poll tax, the difference constituting a small deadweight loss. (Recalling Proposition 5, $t_1 p_1^* \cdot (\tau_1 Z_{11}) = 0.0053$.)

The tax counteracts, in part, the market failure arising from externalities in urban production; see column 5. The tax rate is slightly lower than in the absence of such externalities, exports being slightly higher. The resulting wage rate is also a bit lower, but not enough to prevent a slight increase in the price of good 3. The withdrawal of rural migrant workers from urban production is not large, and notably far

[14] This is the associated Könus price index.

smaller than that under a poll tax. The aggregate EV is 0.1567, a clear improvement over a poll tax: $\Lambda(t_1) < \Lambda$, but the measure $B^\tau - T_1$ remains seriously in error.

Commuting

The base case cannot be exactly the same as that in column 1; for the wedge between w_1 and w introduces a fundamental change in behavioural structure. In order to bring the base cases closer together, the rural labour endowment in the commuting variant is augmented by the time spent commuting (which is endogenous), so that the net endowments in both cases take the value 2.5207 given in Table 1.

The allocation in the absence of the programme is given in column 6, whereby τ_ℓ is set at the stiff value of 0.2. Rural households devote 2.2627 units of their labour endowment to cultivation and 0.3096 units to urban employment and travelling. The latter amount to 15 per cent of urban households' labour endowment. A strict comparison of the wage rate is impossible; but by way of reference, at 1.1666, it is one-sixth higher than its counterpart in the base case with rural–urban migration, thus yielding a price of the non-tradeable higher in the same proportion, together with the lower net rural wage of 0.9333. The output of good 1 is correspondingly higher, as is the level of foreign trade.

In keeping with the programme's assumed effects on the costs of transporting goods, let it halve τ_ℓ. If financed by a rural poll tax, the resulting allocation in the absence of agglomeration economies is given in column 7. Despite the improvement in the rural sector's barter terms of trade, its levels of output and employment fall, by 2.9 per cent and 6.1 per cent, respectively. With more commuters and shorter commuting time, the urban wage falls, also by 2.9 per cent, but the rural net wage w_1 rises, by 12.1 per cent. Boosted by the latter, EV_1 is 8.05 per cent of rural income, some 80 per cent greater than its counterpart under migration (see columns 2 and 7). The fall in the wage is unwelcome to those in the port-city: EV_2 is transformed from the gain of 1.28 per cent of urban income under migration into a loss of almost 1 per cent. For the whole economy, the EV is now 4.04 per cent of GDP, instead of 3.13 per cent—not only larger, but very differently distributed between town and country.

In the presence of agglomeration economies, the greater inflow of labour into the port-city enhances productivity there. Relative to the base case, the wage falls a little less than in their absence (see column 8), as a consequence of the attendant improvement in productivity, which is reflected in the ratio of the wage rate to the producer price of the non-tradeable ($w/p_{32} = 1.1355/1.1293 = 1.0055$). The spillover effects include a slightly larger withdrawal of labour from rural production and a mitigation of the programme's adverse effects on urban production and incomes. At 8.2 per cent of rural income, EV_1 is slightly higher. Urban households are left virtually untouched, the reduction in the wage being almost exactly offset by the fall in the consumer price of the non-tradeable. In aggregate, the programme yields an EV of 4.55 per cent of GDP, 10 per cent higher than in the absence of agglomeration economies.

If, instead, the programme were financed by an export tax, commuting to urban jobs would be even more attractive, with still heavier pressure on the wage rate. In the absence of agglomeration economies, the wage rate is 6.3 per cent lower, at 1.0934. Rural incomes are slightly lower, but this reduction is more than compensated by the reductions in prices: EV_1 is 9.11 per cent of pre-programme income. Urban households, also benefiting from lower prices, do a little less worse than under a rural poll tax. The aggregate EV is 4.72 per cent of GDP, 17 per cent higher than under the poll tax. In the presence of agglomeration economies, there is a smaller fall in the wage, to 1.1183, almost the same aggregate EV, at 4.62 per cent of GDP, but very differently distributed, with improvements now for both village and town residents, of 7.53 and 1.00 per cent, respectively (see Table 3).[15]

[15] Full details of the allocation are available upon request.

6.4 Social profitability: variation over variations

The foregoing results for the programme's social profitability in all variations are drawn together in Table 3. The EV accruing to rural households is not very sensitive to how the programme is financed or the strength, if any, of agglomeration economies. It is rather sensitive to whether the improvement in the network enables commuting to urban jobs. The inhabitants of villages lacking all-weather roads can always migrate permanently. Hence, if rural mobility takes the form only of migration, then the provision of such roads is likely, by improving villagers' standard of living, to induce some rural migrant workers to return home. If rural mobility takes the form of commuting, there will be the direct effect of an increase, *ceteris paribus*, in the urban wage *net* of commuting costs, and hence in the value of rural households' labour endowments, the net wage being their opportunity cost of labour. There is no such direct effect associated with migration, although the programme will affect the wage in all variations.

Table 3: EV as a percentage of income: taxation, agglomeration economies, and rural mobility

	Poll tax		Export tax	
Rural mobility	$\varepsilon = 0$	$\varepsilon = 0.2$	$\varepsilon = 0$	$\varepsilon = 0.2$
Migration				
Rural households	4.49	4.29	4.11	4.06
Urban households	1.28	–0.56	1.65	0.63
All households	3.13	2.24	3.07	2.61
Commuting				
Rural households	8.05	8.20	9.11	7.53
Urban households	–0.96	0.04	–0.74	1.00
All households	4.04	4.55	4.72	4.62

Source: author's calculations.

If the mobility of rural labour takes the form of migration, the presence of agglomeration economies is disadvantageous to urban households, whether the programme be financed by a poll tax or an export tax; for the programme induces return migration and so lowers the level of urban production. Conversely, they do better with agglomeration economies if there is commuting; for the programme then induces greater supplies of labour to urban production.

The other feature of the results in Table 3 that calls for some comment involves aggregate efficiency. If agents are free to choose their place of residence, but not to commute, an export tax is superior to a rural poll tax in the presence of agglomeration economies. In their absence, the setting is first-best. If, however, commuting is an option, but not the place of residence, this restriction is also a distortion. In the numerical examples, an export tax is clearly superior, in aggregate, to a poll tax in the absence of agglomeration economies, but barely so in their presence.

7 Robustness: other factors

Three other factors come to mind when assessing the robustness of the main findings. First, there is the ease of substitution in final consumption. Second, there is the structure of rural families as an influence on the migration decision. Third, there are the numerical values of ε and the programme's input requirements.

7.1 Substitution in consumption

The programme's effects on welfare are mediated largely through changes in the wage rate. By assumption, the non-tradeable good is produced by means of unassisted labour, so that the wage rate is quite strongly connected to the level of demand for that good. There may be dangers, therefore, in drawing general conclusions from results that rest on Cobb–Douglas preferences; for goods are then rather good substitutes and cross-price elasticities are zero. With just three goods, it is essential to investigate more limited substitutability. Consider, therefore, the preferences represented by

$$U_k(\mathbf{X}_k) = \frac{1}{b_{1k}/X_{1k} + b_{2k}/X_{2k} + b_{3k}/X_{3k}}, \quad \sum_{i=1}^{i=3} b_{ik} = 1, \; k = 1,2,$$

for which the elasticity of substitution $\sigma = -0.5$. Income effects now take on a stronger role, whereas the changes in the price of good 3 have a lesser one. The associated Marshallian demand functions are

$$X_{ik} = \frac{(b_{ik}/p_{ik})^{0.5} M_k}{\sum_{j=1}^{j=3}(b_{jk}p_{jk})^{0.5}}, \; i = 1,2,3, \; k = 1,2.$$

In keeping with the values of the taste parameters when preferences are Cobb–Douglas, let $b_{11} = 0.40$, $b_{21} = 0.30$, $b_{31} = 0.30$, and $b_{12} = 0.30$, $b_{22} = 0.40$, $b_{32} = 0.30$.

In the base case, the programme is indeed rather more profitable. The wage rate is somewhat higher, at 1.074, and the level of final consumption of good 3 is also higher in both town and country.[16] Both EV_1 and EV_2 are correspondingly greater than those with Cobb–Douglas preferences: at 4.92 and 1.40 per cent, respectively, with an aggregate improvement of 3.44 per cent, they are about 10 per cent larger.

7.2 Migration

In the foregoing variations, rural workers are footloose in employment, but keep a foot firmly in the extended family though full income sharing. On the supply side, the migration decision is essentially governed by the value of the marginal product of labour in rural production. In an extreme alternative, migrants would lose all claims on the imputed rents from the family's holding in exchange for an exclusive claim on their urban wages. Lacking any claims on the urban fixed factor, their (money) opportunity cost would be the value of the average product of labour in rural production. Let them have the right of return in the event that rural life became more attractive.

Each family worker on the farm receives $m_1 = (p_{11}Y_1 + p_{21}Y_{21} + p_{31}Y_{31} - T_1)/L_1$. Hence, the following condition must hold in equilibrium:

$$v_1(\mathbf{p}_2, w) = v_1(\mathbf{p}_1, m_1), \tag{20}$$

where v_1 denotes a rural worker's indirect utility function, which is assumed not to depend on his or her location. It is clear that the wage will be higher in this variant than in the main alternative, both with and without the programme. The latter's social profitability depends, however, on how much the wage responds. A profitable programme will surely yield an increase in the numerator of m_1. In order to satisfy Equation (20), the wage must rise—unless, counter-intuitively, there were a heavy reduction in the demand for the non-tradeable good. The adjustment of the wage under the main alternative is almost surely smaller; for the concavity of the technology implies that, in any neighbourhood, the value of the marginal product is less sensitive to movements in employment than that of the average product, and the sharing of family income involves all members wherever they are employed, as opposed to those resident in the village. It follows that the results in Sections 6 and 7.1 understate the absolute magnitudes of the programme's effects when migrants give up their family ties.

7.3 Congestion, programme costs, and scale

The movements of goods and people associated with urban production necessarily involve traffic within the restricted space afforded by a city's limits. If the streets are at all congested, heavier traffic will

[16] The full details of the allocation are available upon request.

generate external costs, agglomeration economies or no. Suppose these costs depend only on the level of urban production. Then the parameter ε represents the joint effect of congestion and agglomeration economies. Its value will be negative if the former outweigh the latter.

In a more refined formulation, τ_i would depend on the level of traffic as well as any investment in the road network. This would greatly complicate the analysis, without yielding any obvious insights beyond those offered by the representation through ε. The value 0.2 selected for Table 2 is rather on the high side, and the results indicate that linear interpolation will yield sufficiently accurate results for intermediate values. In the light of the empirical evidence, the half-way value of 0.1 suggests itself.

Another magnitude central to the programme's profitability is the level of its costs, set above at 0.42 per cent of GDP (i.e. 1 per cent of rural households' labour endowments). Suppose this requirement were double that level. In the basic variant in Section 6.1, in which there are no agglomeration economies, the wage rate would indeed be slightly higher, at 1.0560, thereby inducing a slightly greater contraction of urban activities.[17] At 3.70 per cent of rural income, EV_1 would be somewhat more modest, but still substantial. Urban households, in contrast, would experience a slightly larger increase. The aggregate gain would be 2.70 per cent of GDP instead of 3.13 per cent.

Lastly, there is the matter of scalability. Suppose, in the base case, that both the reductions in $\boldsymbol{\tau}$ and the associated cost were halved. Then EV_1 and EV_2 would be 64 and 54 per cent of their base-case values, respectively. The aggregate would be 62 per cent thereof, which implies modest concavity, as expected.

8 Conclusions

This paper has addressed two questions. First, how do improvements in the rural road network of a small open economy affect economic activity and welfare in both town and country? Second, how sensitive are these effects to the presence of urban agglomeration economies? The answer depends, in particular, on what taxes are used to finance the improvements and whether the mobility of rural labourers takes the form of migration or commuting.

If commuting is ruled out, reductions in transport costs will almost surely result in workers shifting from urban to rural production and an increase in deliveries of the rural good—the exportable—to the city. If that increase is not wholly absorbed there, exports will rise and hence imports of the tradeable urban good. The reallocation of the labour force will therefore be accompanied by a contraction in urban activity, almost always with an increase in the wage rate. Theory and the numerical examples indicate that in the absence of agglomeration economies, urban households may benefit quite substantially, whether the programme be financed by a poll tax on rural households or a tax on exports. Although the change in the value, at the economy's producer prices, of the rural sector's net supply vector closely approximates the true, aggregate (money-metric) benefit, its sectoral distribution is sensitive to the form of the tax, so that using this measure could lead to serious errors when assessing the programme's social profitability.

In the presence of agglomeration economies, the contraction in urban output will lower urban factor productivity and hence work against a rise in the wage rate. If agglomeration effects are sufficiently strong, the contractionary effect can be so large as to lower urban welfare when the programme is financed by a poll tax on rural households; for the tax exacerbates the market failure stemming from agglomeration economies. A tax on exports, in contrast, operates as a countervailing distortion, by

[17] The details are available on request.

offsetting part of the improvement in the rural sector's terms of trade. Numerical examples indicate that whereas rural households would do almost as well, urban households would enjoy modest net benefits. The change in the value, at the economy's producer prices, of the rural sector's net supply vector would be a substantial underestimate of the aggregate net benefits.

If, instead, migration is ruled out, but commuting allowed, the programme will put the urban wage under pressure. In the absence of agglomeration economies, rural households will do better still; but urban households may well do worse, and in the numerical examples, that is indeed the outcome, even with an export tax. In the presence of agglomeration economies, easier commuting will promote both urban employment and productivity, the latter relieving the pressure on the wage. In the numerical examples, the effects balance out under a poll tax, leaving urban welfare unchanged; but under an export tax, urban households will gain quite measurably, and rural households somewhat less so. Here, too, using the change in the value, at the economy's producer prices, of the rural sector's net supply vector would yield a serious error.

To sum up: practitioners charged with programme evaluation need to go beyond partial equilibrium effects. When doing so, they should be wary of methods of estimating benefits that fail to allow for the wider ramifications of agglomeration economies; among them are the received procedures for estimating shadow prices, which seek to decentralize the problem of deciding among public investments. Whatever the method chosen, the tax regime and the form of rural labour mobility require special attention.

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Appendix

A1 Proof of Lemma 3

By hypothesis, there is no change in the wage, so that p_{31} falls. Then the increase in the output price p_{11} and the fall in input prices will induce an increase in output, reinforcing the direct effect of the reduction in τ_1. If $B_1^\tau > T_1$, M_1 will increase, but substitution effects will work against the income effect on X_{11}. Recalling Equation (4) and given $dw = 0$, Equation (11) specializes to

$$\begin{aligned} d[(1-\tau_1)Z_{11}] &= -\left(1 - p_{11}\frac{\partial X_{11}}{\partial M_1}\right) Z_{11} d\tau_1 \\ &+ (1-\tau_1)\left[\nabla(Y_1 - X_{11})\cdot \mathbf{dp}_1 - \frac{\partial X_{11}}{\partial M_1}\cdot (p_2^* Z_{21} d\tau_2 + p_3 Z_{31} d\tau_3 - T_1)\right]. \end{aligned}$$

If all goods are normal, $p_{11}\partial X_{11}/\partial M_1 < 1$. The term $p_2^* Z_{21} d\tau_2 + p_3 Z_{31} d\tau_3 - T_1$ is the programme's net benefits at unchanged urban prices, less that component arising from the increase in p_{11}, that is, $B^\tau - T_1 + p_1^* Z_{11} d\tau_1$, where $d\tau_1 < 0$. Hence, a sufficient condition for the delivery of good 1 at the port-city to increase is

$$p_{11}\nabla(Y_1 - X_{11})\cdot \mathbf{dp}_1 > p_{11}\frac{\partial X_{11}}{\partial M_1}\cdot (p_2^* Z_{21} d\tau_2 + p_3 Z_{31} d\tau_3 - T_1).$$

The left-hand side is the increase in the value of the marketed surplus at the farm gate, which will exceed (fall short of) the increase in the value of output if the rural consumption of good 1 decreases (increases) in response to the change in rural prices, at constant income. Since p_{11} rises and p_{21} and p_{31} fall, $\nabla X_{11}\cdot(0, \mathbf{dp}_1) < 0$ if there is sufficient substitutability in rural consumption.[18] The right-hand side is the marginal propensity to spend on good 1 multiplied by the benefits of the reductions in the rural prices of the urban goods, net of the tax T_1, whereby the sum $p_2^* Z_{21} + p_3 Z_{31}$ is the aggregate value of rural demand for those goods, valued at urban prices. □

A2 Proof of Proposition 1

If the wage remains unchanged, the output of good 2 and urban income, prices, and demand will do likewise; so the whole of the increase in the delivery of good 1 will be exported. By assumption, the economy imports good 2 ($E_2 < 0$), so that the increase in E_1 will induce an increase in imports of good 2. Hence, the entire increase in $p_1^* E_1$ must be exactly matched by an increase in rural demand for good 2 valued at p_2^*—a result that will come about only by the merest fluke—or there will be a contradiction.

Suppose, therefore, that the wage were to fall, thus increasing the output of good 2. It would also induce, *ceteris paribus*, not only a fall in M_1, but an increase in Z_{11}. Although the level of rural demand for good 2 might rise in response to the changes in $\mathbf{p}_1$ if substitution effects were strong enough, the hypothesized fall in w is difficult to reconcile with the requirement that an increased delivery of good 1 at the city-port be accompanied by a sufficient increase in the level of aggregate domestic demand for good 2 so as to validate the hypothesis that the output of that good will actually rise. This does not quite rule out the existence of constellations of endowments, technologies, tastes in consumption, and world prices that would yield such an adjustment to the programme, but the outcome appears to be improbable.

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[18] Even in the extreme case of perfect complements, in which the three goods are consumed in the ratio $(1 : b_{21} : b_{31})$, $\nabla X_{11}\cdot \mathbf{dp}_1 \leq 0$ if the associated price index $p_{11} + b_{21}p_{21} + b_{31}p_{31}$ does not fall.

If, on the contrary, the wage were to increase, the converse of all of the above would apply. The accompanying reduction in the output of good 2 would make room for imports to increase in response to the increase in exports of good 1, whereby the latter would be weakened by the attendant income effects. □

A3 Proof of Proposition 2

Suppose the wage does not change, so that $dL^u = 0$. With the normalization $gl_3 = 1$, so that $p_3 = w$ and $p_{31} = (1+\tau_3)w$, Equation (13) becomes

$$dL^r = d[L_1 + (1+\tau_3)(-Y_{31} + X_{31})] = -L_{1p}.$$

Since the output price p_{11} increases and the input prices p_{21} and p_{31} decrease, L_1 will also increase, unless goods 2 and 3 are highly substitutable for labour in the production of good 1. The same holds for the derived demand for good 3 in rural production, $-Y_{31}$, and for rural final demand, X_{31}, provided T_1 is not so large as to reduce M_1 substantially. The latter effect is unlikely to accompany a socially profitable programme, then yielding a contradiction.

Suppose, therefore, and rather counter-intuitively, that the programme increases M_1 but induces a fall in w. By itself, the latter will result in an increase in L_1, albeit with an attendant substitution effect on $-Y_{31}$, and it will reduce M_1, with an attendant substitution effect on X_{31}. If the cross-price effects are sufficiently small, inspection of $M_1 = R_1 + w(\overline{L}_1 - L_{1p})$ indicates that, *ceteris paribus*, the hypothesized fall in w will likely induce a rise in L^r. As for L^u, a fall in w will result in an increase in L_2 and Y_2, but a fall in M_2, thus leaving only the effect of the fall in p_3 on X_{32} to pull in the opposite direction.

Noting that the programme has the direct effect of reducing transportation costs, with consequent effects on w and p_3, Equation (13) may be decomposed into the form

$$\begin{aligned} dL^r + dL^u &\equiv \nabla[L_1 + (1+\tau_3)(-Y_{31} + X_{31})] \cdot d\boldsymbol{\tau} + \nabla[L_1 + (1+\tau_3)(-Y_{31} + X_{31})] \cdot (dw, dp_3) \\ &+ \nabla[L_2 + (a_{32}Y_2 + X_{32})] \cdot (dw, dp_3) = -L_{1p}, \end{aligned}$$

where $dp_3 = dw$ in virtue of $gl_3 = 1$, and L^u depends on $\boldsymbol{\tau}$ only through w and p_3. Under the assumption that the rural technology and preferences are Cobb–Douglas, $wL_1 = \alpha_{\ell 1} p_{11} Y_1, -p_{31}Y_{31} = \alpha_{31} p_{11} Y_1$, and $p_{31}X_{31} = \beta_{31} M_1$, where $\alpha_{\ell 1}, \alpha_{31}$, and β_{31} are the respective, constant cost shares. Suppose w, and hence also p_3, stays unchanged. Then substituting into the foregoing condition and recalling Equation (3), we have

$$(1/w) \cdot d[(\alpha_{\ell 1} + \alpha_{31}) p_{11} Y_1] + \beta_{31}(Y_1 dp_{11} + Y_{21} dp_{21} + Y_{31} d\tau_3 - w dL_1^0 - T_1) = -L_{1p}.$$

If, at worst, the programme would leave rural net incomes unchanged under these conditions, we have a contradiction.

Proceeding to the terms involving changes in w and p_3 with $\boldsymbol{\tau}$ held constant, substitution and some manipulation yield, at length,

$$\begin{aligned} &\nabla[L_1 + (1+\tau_3)(-Y_{31} + X_{31})] \cdot (dw, dp_3) \\ &\quad = -\frac{1}{w}\left[\frac{(\alpha_{\ell 1} + \alpha_{31}) p_{11} Y_1 + \beta_{31} M_1}{w} - Q - \beta_{31}(\overline{L}_1 - L_1 - L_{1p})\right] dw, \end{aligned}$$

where

$$Q \equiv (\alpha_{\ell 1} + \alpha_{31}) p_{11} \left(\frac{\partial Y_1}{\partial w} + (1+\tau_3)\frac{\partial Y_1}{\partial p_{31}}\right) + \beta_{31}(1+\tau_3)^2 \frac{\partial Y_1}{\partial p_{31}} < 0.$$

Since $M_1 = R_1 + w(\overline{L}_1 - L_{1p})$ and $R_1 + wL_1 > 0$, $\nabla[L_1 + (1+\tau_3)(-Y_{31} + X_{31})] \cdot (dw, dp_3)$ and $dw \, (= dp_3)$ have opposite signs.

Turning to L^u, this may be written

$$L^u = (1-\beta_{32})L_2 + \left[(1-\beta_{32})a_{32} + \beta_{32}\left(\frac{p_2^* - a_{12}p_1^*}{w}\right)\right] Y_2 + \beta_{32}\overline{L}_2,$$

where β_{32} will vary if preferences are not Cobb–Douglas. The expression in brackets is decreasing in w, as are L_2 and Y_2. Hence, L^u is decreasing in w if β_{32} varies sufficiently weakly with w.

As a final step, suppose the programme induces a fall in the wage. Then $dL^r + dL^u > 0$, which is a contradiction. □

If urban households' preferences are Cobb–Douglas, analogous calculations yield

$$\begin{aligned} dL^u &= \nabla[L_2 + a_{32}Y_2 + X_{32})] \cdot (dw, dp_3) \\ &= \frac{1}{w}\left[(\alpha_{\ell 2}\hat{p}_2 + a_{32}w)\frac{\partial Y_2}{\partial w} + \alpha_{\ell 2}\left(\frac{\partial \hat{p}_2}{\partial w} - \frac{\hat{p}_2}{w}\right)\cdot Y_2 - \frac{\beta_{32}}{w}\left(a_{32}Y_2 + \frac{1-\alpha_{\ell 2}}{w}\cdot \hat{p}_2 Y_2\right)\right] dw; \end{aligned}$$

all the terms in brackets are negative.

A4 The wage rate in Section 6.1

In the absence of agglomeration economies ($\varphi = 1$), we obtain, upon substituting the relevant values into Equation (5),

$$Y_2^0 = 1.6875(0.9 - 0.1w)/w, \quad L_2^0 = 0.84375 \cdot ((0.9 - 0.1w)/w)^2. \tag{21}$$

Given the calibration of the system so as to yield $w = 1$, Y_1 and Y_2 follow at once from Equations (19) and (21), and hence, from the constant cost-share property of Cobb–Douglas technologies, both aggregate input bundles. Together with the endowments, these yield incomes M_1 and M_2 (whose sum is GDP), whereby workers from rural households supply $\overline{L}_1 - L_1 - L_{1p}$ units of labour to urban production activities. Using the constant expenditure-share property of Cobb–Douglas preferences, the aggregate expenditure bundles $\mathbf{X}_k$ follow immediately. Exports of good 1 are equal to the excess of the delivery at the port-city over urban demand for good 1, and imports of good 2 are equal to the excess of domestic demand over domestic production:

$$E_1 \equiv (1-\tau_1)Z_{11} - (0.1Y_2 + X_{12}); \; -E_2 \equiv X_{22} + (1+\tau_2)(-Y_{21} + X_{21}) - Y_2. \tag{22}$$

Thus, given $p_1^* = p_2^* = 1$, $p_1^*E_1 + p_2^*E_2 = 0$ indeed holds, and all markets clear.

Substituting from Equations (19) and (21) in (22), noting Equation (6) and rearranging, $p_1^*E_1 + p_2^*E_2 = 0$ yields the following equation in w for the constellation of parameter values:

$$\frac{2.8306}{w^{9.5/3}} + \frac{1.6875}{w^2} \cdot (0.585 + 0.035w)(0.9 - 0.1w) = 3.1470.$$