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Fiscal policy, labour market, and inequality

Diagnosing South Africa's anomalies in the shadow of racial discrimination

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Abstract: Inequality in South Africa is the enduring legacy of racial discrimination. We use a dynamic perspective to show the linkages between persistent effects of discrimination in the labour market and the efficacy of redistributive fiscal policy in reducing inequality. We present a machine-learning analysis based on household survey data in the Post-Apartheid Labour Market Series to predict the main drivers of the relationship between workers' heterogeneous socioeconomic characteristics, the behaviour of variables related to labour market status, and labour income inequality. The empirical investigation covers the period 2000–17. Drawing on this preliminary evidence, we build a dynamic stochastic general equilibrium model with a dual labour market and job search frictions that represents the structural features of South Africa's economy, which can be used to assess the effects of fiscal policy on inequality in the post-apartheid period and to simulate the effects of alternative fiscal measures and labour market reforms.

Key words: inequality, discrimination, job search, labour market, general equilibrium

JEL classification: D50, D63, J71, J46

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1 Introduction

Extreme inequality is the most salient problem of post-apartheid South Africa, despite the widening of social security guaranteed by the African National Congress (ANC) government, which won power in 1994 and is still the ruling party today. This issue has received much attention, and even if a number of causes have been analysed in important studies,[1] an extensive literature shows that increasing unemployment and wage differentials in the labour market are the main drivers of overall inequality in the post-apartheid era (see Aguero et al. 2007; Finn and Leibbrandt 2018; Leibbrandt et al. 2018, among many others).

This paper contributes to this literature, providing a broad narrative of a number of stylized facts related to inequality and proposing a theoretical model that can be useful for estimating the sensitivity to economic policy shocks of labour market dynamics and distributional trends. This analysis exploits the recent improvement of the household survey data integrated in the Post-Apartheid Labour Market Series (PALMS), which allows a reliable comparison over time of inequality measures (Merrino 2020).

A body of literature emphasizes that in the post-apartheid period a mix of rising labour costs, skill shortages, and weak international competitiveness has led to capital deepening in the manufacturing sector and a transition to a service-led economy that is high-skill intensive (Bhorat et al. 2020a, 2020b; Rodrik 2008). In this view, the main explanations of structural unemployment and rising wage gaps are wage rigidities, centralized wage bargaining, strong union power at the service of the labour elite, and weaknesses in the educational system that have not been addressed, notwithstanding the increasing government spending in education. Other studies show that corruption and an expanding public sector progressively put fiscal balances under strain, imposing austerity measures that have reduced the efficacy of the redistribution implemented through the expansion of social grants (Bhorat et al. 2020b; Pons-Vignon and Segatti 2013; Scarlato and d'Agostino 2019).

In this paper, we approach this subject stressing a different perspective: the lingering effects of racial discrimination in the labour market. Indeed, a sharp social stratification along racial lines has been sustained over time through a labour market in which members of the middle class and elite are formally employed with a permanent work contract and union coverage, whereas vulnerable workers, who are overwhelmingly African individuals, are employed in precarious relationships or more often are either unemployed or economically inactive (Schotte et al. 2018; Zizzamia et al. 2019). Note that, even if the share of Africans in the middle class has increased from 47 per cent in 2008 to 64 per cent in 2017, only about one-fifth of the population can be considered middle class and Whites are represented disproportionately highly in this class relative to their population share. In addition, the elite is almost homogeneously White (Zizzamia et al. 2019). Similarly, Assouad et al. (2018) show that the legacy of the apartheid system in South Africa is a society characterized by a dualistic structure and absence of a broad middle class comparable in size to that in high-income countries.

A related aspect is the combination of high levels of unemployment for Black workers and worker discouragement (Burns et al. 2010; Kingdon and Knight 2004, 2006). According to Bhorat et al. (2020b), the dualistic nature of the labour market might shape expectations of discouraged work-seekers, reinforcing the advantages of a small portion of highly-skilled workers who easily obtain secure and well-paid jobs in the formal economy while the larger portion of the labour supply have to compete for low-security and low-paid jobs in the informal economy. Zizzamia (2020) complements this evidence, showing that Black, urban youths turn down or quit wage work because they face non-negligible dis-

[1] For example, Alvaredo and Atkinson (2010) explain the evolution of top income shares in South Africa with the former colonial legacy, the effects of apartheid, and the concentration of its natural resource wealth. For a review, see Bhorat et al. (2020a).

incentives to accept low-quality jobs in the formal labour market, given alternative sources of support in the informal economy that increase their outside options. This means that, for disadvantaged South Africans workers, the benefits of employment are not significantly greater than the costs. Interestingly, this study also shows that for urban, unskilled, Black workers with relatively strong outside options (i.e. social networks and state grants), wage employment is often deliberately temporary. On this basis, 'bad' forms of wage employment can be reconceptualized as 'survivalist' just as certain forms of informal self-employment are considered (Zizzamia 2020).

As dualism is reflected in the historical racial segregation in the labour market, and this is the key channel through which inequality persists in South Africa, the theoretical framework of our analysis distinguishes two sides of the economy: the formal economy that is predominantly populated by White individuals; and the informal economy that is populated by Black individuals. What is peculiar in the South African case is that the informal economy integrates informal employment[2] and systems of informal social protection based on the redistribution within households of income earned through precarious wage work in the formal sector and social grants (Burns et al. 2010; Scarlato and d'Agostino 2019).[3]

To underpin these features of South Africa's economy in the theoretical model, we first present a preliminary empirical analysis that uses household survey data and applies age–period cohort analysis and machine-learning methods to highlight the relationship between workers' socioeconomic characteristics, labour market status of Black and White workers in the two segments of the economy, and the correlation of these variables with labour income inequality.

Drawing from this empirical evidence, we then build a dynamic stochastic general equilibrium (DSGE) model with a dual labour market and search frictions that can be used to assess the effects of alternative fiscal policies in such a segmented context. In further research, this model can be applied to test whether a redistributive policy focused on the extension of social grants to the Black population represents an efficient tool to mitigate inequality or may exacerbate the dual structure of the labour market and, hence, indirectly contribute to worsening the cumulative disadvantages of the Black population.

The paper is structured as follows. Section 2 presents the dataset and provides the preliminary empirical analysis. Section 3 proposes a DSGE model for South Africa's dual economy. Section 4 concludes the paper and suggests some policy implications and possible extensions of the paper.

2 Race, hidden employment, and inequality: a data-driven analysis

2.1 Data

We use the PALMS dataset provided by the DataFirst research unit based at the University of Cape Town. The dataset consists of high-quality microdata from 69 household surveys conducted by Statistics South Africa over the period 1993–2019. The harmonized data include the Household Surveys from 1994 to 1999, the Labour Force Surveys from 2000 to 2007, and the Quarterly Labour Force Surveys from 2008 to 2019 (see Finn and Leibbrandt 2018; Kerr and Wittenberg 2019; Merrino 2020). We restrict the analysis to the period 2000–17 for reasons related to data availability.

[2] On the features and size of the informal labour market in South Africa, see Rogan and Skinner (2018).

[3] Social grants benefit about 31 per cent of the country population (SASSA 2019). Over 50 per cent of South Africans live in households with grant income, and grants represent the most important source of income for half of these households (Zizzamia 2020).

In order to analyse the dynamic of inequality, we first transform the data and set-up a pseudo-panel framework that is useful for analysing the time behaviour of the variables of interest when only independent repeated cross-sectional data are available or, as in this case, when we collect data from different sources. Pseudo-panels observe cohorts (i.e. stable groups of individuals) instead of observing individuals over time, and individual variables are replaced by their intra-cohort means. When the aggregation of different data sources introduces a measurement error in the used data, intra-cohort mean values are still consistent if the used survey data are representative of the studied population.

The main concern in applying the pseudo-panel is related to the trade-off between bias and variance in the formation of the cohort. Indeed, the cohort must be large enough to limit the extent of measurement errors in intra-cohort variable means that generate biased and imprecise estimators of the model parameters. Table A1 in the Appendix reports the frequency of observations for each birth cohort (from 1929 to 1999) and calendar year of our sample covering the period 2000–17. The table reveals that the number of individual observations in each year/cohort is large enough to set-up a proper cohort analysis.[4]

After excluding self-employed from the sample, we extracted relevant data on labour income deflated with the 2015 CPI.[5] We restrict the analysis to individuals who are aged between 18 and 65 and exclude individuals in the range 15–17 to avoid problems related to intra-cohort averaging. To further reduce the measurement error in the used data, we use weights to correct for bracket responses and take account of changes in the population distribution, so as to allow for comparisons over time.

The upper part of Table 1 reports some descriptive statistics of monthly labour income and the Gini index by cohort and year. Furthermore, from the PALMS dataset, we extract several variables describing the socioeconomic characteristics of the individuals. In particular, we define the informal sector by using three measures reported in the PALMS dataset, indicating: when the worker has a written contract, when the job is formal, and when the firm operates in the formal market and is VAT registered. Table 1 reports the selected variables along with some descriptive statistics.

Several papers warn that methodological changes in the construction of wages and earnings data occurred from the third quarter of 2012 onwards and discuss the shortcomings of the inequality measures obtained by using the PALMS dataset (Finn and Leibbrandt 2018; Kerr and Wittenberg 2016, 2017). For this reason, our results should be taken with caution.

2.2 Labour income distribution: a cohort analysis

Our starting hypothesis is that racial discrimination and cumulated disadvantages for the Black population have underpinned the division of the labour market in two segments, which we name the formal and informal economy, characterized by different workers' socioeconomic characteristics and structural dynamics. White workers are mainly employed in the formal economy, whereas most Black workers are segregated in the informal economy. The formal economy is regulated by centralized bargaining and high employment protection legislation (EPL) standards, and is characterized by the traditional division between work and non-work, employment and unemployment. Differently, we conceptualize the informal economy referring to the stylized view of labour underutilization that includes labour underutilization according to hours available to work, marginal labour market attachment, and people outside any form of paid employment—the unemployed and the hidden unemployed or discouraged workers (Baum and Mitchell 2010). We coalesce these different labour force states into the informal economy to emphasize the fluid picture of the broad labour market outcomes of Africans.

[4] Following Deaton (2018), the frequency of observations has to be not less than 100 in each year/cohort.

[5] Following the definitions of the PALMS dataset, labour income is measured by the variable 'monthly earnings' generated from data across all waves where earnings amounts were asked and data have been released.

Table 1: Descriptive statistics

Variable	Mean	Standard deviation	Minimum	Maximum
Outcome variable				
Gini index (labour income)	0.56	0.07	0.36	0.81
Covariates				
Age	40	13	18	62
Gender	0.56	0.05	0.44	0.75
Year of education	9.77	1.32	5.8	12.66
Primary education	0.3	0.12	0.09	0.6
Secondary education	0.58	0.17	0.21	0.88
Tertiary education	0.07	0.03	0	0.16
Unmarried	0.38	0.02	0.28	0.98
Married	0.53	0.21	0.02	0.85
African	0.67	0.08	0.37	0.81
Caucasian	0.16	0.06	0.07	0.57
Skilled-occupation	0.23	0.06	0.01	0.39
Informal sector	0.28	0.11	0.13	0.76
Public employment	0.2	0.08	0	0.37
Union member	0.29	0.1	0	0.47
Manufacturing	0.21	0.03	0.12	0.43
Tertiary	0.51	0.1	0.18	0.7

Note: the informal sector comprises individuals working without a written contract with the employer. Skilled-occupation is based on ISCO (International Standard Classification of Occupations).

Source: authors' compilation.

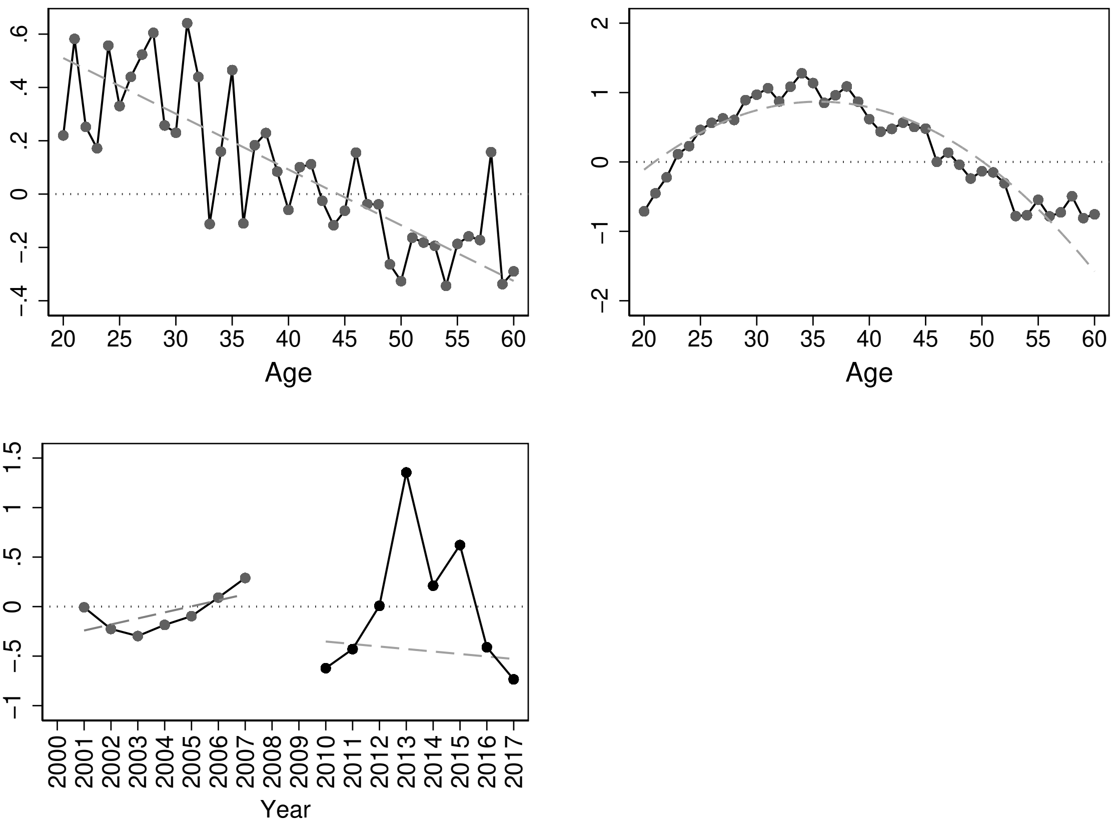
In this section we provide an exploratory analysis to show how worker socioeconomic characteristics and labour market states affect income inequality dynamics over the period 2000–17.

As a first step, following Deaton (2018) we perform the age–period–cohort decomposition of the behaviour of labour income, where the pertaining sets of parameters have a zero-sum and a zero-slope to solve the traditional identification problem (Chauvel 2012). The results are presented in Figure 1. The figure is composed of three panels that disentangle the birth-cohort effect (top-left panel), the age effect (top-right panel), and the time effect (bottom-left panel). This decomposition allows us to distinguish common patterns for all cohorts, the effect related to the life cycle of the individuals, and the effect ascribable to the business cycle.

The first panel indicates that, with very few exceptions, the lines for the younger cohorts are always above the lines for the older cohorts, even when they are observed at the same age. Hence, the rapid economic growth after the end of apartheid, overall, has improved the income of the younger generations despite the persistently high youth unemployment (Yu 2013).[6] The second panel of the figure shows the life cycle profile of the monthly labour income series. We observe that income tends to grow much more rapidly in the early years of the working life of individuals, compared to years after age 40. As a result, the younger cohorts are better off and have experienced a much more rapid income growth compared to the older generations. The last panel of the figure disentangles the time effect and clearly shows a positive trend in labour income before 2013.

[6] According to the Quarterly Labour Force Survey (QLFS), the unemployment rate within the group of young people aged 15–34 years reached 43.2 per cent in the first quarter of 2020. Youth aged 15–24 years are the most vulnerable in the South African labour market as the unemployment rate in this age group was 59.0 per cent in the same period (StatsSA 2020).

Figure 1: Cohort decomposition, labour income



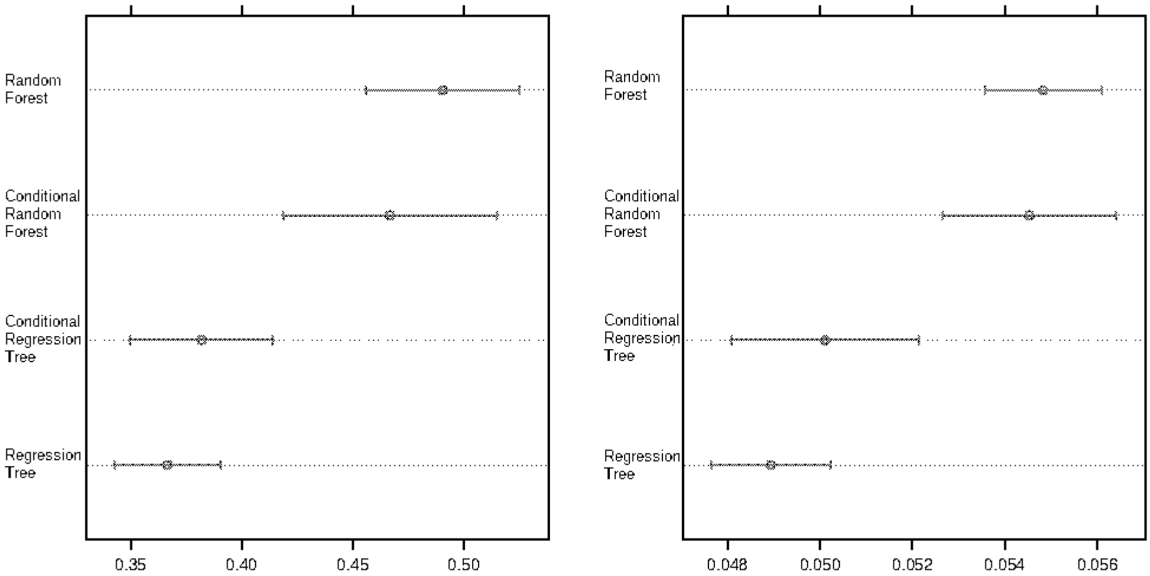
Source: authors' compilation.

We then follow a machine-learning approach to detect a number of basic regularities connecting our variables and draw useful insights for the specification of the theoretical model proposed in the subsequent section. Machine-learning methods have been developed to make out-of-sample predictions of a dependent variable based on a number of observable predictors. They let algorithms automatically select the predictor space, identify non-overlapping regions, and find the best model for predicting the outcome of interest.

The first step of this approach is the model selection and the identification of the machine-learning model that better predicts income inequality, given the selected covariates. In this respect, we compare four models: regression tree, conditional regression tree, random forest, and conditional random forest. Within classification and regression tree methods, the conditional inference tree and conditional inference forest methods show substantial advantages in coping with the shortcomings of traditional non-parametric regression methods in the context of highly correlated variables (Hothorn et al. 2006). Our goal is to identify the model that is able to provide the best prediction of income inequality based on the minimum mean square error and the R^2. The procedure consists of testing how a machine-learning model, estimated on a sub-sample of data (training data), provides a good model approximation when applied to other partitions of the dataset (test data). In the present case, we use 50 bootstrapped partitions as test datasets.

Figure 2 reports the results of the tests for the four chosen machine-learning models. The two graphs show that random forest and conditional random forest are the models with the lowest root-mean-square error and highest R^2 with respect to the other specifications. This result is not surprising because random forest and condition random forest compare 50 different tree specifications to predict the model fit and, hence, they can reduce the influence of outliers. Overall, we see that all the chosen models are very consistent in predicting variations in labour income inequality, given the selected covariates.

Figure 2: Cross validation of the selected models: R^2 (left); RMSE (right)

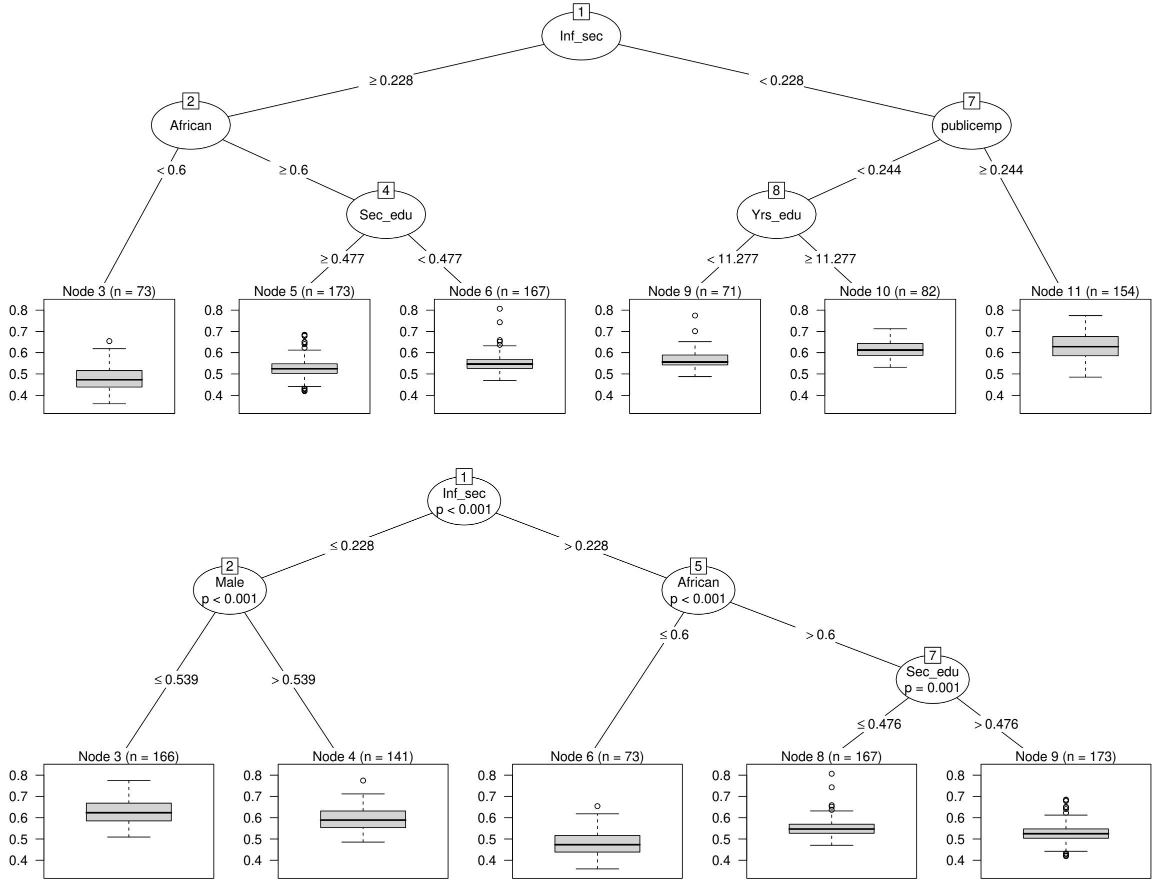


Source: authors' compilation.

Then, we apply the regression tree and conditional regression tree models to the Gini index based on monthly labour income and let an algorithm decide how to partition the population into mutually exclusive types to highlight the main relationships between inequality and the selected socioeconomic variables. The analysis is intuitive as the relevant information can be directly read off the graphical illustration.

Figure 3 compares the diagram of the two models and shows that the labour income inequality structure is mainly determined by the dualism between the formal and informal economies. More in detail, the key explanatory variable is related to working in the informal versus formal economy and, when the presence of workers in the informal economy is prevalent, the other key variables are the racial population group and level of education. These variables are calculated as shares of workers by cohort: the share of workers in the informal sector (Inf_sec), the share of African workers ($African$), and the share of workers with a secondary education degree (sec_educ). It is interesting to note that, when we analyse the birth cohorts with the higher share of informal workers—nodes 1, 2, and 4 in panel (a) and 1, 5, and 7 in panel (b)—the two models predict exactly the same levels of labour income inequality. In the case with a prevalence of informal workers, we also find that, when the share of Africans is below 60 per cent, the final nodes—node 3 in panel (a) and node 6 in panel b)—show the lowest mean value of labour income inequality.

Figure 3: Regression tree models: regression tree (top); conditional regression tree (bottom)



Source: authors' compilation.

Putting together the results obtained from the two models, the evidence shows that labour income inequality in the informal sector is lower, compared to the formal sector, a result that is explained by the polarization between well-paid and high-skilled jobs and bad jobs in the formal economy (Bhorat et al. 2020a, 2020b). In addition, the evidence shows a high heterogeneity in the inequality outcome by racial population groups. We find that labour income inequality is more pronounced for Africans working in the informal economy compared to other racial groups. Interestingly, in this case the level of education does not significantly change the mean outcome. Hence, inequality in the informal economy is mainly driven by race, and level of education is not effective in reducing bad outcomes for African workers who are segregated in the informal economy.

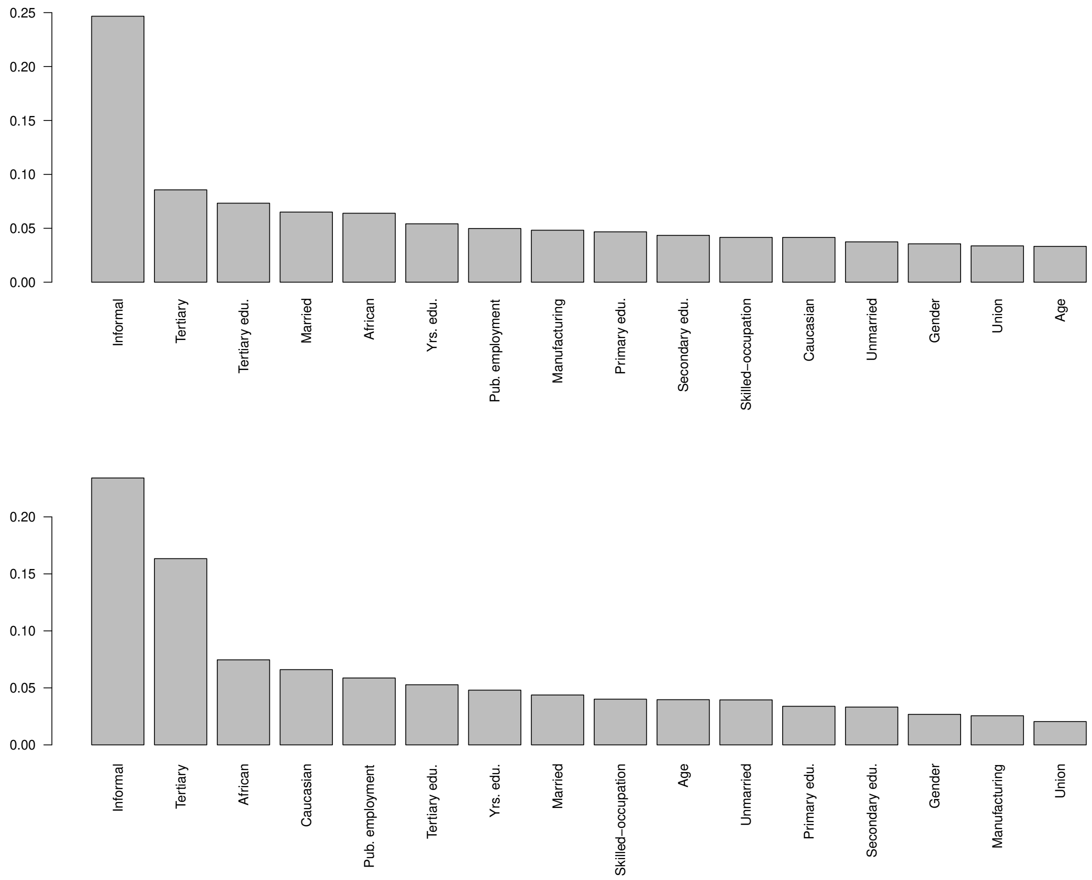
Differently, when formal workers are prevalent (i.e. informal workers are less than 22.8 per cent of the labour force), the regression tree model shows that public employment and years of education characterizes income inequality—see panel (a). It is interesting to note that we find the highest Gini index when the share of employed workers in the public sector exceeds 25 per cent. Indeed, public employment is highly unionized, well-paid, and protected compared to other jobs available to low-skilled workers in the formal economy (Bhorat et al. 2020a). Similarly, the variable related to the level of education indicates the wage polarization that characterizes good and bad jobs in the formal economy, particularly in the tertiary sector. In addition, panel (b) shows the diagram of the conditional regression tree and indicates that gender is the main driving force of labour income inequality for workers who are employed predominantly in the formal economy. This result is well-documented in the literature, showing that gender disparities play an important role in explaining inequality in South Africa (Scarlato and d'Agostino 2019).

We continue the analysis by reporting the results of the random forest and conditional random forest models. In this respect, since random forest models are based on replications of regression tree models, we are not able to present a diagram of each fitted model. Hence, we will use these models to order the selected covariates with respect to their relevance in predicting the distribution of labour income inequality. Figure 4 reports this analysis whereas, to increase the comparability of the two models, we report the importance metric scaled in a range between 0 and 1. More in depth, the importance metric describes how many times a variables is included in a regression tree and hence explains the distribution of inequality.

Comparing the two panels of the figure, we confirm that the dichotomy of the formal/informal economy is the most important aspect describing income inequality in South Africa. In this respect, we find that this variable accounts for about 20 per cent (25 per cent in the conditional random forest model) of the whole set of covariates. A second interesting result is related to the relevance of the tertiary sector in explaining income inequality. As is well known (Bhorat et al. 2020a, 2020b; Rodrik 2008), since the 1990s South African manufacturing lost ground to the tertiary sector. As manufacturing is intensive in less-skilled labour compared to services, this structural change has been a key driver of both unemployment and labour income inequality.

Furthermore, the population group and the variable related to education confirm their good explanatory power. When we account for the share of African workers, we find a discrepancy between the random forest and conditional random forest models. We can explain this result by suggesting that the population group is strongly correlated with other variables, such as the informal sector. As a consequence, the random forest model, which does not account for extremely correlated variables, may attribute part of the effect of the population group to the variable accounting for the informal sector. Differently, the conditional random forest model explicitly accounts for the correlation of the covariates and thus shows that this variable is the third covariate that mainly explains labour income inequality.

Figure 4: Random forest models: random forest (top); conditional random forest (bottom)



Source: authors' compilation.

3 A DSGE analysis

3.1 Theoretical framework

Relying upon the empirical evidence described above, we build a DSGE model that is able to capture the main features of the labour market in South Africa.

Our theoretical framework considers a dual labour market structure in line with the work of Pappa et al. (2015). In particular, we assume that there is a formal sector and an informal sector. However, we extend this setup by considering two representative households, namely White and Black, each consisting of a continuum of infinite living agents. Members of both of these households face endogenous labour decisions and can be formal and informal employees, unemployed job-seekers, and labour force non-participants. The main difference between the members of these two types of households relates to a different probability of joining the formal and informal sectors due to the cumulative effects of past discrimination (Blank 2005). We capture this feature by imposing that, given the same level of search costs sustained by job-seekers, White searchers face a higher probability than Black searchers of filling an open vacancy in the formal sector. The opposite assumption holds for the informal sector. Depending of the prevailing equilibrium we may have that either White and Black workers are employed in both formal and informal sectors or White workers are employed only in the formal sector. Such a segmentation in the labour market could also be the result of a discrimination mechanism arising on the demand side of the labour market as modelled by Holden and Rosen (2014) and Lang and Lehmann (2012).

Wage is Nash-bargained in both formal and informal sectors. Black and White households rent out their private capital to production good firms and purchase the final consumption good. The fiscal authority absorbs part of the gross income of both households by taxing them. More specifically, we consider a consumption tax rate levied on consumption purchases and a tax levied on labour income obtained from the formal sector.

We also assume that formal and informal firms produce and sell two different goods in perfect competitive markets. Production goods in the formal sector are obtained using capital and labour input, whereas informal sector firms use only labour input. Moreover, firms that produce in the informal sector are able to evade the payroll taxes paid on formal employment.

3.2 Labour markets

We account for the imperfections and transaction costs in the labour market by assuming that jobs are created through a matching function. For $j = f, i$ denoting the formal and informal sectors, the total match is given by

$$\begin{aligned} m_t^j &= m^j\left(v_t^j, u_t^{wj}, u_t^{bj}, e_t^{wj}, e_t^{bj}\right) \\ &= m^{wj}\left(v_t^j, u_t^{wj}, e_t^{wj}\right) + m^{bj}\left(v_t^j, u_t^{bj}, e_t^{bj}\right) \end{aligned} \tag{1}$$

where v_t^j is the number of vacancies, u_t^{wj} is the measure of White searchers, and u_t^{bj} denotes the measure of Black searchers. Moreover, e_t^{wj} and e_t^{bj} denote the search intensities for White and Black searchers, respectively.

The matching function for the two groups of individuals is assumed to satisfy

$$m^{wj}\left(v_t^j, u_t^{wj}, e_t^{wj}\right) = \mu_1^{wj}\left(v_t^j\right)^{\mu_2}\left(e_t^{wj}u_t^{wj}\right)^{1-\mu_2} \tag{2}$$

$$m^{bj}\left(v_t^j, u_t^{bj}, e_t^{bj}\right) = \mu_1^{bj}\left(v_t^j\right)^{\mu_2}\left(e_t^{bj}u_t^{bj}\right)^{1-\mu_2} \tag{3}$$

The probability that a vacant job is matched with a searcher depends on the overall labour market tightness as in the standard framework, but also on the relative size of the effective search of Black and White workers. Thus, the probability of a vacancy being filled is defined as

$$\psi_t^j \equiv \frac{m_t^j}{v_t^j} = \left(\frac{u_t^{j*}}{v_t^j}\right)^{1-\mu_2}\left[\mu_1^{wj}\left(e_t^{wj}\frac{u_t^{wj}}{u_t^{j*}}\right)^{1-\mu_2} + \mu_1^{bj}\left(e_t^{bj}\frac{u_t^{bj}}{u_t^{j*}}\right)^{1-\mu_2}\right] \tag{4}$$

where

$$u_t^{j*} = e_t^{wj}u_t^{wj} + e_t^{bj}u_t^{bj} \tag{5}$$

Equation (5) can be defined as the total effective search input in the formal sector.

We define labour market tightness as

$$\theta_t^j = \frac{v_t^j}{u_t^j} \tag{6}$$

The probability of a vacancy being filled by a White worker is given by

$$\psi_t^{wj} = \frac{m^{wj}\left(v_t^j, u_t^{wj}, e_t^{wj}\right)}{v_t^j} = \mu_1^{wj}\left(\frac{e_t^{wj}u_t^{wj}}{v_t^j}\right)^{1-\mu_2} \tag{7}$$

The same probability for a Black worker is given by

$$\psi_t^{bj} = \frac{m^{bj}\left(v_t^j, u_t^{bj}, e_t^{bj}\right)}{v_t^j} = \mu_1^{bj}\left(\frac{e_t^{bj}u_t^{bj}}{v_t^j}\right)^{1-\mu_2} \tag{8}$$

The total job-finding rate per search unit (i.e. the probability of a job-seeker being hired) is defined as

$$\rho_t^j = \left(\frac{v_t^j}{u_t^{j*}}\right)^{\mu_2}\left[\mu_1^{wj}\frac{\left(e_t^{wj}u_t^{wj}\right)^{1-\mu_2}}{\left(u_t^{j*}\right)^{1-\mu_2}} + \mu_1^{bj}\frac{\left(e_t^{bj}u_t^{bj}\right)^{1-\mu_2}}{\left(u_t^{j*}\right)^{1-\mu_2}}\right] \tag{9}$$

The probability to find a job for a White searcher is defined as

$$\rho_t^{wj} = \frac{m^{wj}\left(v_t^j, u_t^{wj}, e_t^{wj}\right)}{e_t^{wj}u_t^{wj}} = \mu_1^{wj}\left(\frac{v_t^j}{e_t^{wj}u_t^{wj}}\right)^{\mu_2} \tag{10}$$

The same probability for a Black searcher is given by

$$\rho_t^{bj} = \frac{m^{bj}\left(v_t^j, u_t^{bj}, e_t^{bj}\right)}{e_t^{bj}u_t^{bj}} = \mu_1^{bj}\left(\frac{v_t^j}{e_t^{bj}u_t^{bj}}\right)^{\mu_2} \tag{11}$$

In each period, jobs are destroyed at a constant fraction, ρ^j. The law of motion of employment, n_t^j, is given by

$$n_{t+1}^j = \left(1-\rho^j\right)n_t^j + \psi_t^j v_t^j \tag{12}$$

where

$$\psi_t^j = \left(\psi_t^{wj} + \psi_t^{bj}\right)$$

and

$$n_t^j = n_t^{wj} + n_t^{bj}$$

The law of motion of White formal employment is given by

$$n_{t+1}^{wf} = \left(1-\rho^f\right)n_t^{wf} + \rho_t^{wf}e_t^{wf}\left(1-s_t^w\right)u_t^w \tag{13}$$

where

$$\rho_t^{wf}e_t^{wf}\left(1-s_t^w\right)u_t^w = v_t^f\psi_t^{wf}$$

The law of motion of Black formal employment is given by

$$n_{t+1}^{bf} = \left(1-\rho^f\right)n_t^{bf} + \rho_t^{bf}e_t^{bf}\left(1-s_t^b\right)u_t^b \tag{14}$$

where

$$\rho_t^{bf}e_t^{bf}\left(1-s_t^b\right)u_t^b = v_t^f\psi_t^{bf}$$

The law of motion of White informal employment is given by

$$n_{t+1}^{wi} = \left(1-\rho^i\right)n_t^{wi} + \rho_t^{wi}e_t^{wi}s_t^w u_t^w \tag{15}$$

where

$$\rho_t^{wi}e_t^{wi}s_t^w u_t^w = v_t^i\psi_t^{wi}$$

The law of motion of Black informal employment is given by

$$n_{t+1}^{bi} = \left(1-\rho^i\right)n_t^{bi} + \rho_t^{bi}e_t^{bi}s_t^b u_t^b \tag{16}$$

where

$$\rho_t^{bi}e_t^{bi}s_t^b u_t^b = v_t^i\psi_t^{bi}$$

3.3 Households

Representative White household

The representative White household maximizes the following utility function with two arguments:

$$U\left(c_t^w, l_t^w\right) = \ln\left(c_t^w - \kappa c_{t-1}^w\right) + \Phi \frac{\left(l_t^w\right)^{1-\varphi}}{1-\varphi} \tag{17}$$

where c_t^w is the private consumption index to be specified below, l_t^w is leisure, $\Phi > 0$ is the relative preference for leisure, and φ is the inverse of the Frisch elasticity of labour supply. The parameter κ measures the degree of external habit formation in consumption.

The representative White household chooses the fraction of the unemployed actively searching for a job, u_t^w, and the fraction that are out of the labour force and enjoying leisure, l_t^w, so that:

$$n_t^{wf} + n_t^{wi} + u_t^w + l_t^w = 1 \tag{18}$$

where n_t^{wf} is the formal employment and n_t^{wi} the informal employment.

Hence,

$$u_t^w = u_t^{wf} + u_t^{wi}$$

The representative White household chooses the fraction of job-seekers searching in each sector: a share s_t^w of job-seekers look for a job in the informal sector, while the remainder, $(1 - s_t^w)$, seek employment in the formal sector. Thus, we have that:

$$u_t^{wi} = s_t^w u_t^w \tag{19}$$

and

$$u_t^{wf} = \left(1 - s_t^w\right) u_t^w \tag{20}$$

Consumption c_t^w is a constant elasticity of substitution (CES) aggregator of formal c_t^{wf} and informal c_t^{wi} goods:

$$c_t^w = \left[(\omega)^{\frac{1}{\varepsilon}} \left(c_t^{wf}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} \left(c_t^{wi}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{21}$$

where $\varepsilon > 0$ represents the intersectoral elasticity of substitution and $\omega \in (0,1)$ is the relative share of formal and informal goods.

The consumption demand conditions are

$$c_t^{wf} = \omega \left(\frac{\left(1+\tau_t^c\right) P_t^f}{P_t}\right)^{-\varepsilon} c_t^w \tag{22}$$

$$c_t^{wi} = (1-\omega) \left(\frac{P_t^i}{P_t}\right)^{-\varepsilon} c_t^w \tag{23}$$

where τ_t^c represent the tax on private consumption paid by households on formal goods.

In Equations (22) and (23), P_t is the aggregate price index:

$$P_t = \left[\omega \left(\left(1+\tau_t^c\right) P_t^f\right)^{1-\varepsilon} + (1-\omega) \left(P_t^i\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{24}$$

where P_t^{wf} and P_t^{wi} are, respectively, the price indices of formal and informal goods.

The representative White household also chooses the level capital stock, which evolves over time according to

$$k_{t+1}^{w} = i_t^w + (1-\delta) k_t^w \tag{25}$$

where i_t^w is investment and δ is a constant depreciation rate.

The intertemporal budget constraint for the representative White household is given by

$$\begin{aligned} c_t^w + P_t^f i_t^w + B_{t+1} = r_t^k k_t^w + (1-\tau_t^n) w_t^f n_t^{wf} + w_t^i n_t^{wi} - P_t^f S_f^w \left(e_t^{wf}\right)(1-s_t^w) u_t^w - P_t^i S_i^i \left(e_t^{wi}\right) s_t^w u_t^w \\ \pi u_t^{wf} + R_t B_t + \Pi_t^p - T_t \end{aligned} \tag{26}$$

where w_t^f is the real wage in the formal sector, w_t^i is the real wage in the informal sector, r_t^k is the real return on capital, π denotes unemployment benefits, available only to formal job-seekers (see, for example, Boeri and Garibaldi 2007), B_t is the real government bond holdings, R_t is the real rate on risk-free one-period government bonds, and Π_t^p are the profits of the formal firms. Moreover, τ_t^c, τ_t^n, and T_t represent tax on private consumption, labour income tax, and lump-sum taxes, respectively.

The representative White household maximizes expected lifetime utility, (Equation 17) subject to Equations (13), (15), (18), and (26). We can write in full the Lagrangian for the representative household's maximization problem:

$$\mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \ln\left(c_t^w - \kappa c_{t-1}^w\right) + \Phi \frac{\left(1-n_t^{wf}-n_t^{wi}-u_t^w\right)^{1-\varphi}}{1-\varphi} \\ -\lambda_t^{wc} \left[\begin{array}{c} c_t^w + P_t^f \left(k_{t+1}^w - (1-\delta) k_t^w\right) + B_{t+1} - r_t^k k_t^w \\ -(1-\tau_t^n) w_t^f n_t^{wf} + P_t^f S_f^w \left(e_t^{wf}\right)(1-s_t^w) u_t^w \\ -w_t^i n_t^{wi} + P_t^i S_i^i \left(e_t^{wi}\right) s_t^w u_t^w - \pi (1-s_t^w) u_t^w \\ -R_t B_t - \Pi_t^p + T_t \end{array} \right] \\ -\lambda_t^{wf} \left[n_{t+1}^{wf} - \left(1-\rho^f\right) n_t^{wf} - e_t^{wf} \rho_t^{wf} (1-s_t^w) u_t^w\right] \\ -\lambda_t^{wi} \left[n_{t+1}^{wi} - \left(1-\rho^i\right) n_t^{wi} - e_t^{wi} \rho_t^{wi} s_t^w u_t^w\right] \end{array} \right\}$$

where e_t^{wj} (for $j = f, i$) denotes the search intensity in the formal and informal sector and $S_j^w \left(e_t^{wj}\right)$ (for $j = f, i$) represents the search costs for the formal sector and informal sector with $S_f^w(.) > S_i^i(.)$.

The first-order condition for c_t^w is given by

$$\frac{1}{c_t^w - \kappa c_{t-1}^w} - \lambda_t^{wc} = 0 \tag{27}$$

The first-order condition for B_{t+1} is given by

$$\lambda_t^{wc} - \beta \lambda_{t+1}^{wc} R_{t+1} = 0 \tag{28}$$

The first-order condition for k_{t+1}^w is given by

$$-\lambda_t^{wc} P_t^f + \beta \lambda_{t+1}^{wc} P_{t+1}^f \left(1 - \delta + r_{t+1}^k\right) = 0 \tag{29}$$

The first-order condition for n_{t+1}^{wf} is given by

$$-\lambda_t^{wf} + \beta \left[\lambda_{t+1}^{wc} \left(1+\tau_{t+1}^n\right) w_{t+1}^f - \Phi \left(l_{t+1}^w\right)^{-\varphi} + \lambda_{t+1}^{wf} \left(1-\rho^f\right)\right] = 0 \tag{30}$$

The first-order condition for n_{t+1}^{wi} is given by

$$-\lambda_t^{wi} + \beta \left[\lambda_{t+1}^{wc} w_{t+1}^i - \Phi \left(l_{t+1}^w\right)^{-\varphi} + \lambda_{t+1}^{wi} \left(1-\rho^i\right)\right] = 0 \tag{31}$$

The first-order condition for u_t^w is given by

$$\lambda_t^{wc}\left[(1-s_t^w)\pi - P_t^f S_f^w\left(e_t^{wf}\right)(1-s_t^w) - P_t^i S_i^w\left(e_t^{wi}\right)s_t^w\right] \tag{32}$$
$$= -\lambda_t^{wf}\left[e_t^{wf}\,\rho_t^{wf}\,(1-s_t^w)\right] - \lambda_t^{wi}\left[e_t^{wi}\rho_t^{wi}s_t^w\right] + \Phi\left(l_{t+1}^w\right)^{-\varphi}$$

The first-order condition for s_t^w is given by

$$\lambda_t^{wc}\left[P_t^f S_f^w\left(e_t^{wf}\right) - P_t^i S_i^w\left(e_t^{wi}\right) - \pi\right]u_t^w = \lambda_t^{wf}\left(e_t^{wf}\rho_t^{wf}u_t^w\right) - \lambda_t^{wi}\left(e_t^{wi}\rho_t^{wi}u_t^w\right) \tag{33}$$

The first-order condition for e_t^{wf} is given by

$$-\lambda_t^{wc}P_t^f S_f^{w\prime}\left(e_t^{wf}\right)(1-s_t^w)u_t^w + \lambda_t^{wf}\rho_t^{wf}(1-s_t^w)u_t^w = 0 \tag{34}$$

The first-order condition for e_t^{wi} is given by

$$-\lambda_t^{wc}P_t^i S_i^{w\prime}\left(e_t^{wi}\right)s_t^w u_t^w + \lambda_t^{wi}\rho_t^{wi}s_t^w u_t^w = 0 \tag{35}$$

Equations (27) and (28) are the arbitrage conditions for the returns to consumption and bonds. Equation 29 shows the standard expression for the real return on capital. Equations (30) and (31) relate the expected marginal value from being employed in each sector to the wage, accounting for the income tax in the regular sector, the utility loss from the reduction in leisure, and the continuation value (which depends on the separation probability).

Equation (32) states that the value of being unemployed (i.e. an active searcher in the labour market) should equal the marginal value of being out of the labour force, given by the utility from leisure minus the expected marginal values of being employed in each sector, weighted by the respective job-finding probabilities and shares of job-seekers.

Equation (33) is the arbitrage condition between being a searcher in the informal or the formal sector according to which the choice of the share, s_t^w, is such that the marginal gain of being a searcher in the informal sector, that is the difference between the savings in formal sector search cost, $S_f^w\left(e_t^{wf}\right)$, minus informal sector search cost, $S_i^w\left(e_t^{wi}\right)$, and the forgone unemployment benefit, π, must equate the marginal cost of being an 'informal searcher', given by the difference in the expected marginal values of being employed in each sector.

Equations (34) and (35) imply that, in both the informal and formal sectors, the optimal level of search intensity is such that the marginal cost of search must be equal to its marginal benefit.

Now we turn to the derivation of the workers' surplus—that is, the value of a match for the family. For the formal sector, we have

$$\kappa_t^{wf} = \frac{\partial \mathcal{U}_t}{\partial n_t^{wf}}\frac{1}{\omega_t^{wc}} \tag{36}$$

For the informal sector, we have

$$\kappa_t^{wi} = \frac{\partial \mathcal{U}_t}{\partial n_t^{wi}}\frac{1}{\omega_t^{wc}} \tag{37}$$

In both Equations (36) and (37), ω_t^{wc} is the marginal utility.

The total value of a match for a worker—that is, the difference between the value of employment (in each sector) and the value of unemployment—is obtained from the household's maximization problem. As in Trigari (2009), we assume that workers value their actions on the basis of the contribution of these actions to the utility of their household. Hence, the surplus that a single worker obtains from employment can be defined as the change in the household's utility from having one additional member employed (in the formal and the informal sectors). In each period, a fraction of household members, n_t^{wf}, is employed, earns a formal market wage, ceases to pay search costs, and incurs the disutility of work; the remaining fraction is searching for a job and sustains the corresponding costs, and for the fraction $(1-s_t^w)\,u_t^w$, earns the unemployment benefit π.

The change in the household's optimal utility of having an additional member employed is given by the solution of the following maximization problem with respect to the state variable n_t^{wf} and n_t^{wi}:

$$\mathscr{U}_t\left(n_t^{wf},n_t^{wi}\right) = \max_{n_t^{wf},n_t^{wi}} \left[\begin{array}{c} \ln\left(c_t^w-\kappa c_{t-1}^w\right)+\Phi\frac{\left(1-n_t^{wf}-n_t^{wi}-u_t^w\right)^{1-\varphi}}{1-\varphi}+ \\ \beta E_t \mathscr{U}_{t+1}\left(n_{t+1}^{wf},n_{t+1}^{wi}\right)\end{array}\right]$$

$$\text{s.t.:}\quad \begin{array}{c} c_t^w = -B_{t+1}+(1-\tau_t^n)\,w_t^f n_t^{wf} - \\ P_t^f S_f^w\left(e_t^{wf}\right)\left(1-n_t^{wi}-n_t^{wf}-s_t^w u_t^w - l_t^w\right)+w_t^i n_t^{wi} - \\ P_t^i S_i^w\left(e_t^{wi}\right)\left(1-n_t^{wi}-n_t^{wf}-(1-s_t^w)\,u_t^w-l_t^w\right)+ \\ \pi\left(1-n_t^{wi}-n_t^{wf}-s_t^w u_t^w-l_t^w\right)+R_t B_t+\Pi_t^p-T_t \end{array}$$

$$\text{and: } n_{t+1}^{wf} = \left(1-\rho^f\right)n_t^{wf}+e_t^{wf}\rho_t^{wf}\left(1-n_t^{wi}-n_t^{wf}-s_t^w u_t^w-l_t^w\right)$$

$$\text{and: } n_{t+1}^{wi} = \left(1-\rho^i\right)n_t^{wi}+e_t^{wi}\rho_t^{wi}\left(1-n_t^{wi}-n_t^{wf}-(1-s_t^w)\,u_t^w-l_t^w\right)$$

The first-order condition n_t^{wf} is given by

$$\begin{aligned}\frac{\partial \mathscr{U}_t}{\partial n_t^{wf}} &= (c_t^w-\kappa c_{t-1}^w)^{-1}\left[(1-\tau_t^n)\,w_t^f+P_t^f S_f^w\left(e_t^{wf}\right)+P_t^i S_i^w\left(e_t^{wi}\right)-\pi\right]- \\ &\Phi\,(l_t^w)^{-\varphi}+\beta(1-\rho^f-e_t^{wf}\psi_t^{wf})E_t\frac{\partial \mathscr{U}_{t+1}}{\partial n_{t+1}^{wf}}\end{aligned}$$

The first-order condition n_t^{wi} is given by

$$\begin{aligned}\frac{\partial \mathscr{U}_t}{\partial n_t^{wi}} &= (c_t^w-\kappa c_{t-1}^w)^{-1}\left[w_t^i+P_t^f S_f^w\left(e_t^{wf}\right)+P_t^i S_i^w\left(e_t^{wi}\right)-\pi\right]- \\ &\Phi\,(l_t^w)^{-\varphi}+\beta(1-\rho^i-e_t^{wi}\rho_t^{wi})E_t\frac{\partial \mathscr{U}_{t+1}}{\partial n_{t+1}^{wi}}\end{aligned}$$

The derivatives $\frac{\partial \mathscr{U}_t}{\partial n_t^{wf}}$ and $\frac{\partial \mathscr{U}_t}{\partial n_t^{wi}}$ represent the surpluses from employment for the household. Hence, according to the previous discussion, the value of a match for the worker, expressed in terms of current consumption units, can be defined as

$$\kappa_t^{wf} = \frac{\partial \mathscr{U}_t}{\partial n_t^{wf}}\frac{1}{\omega_t^{wc}} = \frac{\partial \mathscr{U}_t}{\partial n_t^{wf}}\frac{1}{(c_t^w-\kappa c_{t-1}^w)^{-1}}$$

and

$$\kappa_t^{wi} = \frac{\partial \mathscr{U}_t}{\partial n_t^{wi}}\frac{1}{\omega_t^{wc}} = \frac{\partial \mathscr{U}_t}{\partial n_t^{wi}}\frac{1}{(c_t^w-\kappa c_{t-1}^w)^{-1}}$$

Substituting, we have that

$$\begin{aligned}\kappa_t^{wf} &= \left[(1-\tau_t^n)\,w_t^f+P_t^f S_f^w\left(e_t^{wf}\right)+P_t^i S_i^w\left(e_t^{wi}\right)-\pi\right]- \\ &\frac{\Phi\,(l_t^w)^{-\varphi}}{\omega_t^{wc}}+\beta(1-\rho^f-e_t^{wf}\rho_t^{wf})E_t\frac{\omega_{t+1}^{wc}}{\omega_t^{wc}}\kappa_{t+1}^{wf}\end{aligned} \tag{38}$$

Similarly, we have that

$$\kappa_t^{wi} = \left[w_t^i + P_t^f S_f^w \left(e_t^{wf}\right) + P_t^i S_i^w \left(e_t^{wi}\right) - \pi\right] - \frac{\Phi\left(l_t^w\right)^{-\varphi}}{\omega_t^{wc}} + +\beta(1-\rho^i - e_t^{wi}\rho_t^{wi})E_t \frac{\omega_{t+1}^{wc}}{\omega_t^{wc}}\kappa_{t+1}^{wi} \tag{39}$$

Equation (38) indicates that the gain of a worker that is hired in the formal sector consists of the net income, $(1-\tau_t^n)\, w_t^f$. Moreover, the worker avoids paying search costs in both formal and informal sectors, $P_t^f S_f^w\left(e_t^{wf}\right) + P_t^i S_i^w\left(e_t^{wi}\right)$, but he renounces the unemployment benefit, π, and the utility deriving from leisure, $\frac{\Phi(l_t^w)^{-\varphi}}{\omega_t^{wc}}$. If the worker keeps the job in the next period he obtains the actual value of future surplus. We interpret Equation (39) in a similar way, except that the informal sector is not subject to labour income tax.

Representative Black household

The representative Black household is symmetric to the representative White household. Therefore, the description of the equations that we discussed in the previous section applies in this section.

The instantaneous utility function for the representative Black household is given by

$$U\left(c_t^b, l_t^b\right) = \ln\left(c_t^b - \kappa c_{t-1}^b\right) + \Phi\frac{\left(l_t^b\right)^{1-\varphi}}{1-\varphi} \tag{40}$$

where c_t^b is private consumption and l_t^b is leisure.

We define the fraction of the unemployed actively searching for a job, u_t^b, and the fraction that are out of the labour force and enjoying leisure, l_t^b, so that

$$n_t^{bf} + n_t^{bi} + u_t^b + l_t^b = 1 \tag{41}$$

where n_t^{bf} is formal employment and n_t^{bi} is informal employment.

As above, a share s_t^b of job-seekers look for a job in the informal sector, while the remainder, $\left(1-s_t^b\right)$, seek employment in the formal sector:

$$u_t^{bi} = s_t^b u_t^b \tag{42}$$

and

$$u_t^{bf} = \left(1-s_t^b\right) u_t^b \tag{43}$$

We assume that consumption c_t^b is a CES aggregator of formal c_t^{bf} and informal c_t^{bi} goods:

$$c_t^b = \left[(\omega)^{\frac{1}{\varepsilon}}\left(c_t^{bf}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}}\left(c_t^{bi}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{44}$$

The consumption demand conditions are

$$c_t^{bf} = \omega\left(\frac{(1+\tau_t^c)\,P_t^f}{P_t}\right)^{-\varepsilon} c_t^b \tag{45}$$

$$c_t^{bi} = (1-\omega)\left(\frac{P_t^i}{P_t}\right)^{-\varepsilon} c_t^b \tag{46}$$

The aggregate price index is

$$P_t = \left[\omega\left((1+\tau_t^c)\,P_t^f\right)^{1-\varepsilon} + (1-\omega)\left(P_t^i\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{47}$$

where P_t^{bf} and P_t^{bi} are, respectively, the price indices of formal and informal goods.

The law of capital motion corresponds to

$$k_{t+1}^b = i_t^b + (1-\delta)\,k_t^b \tag{48}$$

where i_t^b is investment.

The intertemporal budget constraint is given by

$$\begin{aligned} c_t^b + P_t^f i_t^b + B_{t+1} = r_t^k k_t^b + (1+\tau_t^n)\,w_t^f n_t^{bf} + w_t^i n_t^{bi} + \\ \pi u_t^{bf} + R_t B_t + \Pi_t^i - T_t - P_t^f S_f^b\left(e_t^{bf}\right)\left(1-s_t^b\right)u_t^b - P_t^i S_i^b\left(e_t^{bi}\right)s_t^b u_t^b \end{aligned} \tag{49}$$

As in the previous section for the representative White household, the expected lifetime utility (Equation 40) is maximized subject to Equations (14), (16), (41), and (49). Thus, we can write the Lagrangian as follows:

$$\mathscr{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \ln\left(c_t^b - \kappa c_{t-1}^b\right) + \Phi \frac{\left(1-n_t^{bf}-n_t^{bi}-u_t^b\right)^{1-\varphi}}{1-\varphi} \\ -\lambda_t^{wc} \left[\begin{array}{c} c_t^b + P_t^f\left(k_{t+1}^b - (1-\delta)\,k_t^b\right) + B_{t+1} - r_t^k k_t^b \\ -(1+\tau_t^n)\,w_t^f n_t^{bf} + P_t^f S_f^b\left(e_t^{bf}\right)\left(1-s_t^b\right)u_t^b \\ -w_t^i n_t^{bi} + P_t^i S_i^b\left(e_t^{bi}\right)s_t^b u_t^b - \pi\left(1-s_t^b\right)u_t^b \\ -R_t B_t - \Pi_t^i + T_t \end{array} \right] \\ -\lambda_t^{bf}\left[n_{t+1}^{bf} - \left(1-\rho^f\right)n_t^{bf} - e_t^{bf}\rho_t^{bf}\left(1-s_t^b\right)u_t^b\right] \\ -\lambda_t^{bi}\left[n_{t+1}^{bi} - \left(1-\rho^i\right)n_t^{bi} - e_t^{bi}\rho_t^{bi}s_t^b u_t^b\right] \end{array} \right\}$$

where e_t^{bj} (for $j=f,i$) denotes the search intensity in the formal and informal sectors and $S_j^b\left(e_t^{bj}\right)$ (for $j=f,i$) represents the search costs for the formal and informal sectors.

We impose that, for any given level of search effort e, we have

$$S_f^w(e) < S_f^b(e) \tag{50}$$

$$S_i^w(e) \geqslant S_i^b(e) \tag{51}$$

Expressions (50) and (51) can be interpreted as follows. Given the same level of search cost sustained by the searchers, the job-finding rate of White searchers in the formal sector is higher with respect to that of Black searchers, whereas the opposite is true for the informal sector. Since the vacancies posted by firms are not directed to a specific group, the majority of White workers end up working in the formal sector, whereas most Black workers will be hired in the informal sector. Given that the proportion of unemployed workers searching for a formal (or informal) job is an endogenous control variable of each household, then different equilibria may exist depending on the prevalence of an interior or a corner solution in this choice.

The first-order condition for c_t^b is given by

$$\frac{1}{c_t^b - \kappa c_{t-1}^b} - \lambda_t^{bc} = 0 \tag{52}$$

The first-order condition for B_{t+1} is given by

$$\lambda_t^{bc} - \beta\lambda_{t+1}^{bc}R_{t+1} = 0 \tag{53}$$

The first-order condition for k_{t+1}^{b} is given by

$$-\lambda_t^{bc} P_t^f + \beta \lambda_{t+1}^{bc} P_{t+1}^f \left(1 - \delta + r_{t+1}^k\right) = 0 \tag{54}$$

The first-order condition for n_{t+1}^{bf} is given by

$$-\lambda_t^{bf} + \beta \left[\lambda_{t+1}^{bc} \left(\left(1+\tau_{t+1}^n\right) w_{t+1}^f\right) - \Phi \left(l_{t+1}^b\right)^{-\varphi} + \lambda_{t+1}^{bf} \left(1-\rho^f\right)\right] = 0 \tag{55}$$

The first-order condition for n_{t+1}^{bi} is given by

$$-\lambda_t^{bi} + \beta \left[\lambda_{t+1}^{bc} \left(w_{t+1}^i\right) - \Phi \left(l_{t+1}^b\right)^{-\varphi} + \lambda_{t+1}^{bi} \left(1-\rho^i\right)\right] = 0 \tag{56}$$

The first-order condition for u_t^b is given by

$$\begin{aligned} &\lambda_t^{bc} \left[\left(1-s_t^b\right)\pi - P_t^f S_f^b\left(e_t^{bf}\right)\left(1-s_t^b\right) - P_t^i S_i^b\left(e_t^{bi}\right) s_t^b\right] \\ &= -\lambda_t^{bf}\left[e_t^{bf}\rho_t^{bf}\left(1-s_t^b\right)\right] - \lambda_t^{bi}\left[e_t^{bi}\rho_t^{bi}s_t^b\right] + \Phi\left(l_{t+1}^b\right)^{-\varphi} \end{aligned} \tag{57}$$

The first-order condition for s_t^b is given by

$$\lambda_t^{bc}\left[P_t^f S_f^b\left(e_t^{bf}\right) - P_t^i S_i^b\left(e_t^{bi}\right) - \pi\right] u_t^b = \lambda_t^{bf}\left(e_t^{bf}\rho_t^{bf}u_t^b\right) - \lambda_t^{bi}\left(e_t^{bi}\rho_t^{bi}u_t^b\right) \tag{58}$$

The first-order condition for e_t^{bf} is given by

$$-\lambda_t^{bc} P_t^f S_f^{b\prime}\left(e_t^{bf}\right)\left(1-s_t^b\right) u_t^b + \lambda_t^{bf}\rho_t^{bf}\left(1-s_t^b\right)u_t^b = 0 \tag{59}$$

The first-order condition for e_t^{bi} is given by

$$-\lambda_t^{bc} P_t^i S_i^{b\prime}\left(e_t^{bi}\right) s_t^b u_t^b + \lambda_t^{bi}\rho_t^{bi} s_t^b u_t^b = 0 \tag{60}$$

The workers' surplus for the formal sector is given by

$$\kappa_t^{bf} = \frac{\partial \mathscr{U}_t}{\partial n_t^{bf}} \frac{1}{\omega_t^{bc}} \tag{61}$$

For the informal sector, we have

$$\kappa_t^{bi} = \frac{\partial \mathscr{U}_t}{\partial n_t^{bi}} \frac{1}{\omega_t^{bc}} \tag{62}$$

where ω_t^{wc} is the marginal utility.

The change in the household's optimal utility of having an additional member employed is given by the solution of the following maximization problem with respect to the state variable n_t^{bf} and n_t^{bi}:

$$\mathscr{U}_t\left(n_t^{bf}, n_t^{bi}\right) = \max_{n_t^{bf}, n_t^{bi}} \left[\begin{array}{c} \ln\left(c_t^b - \kappa c_{t-1}^b\right) + \Phi\frac{\left(1-n_t^{bf}-n_t^{bi}-u_t^b\right)^{1-\varphi}}{1-\varphi} + \\ \beta E_t \mathscr{U}_{t+1}\left(n_{t+1}^{bf}, n_{t+1}^{bi}\right) \end{array}\right]$$

$$\text{s.t.:} \quad \begin{array}{c} \left(1+\tau_t^c\right)c_t^b = -B_{t+1} + \left(1-\tau_t^n\right) w_t^f n_t^{bf} - \\ P_t^f S_f^b\left(e_t^{bf}\right)\left(1-n_t^{bi}-n_t^{bf}-s_t^b u_t^b - l_t^b\right) + w_t^i n_t^{bi} - \\ P_t^i S_i^b\left(e_t^{bi}\right)\left(1-n_t^{bi}-n_t^{bf}-\left(1-s_t^b\right)u_t^b - l_t^b\right) + \\ \pi\left(1-n_t^{bi}-n_t^{bf}-s_t^b u_t^b - l_t^b\right) + R_t B_t + \Pi_t^p + \Pi_t^{cp} - T_t \end{array}$$

$$\text{and: } n_{t+1}^{bf} = \left(1-\rho^f\right)n_t^{bf} + e_t^{bf}\rho_t^{bf}\left(1-n_t^{bi}-n_t^{bf}-s_t^b u_t^b - l_t^b\right)$$

$$\text{and: } n_{t+1}^{bi} = \left(1-\rho^i\right)n_t^{bi} + e_t^{bi}\rho_t^{bi}\left(1-n_t^{bi}-n_t^{bf}-\left(1-s_t^b\right)u_t^b - l_t^b\right)$$

The first-order condition for n_t^{bf} is given by

$$\frac{\partial \mathcal{U}_t}{\partial n_t^{bf}} = (c_t^b - \kappa c_{t-1}^b)^{-1} \left[(1-\tau_t^n) w_t^f + P_t^f S_f^b \left(e_t^{bf} \right) + P_t^i S_i^b \left(e_t^{bi} \right) - \pi \right]$$
$$- \Phi \left(l_t^b \right)^{-\varphi} + \beta (1 - \rho^f - e_t^{bf} \psi_t^{bf}) E_t \frac{\partial \mathcal{U}_{t+1}}{\partial n_{t+1}^{bf}}$$

The first-order condition for n_t^{bi} is given by

$$\frac{\partial \mathcal{U}_t}{\partial n_t^{bi}} = (c_t^b - \kappa c_{t-1}^b)^{-1} \left[w_t^i + P_t^f S_f^b \left(e_t^{bf} \right) + P_t^i S_i^b \left(e_t^{bi} \right) - \pi \right]$$
$$- \Phi \left(l_t^b \right)^{-\varphi} + \beta (1 - \rho^i - e_t^{bi} \psi_t^{bi}) E_t \frac{\partial \mathcal{U}_{t+1}}{\partial n_{t+1}^{bi}}$$

The value of a match for the worker, expressed in terms of current consumption units, can be defined as

$$\kappa_t^{bf} = \frac{\partial \mathcal{U}_t}{\partial n_t^{bf}} \frac{1}{\omega_t^{bc}} = \frac{\partial \mathcal{U}_t}{\partial n_t^{bf}} \frac{1}{(c_t^b - \kappa c_{t-1}^b)^{-1}}$$

and

$$\kappa_t^{bi} = \frac{\partial \mathcal{U}_t}{\partial n_t^{bi}} \frac{1}{\omega_t^{bc}} = \frac{\partial \mathcal{U}_t}{\partial n_t^{bi}} \frac{1}{(c_t^b - \kappa c_{t-1}^b)^{-1}}$$

Substituting, we have that

$$\kappa_t^{bf} = \left[(1-\tau_t^n) w_t^f + P_t^f S_f^b \left(e_t^{bf} \right) + P_t^i S_i^b \left(e_t^{bi} \right) - \pi \right] - \tag{63}$$
$$\frac{\Phi \left(l_t^b \right)^{-\varphi}}{\omega_t^{bc}} + \beta (1 - \rho^f - e_t^{bf} \rho_t^{bf}) E_t \frac{\omega_{t+1}^{bc}}{\omega_t^{bc}} \kappa_{t+1}^{bf}$$

and

$$\kappa_t^{bi} = \left[w_t^i + P_t^f S_f^b \left(e_t^{bf} \right) + P_t^i S_i^b \left(e_t^{bi} \right) - \pi \right] - \tag{64}$$
$$\frac{\Phi \left(l_t^b \right)^{-\varphi}}{\omega_t^{bc}} + \beta (1 - \rho^i - e_t^{bi} \rho_t^{bi}) E_t \frac{\omega_{t+1}^{bc}}{\omega_t^{bc}} \kappa_{t+1}^{bi}$$

3.4 Firms

Production goods are produced by two different firms (one representative firm for each sector) which adopt two different technologies:

$$y_t^f = \left(A_t^f n_t^f \right)^{1-\alpha^f} \left(k_t^f \right)^{\alpha^f}$$
$$y_t^i = \left(A_t^i n_t^i \right)^{1-\alpha^i}$$

where A_t^j denotes total factor productivity in sector j and $A_t^f > A_t^i$. We assume that the informal production technology uses labour inputs only (e.g. Busato and Chiarini 2004). Firms in each sector maximize the discounted value of future profits, subject to Equation (12). In particular, they take the number of workers currently employed in each sector, n_t^j, as given and choose the number of vacancies posted, v_t^j, so as to employ the desired number of workers in the next period, n_{t+1}^j. Here, firms adjust employment by varying the number of workers (extensive margin) rather than the number of hours per worker (intensive margin).

Formal sector firms

Firms in the formal sector choose the number of vacancies to be posted and also decide the amount of private capital, k_t, needed for production. Thus, the optimization problem of a firm is summarized by the following Bellman equation:

$$\mathcal{Q}\left(n_t^f\right) = \max_{k_t, v_t^f} \left[\begin{array}{c} P_t^f y_t^f - (1+\tau_t^s) w_t^f n_t^f - \left(r_t^k + \delta\right) k_t \\ -P_t^f \iota^f v_t^f + \beta E_t \Lambda_{t,t+1} \mathcal{Q}\left(n_{t+1}^f\right) \end{array} \right]$$

$$\text{s.t.: } y_t^f = \left(A_t^f n_t^f\right)^{1-\alpha^f} \left(k_t^f\right)^{\alpha^f}$$

$$\text{and: } n_{t+1}^f = \left(1-\rho^f\right) n_t^f + \psi_t^f v_t^f = \left(1-\rho^f\right) n_t^f + \left(\psi_t^{wf} + \psi_t^{bf}\right) v_t^f$$

From the first-order conditions we obtain the usual job-creation condition and the demand for capital services, respectively:

$$\frac{\iota^f}{\psi_t^f} = \beta E_t \Lambda_{t,t+1} \left[P_{t+1}^f \frac{\partial \left(y_{t+1}^f\right)}{\partial n_{t+1}^f} - \left(1+\tau_{t+1}^s\right) w_{t+1}^f + \left(1-\rho^f\right) \frac{P_t^f \iota^f}{\psi_{t+1}^f} \right] \tag{65}$$

and

$$r_t^k + \delta = P_t^f \left(\frac{\alpha^f y_t^f}{k_t^f} \right) \tag{66}$$

According to Equation (65), the expected marginal cost of hiring a worker in the formal sector should equal the expected marginal benefit. The latter includes the net value of the marginal product of labour minus the wage, augmented by the payroll tax in the formal sector, plus the continuation value. Equation (66) indicates that the net value of the marginal product of private capital should equal the real rental rate.

Informal sector firms

Firms in the informal sector choose the number of vacancies to be posted. Hence the optimization problem of a firm is summarized by the following Bellman equation:

$$\mathcal{Q}\left(n_t^i\right) = \max_{v_t^i} \left[P_t^i \left(y_t^i\right) - w_t^i n_t^i - P_t^i \iota^i v_t^i + \beta E_t \Lambda_{t,t+1}^i \mathcal{Q}\left(n_{t+1}^i\right)\right]$$

$$\text{s.t.: } y_t^i = \left(A_t^i n_t^i\right)^{1-\alpha^i}$$

$$\text{and: } n_{t+1}^i = \left(1-\rho^i\right) n_t^i + \psi_t^i v_t^i = \left(1-\rho^i\right) n_t^i + \left(\psi^{wi} + \psi_t^{bi}\right) v_t^i$$

The job-creation condition is given by

$$\frac{\iota^i}{\psi_t^i} = \beta E_t \Lambda_{t,t+1}^i \left[P_{t+1}^i \frac{\partial y_{t+1}^i}{\partial n_{t+1}^i} - w_{t+1}^i + \left(1-\rho^i\right) \frac{P_t^i \iota^i}{\psi_{t+1}^i} \right] \tag{67}$$

Equation (67) has a similar interpretation to Equation (65). However, in this case the payroll tax is absent.

3.5 Wage determination

Wages in both sectors are determined by ex-post (after matching) Nash bargaining. White and Black workers as well as formal and informal firms split rents and the part of the surplus that they receive depends on their bargaining power.

In the formal sector, we assume that White workers are the major representatives of the union. In this sector, the wage is determined through a Nash-bargaining process with White workers. Thus, the Nash optimization problem is given by

$$\max_{w_t^f} (\kappa_t^{wf})^{df} (\digamma_t^f)^{1-df}$$

where df and $1-df$ denote the bargaining weights.

We know that

$$\begin{aligned}\kappa_t^{wf} = & \left[(1-\tau_t^n) w_t^f + P_t^f S_f^w \left(e_t^{wf}\right) + P_t^i S_i^w \left(e_t^{wi}\right) - \pi\right] \\ & - \frac{\Phi \left(l_t^w\right)^{-\varphi}}{\omega_t^{wc}} + \beta(1-\rho^f - e_t^{wf}\rho_t^{wf}) E_t \frac{\omega_{t+1}^{wc}}{\omega_t^{wc}} \kappa_{t+1}^{wf}\end{aligned}$$

and

$$\digamma_t^f = P_t^f \frac{\partial y_t^f}{\partial n_t^f} - (1+\tau_t^s) w_t^f + \left(1-\rho^f\right) \frac{P_t^f \iota^f}{\psi_t^f}$$

By solving the Nash bargaining, the wage in the formal sector is

$$\begin{aligned}w_t^f = & \frac{df}{(1+\tau_t^s)} \left(P_t^f \frac{\partial \left(y_t^f\right)}{\partial n_t^f} + e_t^{wf} \rho_t^{wf} \frac{P_t^f \iota^f}{\psi_t^f} \right) + \\ & (1-df) \frac{1}{(1-\tau_t^n)} \left(\frac{\Phi \left(l_t^w\right)^{-\varphi}}{\omega_t^{wc}} + \pi - P_t^f S_f^w \left(e_t^{wf}\right) - P_t^i S_i^w \left(e_t^{wi}\right) \right)\end{aligned} \tag{68}$$

The last expression can be interpreted as follows. If formal workers have all the bargaining power (i.e. $df \to 1$), the wage tends to the so-called 'firm reservation wage'—that is, the sum of the marginal product of labour and the continuation value. If formal firms have all the bargaining power $(df \to 0)$, the wage tends to the worker reservation wage—that is, the sum of the disutility from working and the unemployment benefit net of the search costs paid by the searcher. In all other cases, the prevailing wage is going to be a weighted average of these two extreme cases.

In the informal sector, we assume that Black workers are the major representatives of the union. In this sector, the wage is determined through a Nash-bargaining process with Black workers. Thus, the Nash optimization problem is given by

$$\max_{w_t^i} (\kappa_t^{bi})^{di} (\digamma_t^i)^{1-di}$$

where di and $1-di$ denote the bargaining weights.

We know that

$$\begin{aligned}\kappa_t^{bi} = & \left[w_t^i + P_t^f S_f^b \left(e_t^{bf}\right) + P_t^i S_i^b \left(e_t^{bi}\right) - \pi\right] - \\ & \frac{\Phi \left(l_t^b\right)^{-\varphi}}{\omega_t^{bc}} + \beta(1-\rho^i - e_t^{bi}\rho_t^{bi}) E_t \frac{\omega_{t+1}^{bc}}{\omega_t^{bc}} \kappa_{t+1}^{bi}\end{aligned}$$

and

$$\digamma_t^i = P_t^i \frac{\partial \left(y_t^i\right)}{\partial n_t^i} - w_t^i + \left(1-\rho^i\right) \frac{P_t^i \iota^i}{\psi_t^i}$$

Finally, the wage in the formal sector is given by

$$\begin{aligned}w_t^i = & di \left[P_t^i \frac{\partial \left(y_t^i\right)}{\partial n_t^i} + e_t^{bi} \rho_t^{bi} \frac{P_t^i \iota^i}{\psi_t^i} \right] + \\ & (1-di) \left[\frac{\Phi \left(l_t^b\right)^{-\varphi}}{\omega_t^{bc}} + \pi - P_t^f S_f^b \left(e_t^{bf}\right) - P_t^i S_i^b \left(e_t^{bi}\right) \right]\end{aligned} \tag{69}$$

Equation (69) has a similar interpretation as Equation (68). However, in this case the labour income tax and the payroll tax are absent.

3.6 Fiscal sector

Government budget constraints assume that the financing of consumption purchases and unemployment benefits takes place through tax revenues and issuing bonds:

$$P_t^f G_t + \pi\left(\left(1-s_t^w\right)u_t^w + \left(1-s_t^b\right)u_t^b\right) + B_{t-1} = TR_t + \frac{B_t}{R_t} \tag{70}$$

where tax revenues are given by

$$TR_t = \left(\tau_t^n + \tau_t^s\right) w_t^f n_t^f + \tau_t^c\left(c_t^{wf} + c_t^{bf}\right) + T_t \tag{71}$$

We use fiscal policy rules that are in line with Leeper et al. (2010):

$$\hat{\tau}_t^n = \varphi^{yn}\hat{y}_t + \gamma^{bn}\hat{b}_{t-1} + \hat{\varepsilon}_t^n \tag{72}$$
$$\text{where: } \hat{\varepsilon}_t^n = \rho^n\hat{\varepsilon}_{t-1}^n + \eta_t^n \tag{73}$$
$$\hat{\tau}_t^s = \varphi^{ys}\hat{y}_t + \gamma^{bs}\hat{b}_{t-1} + \hat{\varepsilon}_t^s \tag{74}$$
$$\text{where: } \hat{\varepsilon}_t^s = \rho^s\hat{\varepsilon}_{t-1}^s + \eta_t^s \tag{75}$$
$$\hat{\tau}_t^c = \varphi^{yc}\hat{y}_t + \gamma^{bc}\hat{b}_{t-1} + \hat{\varepsilon}_t^c \tag{76}$$
$$\text{where: } \hat{\varepsilon}_t^c = \rho^c\hat{\varepsilon}_{t-1}^c + \eta_t^c \tag{77}$$
$$\hat{t}_t = -\varphi^{yt}\hat{y}_t - \gamma^t\hat{b}_{t-1} + \hat{\varepsilon}_t^t \tag{78}$$
$$\text{where: } \hat{\varepsilon}_t^t = \rho^t\hat{\varepsilon}_{t-1}^t + \eta_t^t \tag{79}$$
$$\hat{g}_t = -\varphi^{yg}\hat{y}_t - \gamma^g\hat{b}_{t-1} + \hat{\varepsilon}_t^g \tag{80}$$
$$\text{where: } \hat{\varepsilon}_t^{g^u} = \rho^{g^u}\hat{\varepsilon}_{t-1}^{g^u} + \eta_t^{g^u} \tag{81}$$

In Equations (72)–(81) the small hatted letters denote that the variables are expressed in terms of log deviations around the deterministic steady state. Moreover, $\hat{\varepsilon}_t^n$, $\hat{\varepsilon}_t^s$, $\hat{\varepsilon}_t^c$, $\hat{\varepsilon}_t^t$, and $\hat{\varepsilon}_t^g$ are assumed to follow distinct AR(1) processes and each of the η is distributed i.i.d. N (0,1). All our fiscal policy rules have two characteristics. First, we assume that the fiscal variables respond to contemporaneous variations of output ($\varphi^{yn} \geq 0$, $\varphi^{ys} \geq 0$,$\varphi^{yc} \geq 0$, $\varphi^{yt} \geq 0$, and $\varphi^{yg} \geq 0$). Second, our rules allow for dynamic responses to changes in government debt ($\gamma^{bn} \geq 0$, $\gamma^{bs} \geq 0$, $\gamma^{bc} \geq 0$, $\gamma^{bt} \geq 0$, and $\gamma^{bg} \geq 0$). Moreover, in order to include the persistence in taxes, transfers, and expenditures, we allow for the shocks to be serially correlated ($\rho^n \in [0,1]$, $\rho^s \in [0,1]$, $\rho^c \in [0,1]$, $\rho^t \in [0,1]$, and $\rho^g \in [0,1]$). In order to capture unexpected changes in distortionary taxes, lump-sum transfers, and spending, we assume that fiscal rules include exogenous processes ($\hat{\varepsilon}_t^n$, $\hat{\varepsilon}_t^s$, $\hat{\varepsilon}_t^c$, $\hat{\varepsilon}_t^t$, and $\hat{\varepsilon}_t^g$, respectively).

3.7 Aggregation and good market equilibrium

Aggregate capital stock, investment, employment, consumption, and search costs for formal and informal sectors are given by

$$k_t = k_t^w + k_t^b \tag{82}$$
$$i_t = i_t^w + i_t^b \tag{83}$$
$$n_t = n_t^w + n_t^b \tag{84}$$
$$c_t = c_t^w + c_t^b \tag{85}$$
$$S_{Tot}^f\left(e_t\right) = S_f^w\left(e_t^{bf}\right) + S_f^b\left(e_t^{bf}\right) \tag{86}$$
$$S_{Tot}^i\left(e_t\right) = S_i^w\left(e_t^{bi}\right) + S_i^b\left(e_t^{bi}\right) \tag{87}$$

Finally, the equilibrium in the goods market in each sector requires that

$$\left(A_t^i n_t^i\right)^{1-\alpha^i} = c_t^{wi} + c_t^{bi} + S_{Tot}^i\left(e_t\right) + \iota^i v_t^i \tag{88}$$

and

$$\left(A_t^f n_t^f\right)^{1-\alpha^f}\left(k_t^f\right)^{\alpha^f} = c_t^{wf} + c_t^{bf} + i_t^w + i_t^b + G_t + S_{Tot}^f\left(e_t\right) + \iota^f v_t^f \tag{89}$$

3.8 Model implications and potential extensions

Following Leeper et al. (2010), our model assumes several policy rules for the distortive taxes that affect households and firms. The government collects the taxes from the formal sector and uses these revenues in order to finance its expenditure and implement labour market policies such as unemployment benefits and income support schemes (Zanetti 2011).

Given our rich theoretical framework, we can provide simulations of our model for policy analysis. More specifically, an impulse response analysis can be carried out. Assuming exogenous shocks to the several taxes and government consumption expenditure, we can assess the response of the main macroeconomic aggregates in South Africa. In a different exercise, we can quantify the effects of different search costs for White and Black workers in the South African labour market. We may also analyse the effects of labour market reforms such as changes in the efficiency of the labour-matching functions or in the Nash-bargaining weights. Finally, we can provide a variance decomposition analysis in order to study the importance of different shocks in the South African economy.

Hence, our analysis is able to capture the specific features of South Africa's dual economy and to consider the institutional reforms and changes in fiscal policy in the post-apartheid period.

Finally, we believe that the current theoretical framework could be extended in order to consider the effect of rent-seeking. In line with the studies of Angelopoulos et al. (2009, 2010), we could incorporate rent-seeking competition from state coffers into our DSGE model. As argued by Ivanyna et al. (2018: 7) 'rent seeking diverts funds that could be used for investment toward transfer payments and government consumption ... it can also cause the funds that are budgeted for investment to be misallocated, as political considerations dominate economic ones'. Accordingly, we could assume that the government collects tax revenues to finance public goods and services, but private agents use a part of these resources to extract a fraction of that revenue for their own personal benefit.

4 Concluding remarks

This paper provides an empirical and a theoretical contribution to the analysis of inequality in South Africa. The empirical investigation is based on the PALMS dataset recently made available by the DataFirst research unit of the University of Cape Town. After transforming the dataset in a pseudo-panel framework, we follow a machine-learning approach to identify the main drivers of labour income inequality over the years 2000–17. The analysis reveals that labour income distribution is determined by the structure of the economy—that is, the dualism between the formal and informal sectors. Gender and public sector employment are the main drivers of inequality in the formal economy, whereas the key variable in the informal economy is the racial population group. In both cases, level of education doesn't significantly change bad outcomes in the labour market.

Overall, the empirical analysis shows that inequality mainly depends on the high share of Africans segregated in the informal economy and in low-paid jobs in the formal economy, which is characterized by a high polarization between good and bad jobs. The structural nature of inequality explains the apparent paradox that the expansion of the system of social grants has been effective in reducing poverty but not in breaking the intergenerational transmission of poverty within the Black majority—that is, the African population.

This analysis suggests as a policy implication that to reduce inequality in South Africa both fiscal policy and a thorough reform of the labour market are needed. Fiscal policy should not be limited to redistributive measures based on social grants, but should rather increase productive public investment (e.g., universal social services, transport infrastructure, public housing) that support the intensity of Africans' job search in the formal economy and increases their job-finding rate. Similarly, labour market reforms should be aimed at strengthening the social insurance component of the social security system and public training for young African people in order to reinforce the attachment of Africans to the formal labour market and their access to good jobs. Abolishment of the most precarious work, adoption of minimum-wage laws for occupations characterized by the presence of a large, low-skilled labour force, and improvements in social rights in the formal economy are also important for reducing the effects of the segregation of Africans into bad jobs. Last, a system of racial quotas to support African workers could be experimented with to overcome the lingering effect of segregation of Black people, favouring their access to the most protected and well-paid jobs, such as those in the public sector.

Relying on this empirical evidence, we have built a DSGE model that captures the specific features of South Africa's economy: dualism and the lingering effects of racial segregation. The model is novel in a number of assumptions: the setup considers a formal and an informal sector and two representative households, namely White and Black, each consisting of a continuum of infinite living agents. We also account for imperfections and frictions in the labour market using a matching function and for the dualism between formal/informal sectors and Black/White workers, assuming different hypotheses concerning the functioning of the labour market segments and agents' search effort.

The important contribution of this model is that it is based on our previous microeconomic evidence and can be simulated for policy analysis and to test our proposed policy implications. In further research, we may simulate the effect of the fiscal policy and labour market reforms implemented in the post-apartheid period to evaluate their effectiveness in reducing inequality, and we may compare the impact of alternative policies. An interesting issue would also be to extend the theoretical model to account for corruption and rent-seeking, and to simulate their impacts on the effectiveness of South Africa's redistributive policy based on the system of social grants.

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Appendix

Table A1: Cohort distribution by year

	Birth cohort														
Calendar year	1929	1934	1939	1944	1949	1954	1959	1964	1969	1974	1979	1984	1989	1994	1999	Total
2000	0	126	1,211	1,884	2,965	3,902	4,719	4,859	4,795	3,599	1,208	0	0	0	0	29,268
2001	0	0	1,484	2,518	4,292	5,133	7,047	6,930	7,412	5,540	2,435	0	0	0	0	42,791
2002	0	0	1,064	2,300	4,065	5,001	6,893	6,772	6,959	5,903	3,181	237	0	0	0	42,375
2003	0	0	651	1,979	3,621	4,564	6,144	6,580	6,529	5,771	3,656	525	0	0	0	40,020
2004	0	0	325	1,850	3,379	4,294	5,631	6,452	6,639	6,105	4,411	1,025	0	0	0	40,111
2005	0	0	177	1,597	3,194	4,226	5,613	6,052	6,910	6,297	5,128	1,821	0	0	0	41,015
2006	0	0	0	1,369	2,830	4,182	5,145	6,230	6,468	6,494	5,800	2,848	0	0	0	41,366
2007	0	0	0	1,038	2,701	4,006	4,959	6,039	6,732	6,476	6,221	3,664	237	0	0	42,073
2008	0	0	0	1,311	4,773	7,447	9,814	11,270	12,344	12,314	12,910	8,978	1,113	0	0	82,274
2009	0	0	0	690	4,040	6,472	8,973	10,212	11,631	11,393	11,519	8,936	1,691	0	0	75,557
2010	0	0	0	297	3,187	5,678	7,933	9,527	10,419	10,130	10,361	8,776	2,355	0	0	68,663
2011	0	0	0	0	2,830	5,500	7,399	8,767	9,820	9,822	10,391	9,223	3,463	0	0	67,215
2012	0	0	0	0	2,143	5,270	7,611	8,785	10,141	10,636	10,906	9,888	4,716	133	0	70,229
2013	0	0	0	0	1,463	4,941	7,253	9,030	10,185	10,593	11,434	11,024	6,384	505	0	72,812
2014	0	0	0	0	852	4,518	7,189	8,614	9,886	9,996	10,810	11,223	7,404	1,162	0	71,654
2015	0	0	0	0	345	3,186	5,411	6,845	8,506	9,422	10,346	10,698	7,779	1,834	0	64,372
2016	0	0	0	0	0	2,549	4,885	6,356	7,733	8,654	9,934	10,297	8,018	2,481	0	60,907
2017	0	0	0	0	0	1,932	4,654	6,064	7,688	8,651	9,980	10,230	8,343	3,561	154	61,257

Source: authors' compilation.