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Rural roads and urban agglomeration economies

Benefits for town and country?

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Abstract: Do urban agglomeration economies enhance the social profitability of rural roads? When all goods are traded at parametric world prices, lower transport costs benefit villagers. Urban activities and welfare are unaffected if labour is immobile, but their levels fall when rural workers move freely to take up urban jobs while remaining members of their extended families. In a closed, two-good economy with mobile labour, the effects of agglomeration economies depend on the substitutability of rural and urban goods. With a Cobb–Douglas rural technology, aggregate benefits are substantially greater in the presence of empirically plausible elasticities of agglomeration economies when preferences are Cobb–Douglas and urban households' tastes for urban goods are somewhat stronger than those of rural households. When the goods are rather poor substitutes, these enhancing effects are quite small. In an open economy with a single non-tradeable whose production is relatively labour-intensive, improved rural roads will likely induce a fall in urban welfare in the presence of agglomeration economies, even with Cobb–Douglas preferences and immobile labour.

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1 Introduction

Providing villages with all-weather roads has its attractions as a way of improving rural welfare in less developed countries. Farmers should enjoy improved net terms of trade, and all villagers, as consumers, will pay less for urban goods. There are also potential benefits in the form of better schooling and a faster trip to a clinic or hospital, especially in an emergency. While estimating all these benefits is a central and demanding task,[1] what has been rather neglected are the consequences for towns and their inhabitants of all the associated movements of goods and people between town and country. This neglect may lead to serious errors when evaluating the social profitability of rural roads programmes, for it is highly improbable that their effects are confined to villages. By increasing villagers' purchasing power, such programmes enlarge the market for some urban goods, thus generating additional benefits when there are agglomeration economies. Yet, by making rural life more attractive, they will also stem rural–urban migration, thus putting a brake on urban production. Is it safe to assume that the net effect is small? The authors of *Reshaping Economic Geography* (World Bank 2009), for example, do not address this question directly—though their strictures on the folly of limiting rural–urban migration (World Bank 2009:140–42) rather lead one to infer that investing in rural roads, in whose financing the Bank has been heavily involved, has at least one serious drawback.[2]

The aim of this paper is to analyse how strongly urban agglomeration economies influence the ensuing changes in economic activity and welfare in both town and country. It therefore marries reductions in transport costs between two locations with increasing returns in one of them. This calls for a general equilibrium treatment, in which the reallocation of resources induced by improved rural roads depends, first, on the extent to which goods are internationally tradeable at parametric world prices; for with such market opportunities, there is never a shortage of demand. Two other salient factors are the mobility of rural labour and differences in tastes between town and country. Several variations on a common structure are analysed, with illustrative numerical examples aimed at establishing the likely magnitude of the main effects.

In order to keep things tractable, certain effects are ruled out. Urban production stays in the towns already established before any improvements in the rural road network are made—that is, villages produce only rural goods. Schooling and health are unaffected, whatever the levels of air and water pollution, as well as those personal contacts that further the propagation of communicable diseases.[3] Given the general setting and particular object of this paper, the representation of urban production structure and external economies is similarly parsimonious.[4] With the exception of the benchmark case in Section 2, wherein all goods are internationally traded, there is no substitutability among inputs in urban produc-

[1] Studies of the effects of rural roads programmes on rural output, incomes, and poverty in various developing countries include Fan et al. (2000), Jacoby (2000), Escobal and Ponce (2002), Jacoby and Minten (2009), Khandker et al. (2009), and Warr (2010). For a unified, partly speculative, estimate of the chief rural benefits generated by India's *Pradhan Mantri Gram Sadak Yojana* (PMGSY), see Bell (2012). This very large scheme will eventually cover some 170,000 habitations, take at least 20 years to implement, and cost in excess of US$40 billion (World Bank 2010).

[2] The authors of *Infrastructure for Development* (World Bank 1994) are largely concerned with improving efficiency in the provision of infrastructure. Rural roads receive little attention, and rural–urban migration is hardly mentioned in any connection.

[3] In a broad-ranging survey of urbanization that dwells on the distinct possibility that there are too many mega-cities, Henderson (2002) emphasizes the costs of congestion and pollution.

[4] For extensive surveys of the theoretical and empirical literature on agglomeration economies, see Behrens and Robert-Picoud (2016) and Combes and Gobillon (2016), respectively. Cottineau et al. (2016) demonstrate, using French data, that the size of agglomeration economies depends on the definition of what is 'urban'.

tion. Congestion is treated, parenthetically in Section 3.3, as an external diseconomy, and agglomeration economies are represented as net of the latter. Trade and transport are competitively organized.[5]

The extent to which external economies are exploited depends on the extent of the market and supplies of labour. Consider a small, open economy that faces parametrically given world prices for tradeable goods. If all goods are thus traded, labour is intersectorally immobile and unit transport costs between the ports, border crossings, and towns are constant, then prices in towns will be independent of the condition of the rural road network, so that improving the latter will improve rural welfare, but leave urban activity and welfare unchanged. When rural–urban migration relaxes the supply of labour to urban firms, improvements in rural welfare can well result in lower urban output and welfare by making migration less attractive. The formal argument is set out in Section 2.

In practice, transport costs—and other barriers to trade—are so high that a whole variety of goods are neither exported nor imported, nor are they likely to be under any conceivable constellation of domestic productivity levels and tastes. In order to give demand full play, Section 3 deals with the other extreme of a closed dual economy, in which villages produce a rural good, a single city produces an urban one, and both goods are freely traded internally, with 'iceberg' transport costs. Labour is fully mobile, with a fixed urban wage; and the opportunity cost of rural labour is set to zero, thereby creating especially favourable conditions for urban agglomeration economies to exert their influence. The effects of a rural roads programme on the system are analysed, first, in the absence of agglomeration economies (Section 3.1), and then in their presence (Section 3.2). The programme's effects on welfare depend on the substitutability of the two goods in consumption and the degree to which the urban good is substitutable for the fixed factor in rural production. The numerical examples in Section 3.4 are based on a Cobb–Douglas technology in rural production and two values of the elasticity of substitution in consumption, namely, -1 (Cobb–Douglas) and the alternative -0.5, in which the goods are rather poor substitutes. These assumptions, together with a constellation of arguably plausible parameter values, permit computation of the equivalent variation under varying strengths of the economies of urban agglomeration. In this iso-elastic world, improving rural roads will very likely generate substantially greater aggregate benefits in the presence of such externalities when preferences are Cobb–Douglas, but rather small enhancing effects under the alternative.

Intuition suggests that when only some goods are internationally tradeable, a rural roads programme will have effects that lie in between those for these extreme cases. Section 4 addresses this conjecture in a third variant, with two internationally tradeable goods and one non-tradeable, which is producible only in the city but freely traded domestically. A key factor governing the effects of such a programme in the presence of agglomeration economies is the difference between the labour intensities of the two goods produced in the city. If, as is commonly the case in practice, the non-tradeable is relatively labour-intensive, agglomeration economies may well lead to modestly lower urban welfare, while yielding slight additional benefits for villagers, relative to the levels that would hold in the absence of such economies. The paper concludes with a brief discussion in Section 5.

2 The open economy: all goods tradeable

A small open economy comprises a city, its rural hinterland, and a port. The villagers produce good 1, whose world price, p_1^*, is parametrically given. What they do not consume themselves, they sell to agents in the city, where good 2 is produced. The marketed surplus can be consumed, used as an input in the

[5] Casaburi et al. (2013) analyse various market structures with reference to rural Senegal. In their partial equilibrium framework, rural output is assumed to be fixed, as are the only urban variables, namely the urban prices of rural goods.

production of good 2, or transported to the port for export. The world CIF (costs, insurance, and freight) price of good 2, p_2^*, is also parametrically given. Like land in the rural sector, let there be a specific fixed factor which is used in the production of good 2, so that both goods will be produced domestically; and let the endowments of the specific factors and world prices be such that in equilibrium, good 1 is indeed exported and good 2 imported.

The three locations—rural hinterland, city, and port—are denoted by the index $k = 1,2,3$, respectively, and the price of good i in location k by p_{ik}. With domestic prices tethered to world prices by arbitrage and domestic transport costs, the farm-gate (producer) price of good 1 is $p_{11} = p_1^* - t_{112} - t_{123}$ and the producer price of good 2, that is at the factory gate in the city, is $p_{22} = p_2^* + t_{232}$, where $t_{ikk'}$ is the unit cost of transporting good i from location k to location k'. Villagers pay

$$p_{21} = p_{22} + t_{221} \tag{1}$$

for good 2, whether they consume it or use it as an input in the production of good 1; and firms and households in the city pay

$$p_{12} = p_{11} + t_{112} = p_1^* - t_{123} \tag{2}$$

for good 1.

2.1 Immobile labour

Let there be no rural–urban migration, so that labour forms part of the specific factor endowment in each sector, which is supplied perfectly inelastically. Rural households, which are assumed to be identical, choose inputs of good 2 so as to maximize their net revenues, taking prices as parametrically given. In aggregate, they obtain $p_{11}Y_1 + p_{21}Y_{21}$, where Y_1 denotes their aggregate output of good 1 and, with the usual convention that inputs have a negative sign, $Y_{21}(< 0)$ denotes the aggregate output of good 2 in sector 1. The corresponding derived demand function for good 2 is (algebraically) increasing in p_{12}, the aggregate supply of good 1, $Y_1(p_{11}, p_{12})$, is decreasing therein, and the net revenue, or profit, function $p_{11}Y_1(p_{11}, p_{12}) + p_{12}Y_{21}(p_{11}, p_{12})$ is likewise and also convex.

Firms in the city combine inputs of good 1 with the specific factor to produce good 2, choosing inputs so as to maximize profits at the parametrically given prices at that location. The resulting profits are distributed to urban households as income. Production of good 2 is subject to Marshallian external economies: the higher is the level of aggregate output, Y_2, the lower are each firm's costs of production. Consider firm j. Its profits are $p_{22}y_{2j} - c_j(p_{12}, y_{2j}, Y_2)$, where $c_j(p_{12}, y_{2j}, Y_2)$ is its cost function. Let firms be numerous and, when choosing a production plan, let each of them make Nash conjectures concerning the plans of the rest. Then, ignoring its contribution to Y_2, firm j will choose y_{2j} so as to equate marginal revenue with marginal (private) cost: $p_{22} = \partial c_j(p_{12}, y_{2j}, Y_2)/\partial y_{2j}$. The market failure that arises from this external economy results in a level of aggregate output of good 2 that is socially too small.

The government now undertakes a rural roads programme, which reduces the unit transport costs t_{112} and t_{221}. If there are no accompanying changes in the unit transport costs along the trunk route between the city and the port, namely t_{123} and t_{232}, and the reductions in t_{112} and t_{221} are passed on, at least in part, rural households will be better off. Suppose, further, that unit transport costs are independent of the volume of traffic. Then competition and arbitrage will yield an increase in p_{11} and a decrease in p_{21} by the full amount of the reductions in t_{112} and t_{221}, respectively. In the absence of any changes in the unit costs on the trunk route, however, there will be no changes in the prices of goods 1 and 2 in the city. Hence, firms will have no cause to change their production plans, and constructing the rural roads will have no effect on the activities of urban firms or the welfare of urban households.

If both goods are normal in consumption, the improvement in rural households' terms of trade will be accompanied by a greater volume of both goods transported. With no change in urban production, exports of good 1 and imports of good 2 will both increase. In virtue of Walras' Law, the balance of trade is preserved at world prices: $p_1^* E_1 + p_2^* E_2 = 0$ always holds in equilibrium, where E_i denotes the net exports of good i. One may well ask whether these movements do not constitute an increase in the level of urban production, broadly construed. The answer is that there is no transport sector as such in the above structure. Transport costs should be thought of as 'iceberg costs': a fixed fraction of each shipment of good i between locations k and k' simply disappears along the way. The rural roads programme will then have no spillover effects on the city, except that freight movements will increase; and in the absence of any congestion, the city's inhabitants will experience no changes in anything that matters to them.

The failure of the rural roads programme to generate any spillover effects on the urban sector under the above assumptions stems from the fact that changes in domestic demand have no effect on domestic production when all goods are traded at parametric world prices and labour is immobile. Even if transport activities were so organized that urban agents could extract profits by serving rural households' needs for transportation services, prices in the city would remain tethered to parametric world prices by arbitrage along the trunk route between the port and the city.

2.2 Rural-urban migration

Various possibilities—and complications—arise when labour is intersectorally mobile. If villagers commute to urban jobs, they pay fares and lose time in travelling; and if they buy goods in the towns, part of their families' total expenditure is made at urban prices. If, instead, they move to towns, they may lose their claims on the imputed rents arising from the family's fixed endowments (principally land); and in that event, a new urban household is formed, but without claims on the incomes derived from the urban fixed factor. Then again, the rural household may remain an extended family unit, pooling all income, but making some expenditures at urban prices.

There is no space here to go through such variations; the following one will serve for present purposes. Suppose migrants remain members of the extended rural family and, for simplicity, that all rural family expenditures are made at village prices. Daily commuting by villagers closely resembles such an arrangement; but urban residence may also be a fair approximation if migrants make up a small fraction of the population composed of rural households. The latter decide how to allocate their labour between the family farm and urban jobs paying the wage rate w, as well as the level of the urban good to be used in rural production. Since they are assumed to be identical, the individual household's decision problem may be written in the form

$$\max_{(\mathbf{X}_1, L_1, Y_{12})} U_1(\mathbf{X}_1) \text{ s.t. } M_1 \equiv p_{11}Y_1 + p_{21}Y_{21} + w(\bar{L}_1 - L_1) \geq \mathbf{p}_1\mathbf{X}_1, (\mathbf{X}_1, L_1, Y_{12}) \geq \mathbf{0}, \tag{3}$$

where $\mathbf{X}_1 = (X_{11}, X_{21})$ denotes the aggregate rural consumption bundle, M_1 aggregate rural income, $\bar{L}_1$ the aggregate rural labour endowment, L_1 the input of labour on family farms, and $\bar{L}_1 - L_1 \geq 0$ is the labour supply of those who commute to, or are resident in, the city. The rural roads programme leads to an increase in p_{11} and a decrease in p_{21}. It follows from the envelope theorem and $Y_1 > X_{11}$ that the programme will make villagers better off if it does not also lead to a sufficiently sharp fall in w whenever some villagers already have urban jobs.

The urban wage rate is equal to the supply price of rural labour to firms, namely the opportunity cost of rural labour, $p_{11} \cdot \partial Y_1 / \partial L_1$. The programme increases p_{11} and reduces p_{21}, thereby making heavier use of good 2 in rural production more attractive. Hence, there is scope for labour to be reallocated in favour of rural production without necessarily inducing a fall in w. If Y_1 is sufficiently weakly concave in L_1, or the cross-derivative $\partial^2 Y_1/\partial Y_{21}\partial L_1$ is sufficiently large, the marginal product $\partial Y_1/\partial L_1$ will fall, if at all, only a little after such a reallocation, with the result that w will increase.

Urban firms' and households' decisions are separable, the former maximizing profits and the latter utility, given prices and income. Let these households supply their endowments of labour completely inelastically to urban production alone. Firms choose inputs of good 1 and labour optimally, returning all profits to urban households, whose aggregate income is therefore

$$M_2 = p_{22}Y_2 + p_{12}Y_{12} - wL_{12}, \tag{4}$$

where $L_2 = \bar{L}_2 + L_{12}$ is the total input of labour in urban production and $L_{12} = \bar{L}_1 - L_1$ is the labour supplied by migrants, which is assumed to be non-negative—that is, urban households supply no labour to rural production. In equilibrium, $L_2 = \bar{L}_1 + \bar{L}_2 - L_1$.

The programme has no effect on the urban prices of goods. From the firms' first-order conditions, it follows that the marginal product of good 1 is also constant. In the absence of any agglomeration economies, Y_2 will be strictly concave in good 1 and labour alone, so that L_2, and hence L_{12}, is decreasing in w. The entire allocation in the city results from the decision problem

$$\max_{(\mathbf{X}_2, L_2, Y_{21})} U_2(\mathbf{X}_2) \quad \text{s.t.}\, M_2 \geq \mathbf{p}_2\mathbf{X}_2,\ (4),\ (\mathbf{X}_2, L_2, Y_{21}) \geq \mathbf{0}. \tag{5}$$

It follows from the envelope theorem that the programme will lower urban welfare if w rises and, as just assumed, firms were hiring rural labour before the programme is introduced; and if w does rise, urban output will contract. Since a rural roads programme will lead to an increase in both p_{11} and the relative price of goods 1 and 2 in the hinterland, p_{11}/p_{21}, the wage rate will indeed rise.

Proposition 1 *If both goods are internationally traded, their production technologies are strictly concave in variable inputs, and migration takes the form described above, then in the absence of agglomeration economies, the programme will induce an increase in the wage rate, and hence a contraction of urban output and a fall in urban welfare.*

Proof. See Appendix.

When agglomeration economies are at work, any changes in Y_2 will have additional effects on urban productivity. By continuity, however, Proposition 1 will still hold if such economies are sufficiently weak. Yet it remains to be established whether agglomeration economies intensify or mitigate the adverse effects on the urban sector.

Suppose agglomeration economies are sufficiently weak, so that the programme leads to a fall in urban output at unchanged urban goods prices $\mathbf{p}_2$ and a higher wage. Hence, ignoring for the moment the increase in w, firms' marginal costs will increase somewhat more strongly than in the absence of such economies, thereby inducing a sharper decrease in Y_2 relative to the latter case. If the marginal product of labour in rural production is locally rather insensitive to changes in rural employment, the implied additional return migration will have little effect on w, so that the sharper fall at a postulated unchanged wage will also hold after allowing for any adjustment that does occur. There is, however, a *caveat*: the equilibrium allocation in the presence of agglomeration economies will differ from that in their absence, so that the sharper fall relates to a different starting level. If the two allocations are sufficiently alike, it can be claimed that the adverse effects on the urban sector are stronger in the presence of agglomeration economies. The numerical example in Section 4.1 illustrates precisely this possibility, albeit in a somewhat different setting.

3 A closed dual economy

Let good 1 at the farm gate be the numéraire : $p_{11} = 1$. There is now neither the port nor the trunk road, so that with fractional iceberg transport costs between the city and its hinterland of $\tau_i\,(i = 1,2)$, (1) and (2) become, for good 2 in the village,

$$p_{21} = (1+\tau_2)p_{22}, \tag{6}$$

and, for good 1 in the city,

$$p_{12} = (1+\tau_1). \tag{7}$$

Let good 2 be produced by means of good 1 and labour; and since the aim is to investigate the size of spillover effects, let the technology be Leontief in form. There are constant returns to scale at the level of the individual firm, but if there are Marshallian external economies, each firm's input–output coefficients will depend on the aggregate level of output, Y_2. Let all firms have access to the same technology. In equilibrium, all will make zero profits. We confine our attention to symmetric equilibria, so that the price of good 2 at the factory gate is

$$p_{22} = wl_2(Y_2) + (1+\tau_1)a_{12}(Y_2), \tag{8}$$

where $Y_2 = \sum_j y_{2j} = ny_2$ and the coefficients l_2 and a_{12} depend on Y_2.

Since urban profits are zero, rural migrants are indistinguishable from native townies where urban claims are concerned, so it is simplest to treat all urban residents as identical in incomes and tastes by cutting migrants' rural ties, so denying them any claims on the imputed rents of the rural fixed factor. Let labour be fully mobile, as in Section 2.2, and let firms face a perfectly elastic supply of labour à la Lewis (1954), where the urban wage rate reflects productivity and social organization in the rural sector. In order to boost demand in general, and for the urban good in particular, let the marginal product of labour in rural production be zero over the range of outcomes in question. With family ties cut by migration, villagers require some incentive to move, as represented by the level of w, whereby it is also assumed that those who seek, but do not obtain, urban jobs return at once to their home villages.[6]

Rural households have net revenues of

$$M_1 = Y_1(p_{21}) + p_{21}Y_{21}(p_{21}), \tag{9}$$

where the right-hand side (RHS) is the profit function. In virtue of the above assumptions, Y_1 and M_1 are independent of changes in the numbers of migrants. Denoting by $\bar{L} \equiv \bar{L}_1 + \bar{L}_2$ the economy's aggregate endowment of labour, per capita income is $m_1 = M_1/(\bar{L} - L_2)$. Rural households' aggregate demand for good i is $X_{i1} = (\bar{L} - L_2)x_{i1}(1, p_{21}, m_1)$, where $x_{i1}(1, p_{21}, m_1)$ is the individual household's demand for i.

Urban households' incomes comprise only wages, profits being zero in equilibrium:

$$M_2 = wl_2(Y_2)\cdot Y_2. \tag{10}$$

Their aggregate demand for good i is $X_{i2} = L_2 \cdot x_{i2}(1+\tau_1, p_{22}, w)$. The markets for goods 1 and 2 will clear when

$$Y_1 = X_{11} + (1+\tau_1)(X_{12} + a_{12}(Y_2)\cdot Y_2)$$

and

$$Y_2 = X_{22} + (1+\tau_2)(X_{21} - Y_{21}).$$

[6] This assumption rules out, inter alia, Harris and Todaro's (1970) migratory mechanisms.

Since the labour market clears at the fixed level of w, it follows from Walras' Law that if the market for good 1 clears, then so too will that for good 2. From (9) and (10) and the choice of numéraire, the clearing condition for good 1 may be written in extensive form as

$$\begin{aligned} Z_1(p_{21}) &\equiv Y_1(p_{21}) - X_{11}(1, p_{21}, M_1(p_{21})) \\ &= (1+\tau_1)(X_{12}(1+\tau_1, p_{22}, M_2) + a_{12}(Y_2)Y_2) \equiv D_{12}, \end{aligned} \tag{11}$$

where D_{12} is the total urban demand for the rural good, inclusive of transport costs. In general, $Z_1(p_{21})$ is a correspondence: a 'perverse' response to an improvement in the rural sector's terms of trade, $1/p_{21} = 1/(1+\tau_2)p_{22}$, cannot be ruled out without further assumptions. Given that w is fixed, any non-negative pair $(p_{22}, Y_2 = ny_2)$ satisfying the zero-profit condition (8) and the market-clearing condition (11) is an equilibrium, with all other prices and quantities following from (6) and (7), and rural and urban households' optimal choices in the light thereof. Such an equilibrium, if one exists, may not be unique.

3.1 No externalities

Prices are independent of quantities; with w and τ_i given, and l_2 and a_{12} constant, (8) reduces to $p_{22} = wl_2 + (1+\tau_1)a_{12}$, a constant. Y_1 then follows at once from rural households' optimal choice of good 2 as an input into production; and

$$Z_1 = Y_1((1+\tau_2)p_{22}) - (\bar{L} - L_2) \cdot x_{11}(1, (1+\tau_2)p_{22}, m_1), \tag{12}$$

where $l_2 Y_2 = L_2$. If good 2 is necessary in either production or consumption in the rural sector, then Z_1 will be positive.

The level of urban demand for good 1 becomes

$$D_{12} = (1+\tau_1)(x_{12}(1+\tau_1, p_{22}, w) \cdot l_2 + a_{12})Y_2 .$$

Any non-negative Y_2 satisfying $Z_1 = D_{12}$ will, in conjunction with the condition $p_{22} = wl_2 + (1+\tau_1)a_{12}$, be an equilibrium level of output of good 2. Since D_{12} vanishes when $Y_2 = 0$, there may be more than one equilibrium.

With this reservation, we turn to comparative statics, in the form of a rural roads programme that reduces $\tau_i\,(i = 1,2)$. The fall in the delivered price of good 1 at the factory gate leads to a reduction in p_{22}, and hence in the delivered price of good 2, $(1+\tau_2)p_{22}$, in the villages. Rural output and net revenues both increase, but whether these households are better off depends on the level of induced return migration. Urban households face a lower price for good 1, $(1+\tau_1)$, as well as for good 2. With w fixed by assumption, they will be better off.

The change in the level of migration is bound up with how Z_1 and D_{12} accommodate one another, with changes in prices also affecting Y_2 and L_2. Differentiating (12) totally, noting that $L_2 = l_2 Y_2$, and using Shepard's lemma, we have

$$dZ_1 = \left(\frac{\partial Y_1}{\partial p_{21}} - (\bar{L} - L_2) \cdot \frac{\partial x_{11}}{\partial p_{21}} - \frac{\partial x_{11}}{\partial m_1} \cdot Y_{21} \right) \cdot dp_{21} + x_{11} \left(1 - \frac{m_1}{x_{11}} \cdot \frac{\partial x_{11}}{\partial m_1} \right) \cdot dL_2, \tag{13}$$

where $dp_{21} = d[(1+\tau_2)p_{22}] = (1+\tau_2)a_{12} \cdot d\tau_1 + (wl_2 + (1+\tau_1)a_{12}) \cdot d\tau_2$. Both Y_1 and M_1 are decreasing in p_{21}; but the reductions in prices will also induce changes in L_2, which will affect Z_1 if rural households' income elasticity of demand for good 1 is other than 1. Differentiating D_{12} totally, we have

$$dD_{12} = \left(x_{12} + \frac{a_{12}}{l_2} \right) L_2 \cdot d\tau_1 + (1+\tau_1) \left[\left(\frac{\partial x_{12}}{\partial p_{12}} \cdot d\tau_1 + \frac{\partial x_{12}}{\partial p_{22}} \cdot dp_{22} \right) L_2 + \left(x_{12} + \frac{a_{12}}{l_2} \right) \cdot dL_2 \right]. \tag{14}$$

By reducing τ_1, the programme reduces D_{12} directly. Since $Z_1 = D_{12}$ in equilibrium, this direct effect may result in some migrants returning to their villages, thus shifting the demand for good 1 from town to country, and so reducing Z_1. In the event of some return migration, however, Y_2 and L_2 will also contract, so that, in equilibrium, the direction of their response is unclear.

Combining (13) and (14) and rearranging terms, we obtain

$$\left[((1+\tau_1)(x_{12}+a_{12}/l_2)-x_{11})+m_1\cdot\frac{\partial x_{11}}{\partial m_1}\right]\cdot dL_2$$
$$=-\left[x_{12}\left(1+\frac{p_{12}}{x_{12}}\cdot\frac{\partial x_{12}}{\partial p_{12}}\right)+\frac{a_{12}}{l_2}\right]L_2\cdot d\tau_1$$
$$+\left[\frac{\partial Y_1}{\partial p_{21}}-\frac{\partial x_{11}}{\partial m_1}\cdot Y_{21}-(\bar{L}-L_2)\cdot\frac{\partial x_{11}}{\partial p_{21}}\right]\cdot dp_{21}-(1+\tau_1)\frac{\partial x_{12}}{\partial p_{22}}\cdot L_2\cdot dp_{22}\,.$$

The term $(1+\tau_1)(x_{12}+a_{12}/l_2)-x_{11}$ is the difference between per capita use of good 1 in town and country, respectively. In view of $\tau_1 > 0$ and the intermediate demand for good 1 in producing good 2, this is almost surely positive in practice. Hence, the whole multiplicand in brackets on the left-hand side (LHS) is likewise if good 1 is non-inferior in consumption.

Turning to the RHS, $d\tau_1$, dp_{21}, and dp_{22} are all negative. Taking up their multiplicands in that order, the sign of the expression in brackets multiplying $d\tau_1$ is the combined level, for each urban worker, of final and intermediate demand for good 1, with an adjustment for any change in its price at that location. It is positive if the absolute value of the own-price elasticity of urban final demand for good 1 is at most 1, a condition that surely holds in practice. In that event, the first term on the RHS will be positive if τ_1 falls.

The expression multiplying dp_{21} is the change in the net supply—at the farm gate—of good 1 to the town. Its sign is ambiguous. The output of good 1 is decreasing in p_{21}; but the second term in brackets is positive if good 1 is normal in consumption, and $Y_{12} < 0$. If the goods are sufficiently good substitutes in rural consumption, the third, cross-price term in brackets will be positive. (It will be zero if preferences are Cobb–Douglas.) The sign of the (cross-price) term multiplying dp_{22} likewise depends on the substitutability of goods 1 and 2 in urban consumption.

Suppose, therefore, that preferences in both locations are Cobb–Douglas, but not necessarily with the same taste parameter. Then the above condition simplifies to

$$\left[((1+\tau_1)(x_{12}+a_{12}/l_2)-x_{11})+m_1\cdot\frac{\partial x_{11}}{\partial m_1}\right]\cdot dL_2$$
$$=-a_{12}Y_2\cdot d\tau_1+\left(\frac{1}{Y_{21}}\cdot\frac{\partial Y_1}{\partial p_{21}}-\frac{\partial x_{11}}{\partial m_1}\right)Y_{21}\cdot dp_{21}\,.$$

The aggregate level of urban intermediate demand for good 1 is $a_{12}Y_2$, and if the fraction lost on the way falls by $d\tau_1$, this will work to increase the output of good 2. The first term in the expression in parentheses multiplying dp_{21} is the inverse of the absolute value of the own elasticity of the marginal product of good 2 in rural production.[7] The said inverse exceeds 1 for all members of the CES (constant elasticity of substitution) family, the absolute value of whose elasticity of substitution between good 2 and the fixed factor is at least 1, and it exceeds 0.5 even when the value of the latter is 0.5, when the inputs are rather poor substitutes. The term $\partial x_{11}/\partial m_1$ is rural households' marginal propensity to consume good 1, which is unlikely to exceed 0.5, even in a peasant economy. Noting that $Y_{12} < 0$, we have therefore established the following result.

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[7] This follows by differentiating the first-order condition totally and rearranging.

Proposition 2 *If, in the dual-economy setting described above, (1) preferences in town and country are Cobb–Douglas, though possibly with different taste parameters, and (2) the rural production technology is CES with an elasticity of substitution whose absolute value is at least 0.5, then reductions in transport costs between town and country will induce an increase in urban output if rural households' marginal propensity to consume the rural good is at most 0.5.*

Remark 1 *In view of the fact that* $a_{12}Y_2$ *is surely quite large in practice, the two rather weak auxiliary conditions can be weakened further. By continuity, the result will also hold for all preferences that are sufficiently close to Cobb–Douglas in form.*

To summarize the results thus far, urban households certainly will be better off as a result of the reduction in transport costs. The same will hold for rural households under the following sufficient conditions. First, that the goods be sufficiently good substitutes in consumption: Cobb–Douglas preferences, though not necessarily identical in town and country, will serve. Second, that the rural fixed factor and the urban good be sufficiently good substitutes in rural production, whereby Cobb–Douglas is much stronger than needed. Under these two conditions, reductions in transport costs will induce more villagers to leave for urban employment, thereby increasing rural per capita income as well as reducing the village price of the urban good. If, however, the goods are poor substitutes, urban output may fall, and the associated return migration will increase the claimants on aggregate rural income. Such an outcome will have adverse effects on urban activities in the presence of agglomeration economies.

3.2 Agglomeration economies

In the presence of such externalities in urban production, the input–output coefficients l_2 and a_{12} will depend on Y_2. The empirical literature on agglomeration economies is much concerned with arriving at estimates of the elasticity of unit costs or productivity in particular industries with respect to the total employment in that branch in the location in question—so-called localization economies. Equally important, in principle, are urbanization economies, which arise from the potentially greater diversity of activities and drawing power that accompany a city's growth. In the present setting, with only a single city and urban good, these two forms of local external economies are necessarily conflated. Let their influence on l_2 and a_{12} be iso-elastic, with parameter $\varepsilon > 0$: in a small abuse of notation, therefore,

$$l_2(Y_2) = l_2 \cdot Y_2^{-\varepsilon},\ a_{12}(Y_2) = a_{12} \cdot Y_2^{-\varepsilon},$$

where l_2 and a_{12} now denote the respective coefficients for the scale factor $Y_2^{-\varepsilon}$.

The zero-profit condition (8) specializes to

$$p_{22} = (wl_2 + (1+\tau_1)a_{12})Y_2^{-\varepsilon}, \tag{15}$$

so that price and output move in opposite directions and the system is no longer recursive. The market-clearing condition (11) becomes

$$Z_1((1+\tau_2)p_{22}) = (1+\tau_1)(x_{12}(1+\tau_1, p_{22}, w) \cdot l_2 + a_{12})Y_2^{1-\varepsilon}. \tag{16}$$

Any positive pair (p_{22}, Y_2) satisfying (15) and (16) is an equilibrium allocation, with all other prices and quantities following as before. There may exist no equilibrium at all or more than one; but by continuity, the findings in Section 3.1 must also hold for all values of ε sufficiently close to zero.

According to condition (15), p_{22} is a continuous, decreasing and strictly convex function of Y_2 for all $\varepsilon > 0$. If $\varepsilon = 0$, as in Section 3.1, it is a horizontal line, with $p_{22} = wl_2 + (1+\tau_1)a_{12}$. A reduction in τ_1 will shift the function downwards equiproportionally. Turning to condition (16), suppose the reductions in τ_i induce an increase in Z_1, the sufficient conditions for which established in Section 3.1 will also hold if ε is sufficiently small. The scale effect represented by ε weakens the effects of changes in Y_2 on the RHS, relative to those in the absence of agglomeration economies. This suggests that urban output will respond more strongly to reductions in transport costs than in the absence of such economies; but p_{22} is decreasing in Y_2, so that further substitution effects are at work in determining x_{12}. If, however, the goods are sufficiently poor substitutes, L_2 may well contract, thereby increasing unit costs in urban production.

Consider, for example, the utterly extreme value $\varepsilon = 1$, so that a doubling of urban output yields a doubling of productivity. Then the assumption that firms face perfectly elastic supplies of labour implies that aggregate urban income is fixed, at wl_2. The derived demand for good 1 as an intermediate input is also fixed, at $(1+\tau_1)a_{12}$, and the value of output at the factory gate, $p_{22}Y_2$, is likewise, at $wl_2 + (1+\tau_1)a_{12}$. A reduction in τ_1 will have no effect on urban incomes, and it will reduce both the derived demand for good 1 and the value of output at the factory gate. Final demand for goods in the city will depend only on p_{12}, which falls with τ_1, and p_{22}, which need not fall, since aggregate urban demand for good 2 may fall. If urban output contracts, so inducing an increase in p_{22}, the delivered price of good 2 in the villages, p_{21}, may also increase, despite the reduction in τ_2. In practice, of course, ε will be fairly close to zero; but the findings for $\varepsilon = 1$ serve as a warning that there are no immediately compelling reasons to suppose that the spillover effects of a rural roads programme on urban activities and income are necessarily larger in the presence of agglomeration economies. This motivates an examination of some numerical examples, following a brief discussion of congestion.

3.3 Congestion

The movements of goods and people associated with urban production necessarily involve traffic within the restricted room afforded by a town's limits. If the streets are at all congested, heavier traffic will generate external costs, agglomeration economies or no. Suppose these costs depend only on the level of urban production. Then the parameter ε represents the joint effect of congestion and agglomeration economies. Its value will be negative if the former outweigh the latter, and the analysis of Section 3.2 applies, *mutatis mutandis*.

In a more refined formulation, τ_i would depend on the level of traffic as well as any investment in the road network. This would greatly complicate the analysis, without yielding any obvious insights beyond those offered by the representation through ε.

3.4 Numerical examples

Suppose households' preferences are homothetic, so that all income elasticities of demand are unity and individual demand functions take the form $x_{ik} = f_{ik}(\mathbf{p}_k)\cdot m_k$ $(i,k = 1,2)$. Then $X_{i1} = (\bar{L} - L_2) f_{i1}(\mathbf{p}_k)\cdot m_1 = f_{i1}(\mathbf{p}_1)\cdot M_1$, and likewise $X_{i2} = f_{i2}(\mathbf{p}_2)\cdot M_2$. There will be a unique equilibrium, albeit at the cost of imposing rather strong income effects on the rural demand for good 1, thus implying, *ceteris paribus*, a smaller response of Z_1 to changes in τ_i, and hence also a dampening effect on Y_2.

To begin with, and informed by Proposition 2, let households' preferences in town and country be Cobb–Douglas. Let the production technology in the rural sector be likewise. The elasticity of the output of good 1 with respect to inputs of good 2 is denoted by α_1, the taste parameter for households' consumption of good 1 in town and country, respectively, by β_k $(k = 1,2)$. Rural households' aggregate income is $M_1 = (1-\alpha_1)Y_1$, and recalling that $p_{11} = 1$ and $p_{12} = (1+\tau_1)$, the market-clearing condition for good 1 specializes to

$$(1-\beta_1(1-\alpha_1))Y_1 = (\beta_2 w l_2 + (1+\tau_1)a_{12})Y_2^{1-\varepsilon}. \tag{17}$$

Since $M_2 = wl_2Y_2^{1-\varepsilon}$, it follows from these assumptions that aggregate rural and urban incomes stand in a fixed relationship to one another. The output of good 1 is $Y_1 = A_1 \cdot (-Y_{21})^{\alpha_1}$, so that households' maximization of net revenues yields the aggregate supply function:

$$Y_1(p_{21}) = \left(\frac{\alpha_1 A_1}{p_{21}}\right)^{\alpha_1/(1-\alpha_1)}. \tag{18}$$

Any (Y_1, Y_2, p_{22}) satisfying (17), (18), and the zero-profit condition (15) is an equilibrium.

Turning to the parameters' numerical values, peasant farmers use artificial fertilizers, other chemicals, fuel, and certain urban services, but on a rather limited scale. A cost-share in the region of 10 per cent is plausible: $\alpha_1 = 0.1$. The scalar A_1 should be chosen such that the values of Y_1 are convenient for discussion: with $\alpha_1 = 0.1$, let $A_1 = 10$. Rural households consume a substantial fraction of their own output: let the expenditure share $\beta_1 = 0.5$. Urban households' tastes for good 1 are arguably not as strong: let $\beta_2 = 0.4$. Since $M_2 = wl_2 \cdot Y_2^{1-\varepsilon}$ and the wage is fixed, let $w = l_2 = 1$. Inputs of the rural good in urban production are quite substantial: let $a_{12} = 0.2$. Where agglomeration economies are concerned, three values of ε will be employed. Their wholesale absence, $\varepsilon = 0$, is the benchmark. Henderson (2002) reports localization elasticities for various industries in the range 0.05 to 0.08, to which must be added the contribution of general economies of urbanization. In the light of the extreme aggregation of a single urban good in the present structure, $\varepsilon = 0.2$ would represent strong economies of agglomeration. The third value, 0.5, is utterly implausible; but it is useful to compare the associated allocation with those yielded by the other two, by way of sensitivity analysis.

The transport cost parameters τ_i complete each constellation. In one alternative, $\tau_1 = \tau_2 = 0.1$ before the roads programme is undertaken, and the programme halves both. In the second, more striking alternative, the reduction is three times as large: $\tau_1 = \tau_2 = 0.2$ beforehand, again with $\tau_1 = \tau_2 = 0.05$ upon completion of the programme.

Beginning with the benchmark case $\varepsilon = 0$, the halving of unit transport costs reduces the delivered price of good 1 in the city by 4.5 per cent, which induces a reduction of 0.8 per cent in the price of good 2 at the factory gate, and hence a reduction of just over 5 per cent in the village price (see Table 1). Aggregate output and incomes in town and country each increase by 0.61 per cent and 2.24 per cent, respectively. By assumption, urban households' labour supply is completely inelastic, so that the programme induces some rural–urban migration. In the presence of rather strong economies of agglomeration ($\varepsilon = 0.2$), the proportional reduction in the factory gate price is a little greater, at 1.39 per cent (see the Appendix for the details of the calculations). Aggregate output and incomes in town and country now increase by 0.67 per cent and 4.95 per cent, respectively. For the implausible value $\varepsilon = 0.5$, the corresponding figures are 0.89 per cent and 2.55 per cent, respectively, with the programme yielding a reduction in the factory gate price of 3.28 per cent.

Under the alternative programme in which the initial value of τ_i is 0.2, the effects are correspondingly stronger, but the general pattern is much the same. Rural output and incomes each increase by 1.77 and 1.96 per cent when $\varepsilon = 0$ and 0.2, respectively. The corresponding increases for urban output are 6.87 and 8.79 per cent, respectively; for incomes, 6.87 and 9.72 per cent, respectively.

In order to draw firm conclusions about whether agglomeration economies matter much in comparison with the direct effects of reductions in transport costs, it is essential to use an exact measure of the programme's welfare effect, as the changes in consumer prices also depend on the strength of such economies. The natural candidate is the equivalent variation (EV), namely the lump-sum transfer such that households would be indifferent between having that sum with the initial transport costs and enjoying the programme to reduce τ_i to 0.05.

Table 1: Allocations in the closed economy: various constellations

	$\tau_i = 0.2$		$\tau_i = 0.1$		$\tau_i = 0.05$	
	$\lvert\sigma\rvert = 1$	$\lvert\sigma\rvert = 0.5$	$\lvert\sigma\rvert = 1$	$\lvert\sigma\rvert = 0.5$	$\lvert\sigma\rvert = 1$	$\lvert\sigma\rvert = 0.5$
$\varepsilon = 0$:						
p_{21}	1.4880	1.4880	1.3420	1.3420	1.2705	1.2705
Y_1	0.9568	0.9568	0.9678	0.9678	0.9738	0.9738
M_1	0.8611	0.8611	0.8711	0.8711	0.8764	0.8764
X_{11}	0.4306	0.3879	0.4355	0.4036	0.4382	0.4112
X_{21}	0.2894	0.3180	0.3245	0.3484	0.3449	0.3655
p_{22}	1.2400	1.2400	1.2200	1.2200	1.2100	1.2100
Y_2	0.8223	0.8299	0.8586	0.8599	0.8780	0.8750
M_2	0.8223	0.8299	0.8586	0.8599	0.8780	0.8750
X_{12}	0.2741	0.3081	0.3122	0.3411	0.3345	0.3600
X_{22}	0.3979	0.3712	0.4223	0.3967	0.4354	0.4107
$\varepsilon = 0.2$:						
p_{21}	1.5648	1.5553	1.3957	1.3906	1.3137	1.3111
Y_1	0.9515	0.9521	0.9636	0.9640	0.9701	0.9704
M_1	0.8563	0.8569	0.8673	0.8676	0.8731	0.8733
X_{11}	0.4282	0.3813	0.4336	0.3981	0.4366	0.4071
X_{21}	0.2736	0.3058	0.3245	0.3376	0.3323	0.3556
p_{22}	1.3040	1.2961	1.2688	1.2642	1.2512	1.2486
Y_2	0.7775	0.8016	0.8220	0.8371	0.8459	0.8546
M_2	0.8177	0.8379	0.8548	0.8674	0.8971	0.8819
X_{12}	0.2726	0.3072	0.3109	0.3409	0.3418	0.3596
X_{22}	0.3763	0.3620	0.4043	0.3895	0.4302	0.4039
$\varepsilon = 0.5$:						
p_{21}	1.8545	1.7266	1.5931	1.5128	1.4708	1.4109
Y_1	0.9337	0.9412	0.9496	0.9550	0.9580	0.9625
M_1	0.8403	0.8470	0.8546	0.8595	0.8622	0.8662
X_{11}	0.4202	0.3661	0.4273	0.3855	0.4311	0.3959
X_{21}	0.2266	0.2786	0.2682	0.3134	0.2931	0.3333
p_{22}	1.5454	1.4384	1.4483	1.3753	1.4008	1.3438
Y_2	0.6438	0.7431	0.7096	0.7869	0.7462	0.8108
M_2	0.8024	0.8621	0.8424	0.8871	0.8638	0.9005
X_{12}	0.2675	0.3069	0.3063	0.3404	0.3291	0.3595
X_{22}	0.3115	0.3433	0.3490	0.3728	0.3700	0.3892

Notes: good 1 at the farm gate is the numéraire ($p_{11} = 1$).
$|\sigma| = 1$: Cobb–Douglas preferences.
The calculations for $|\sigma| = 1$, $\varepsilon = 0.2$, and $\tau_i = 0.1, 0.05$ are in the Appendix.
Source: author's choice of functional forms and parameter values, as discussed in the text.

Rural–urban migration introduces some complications. By assumption, 'native' urban households supply their labour endowments to urban production completely inelastically at the fixed wage w. Their money-metric welfare is therefore inversely proportional to the level of the true cost-of-living index, $\kappa(\mathbf{p}_2)$. Since the utility function is Cobb–Douglas, with $\beta_2 = 0.4$, $\kappa(\mathbf{p}_2) = p_{12}^{0.4} p_{22}^{0.6}$. For $\varepsilon = 0.2$, $\kappa(\mathbf{p}_2)$ falls from 1.1984 when $\tau_i = 0.1$ to 1.1665 when $\tau_i = 0.05$. Such households therefore enjoy an improvement of 2.737 per cent in (money-metric) welfare. The corresponding figure for $\varepsilon = 0$ is 2.379 per cent—that is, 15 per cent smaller. These and the values for other constellations are set out in the columns denoted by $-\Delta\kappa(\mathbf{p}_2)/\kappa(\mathbf{p}_2)$ in Table 2. They are increasing in ε, but only rather modestly for plausible values of the latter.

Precise conclusions about changes in rural welfare can be drawn only if the numbers of claimants on income are known—and with mobile labour, these are endogenous. The results in Table 1 implicitly contain changes in the numbers of migrants in the form of changes in $l_2 Y_2^{1-\varepsilon} = M_2$ when $w = 1$; but the absolute numbers of 'native' rural and urban individuals are not necessary for the derivation of those results. Consider, for example, the case $\varepsilon = 0$. When τ_i falls from 0.1 to 0.05, Y_2 rises from 0.85857 to 0.87797. Since $l_2 = 1$, additional villagers endowed with 0.0194 units of labour must have migrated to urban jobs. When τ_i is initially 0.2, the corresponding figure is close to threefold larger, at 0.0557. By assumption, migration is voluntary, so whatever the number of ‘native’ villagers, these migrants must be better off earning $w = 1$ and facing $\mathbf{p}_2(\tau_i = 0.05)$ than staying in the village, even though the programme turns out to yield a positive aggregate EV_1. The level of the latter will under- or overstate the changes in rural individuals’ welfare accordingly as the programme induces more to leave or some earlier migrants to return.

Table 2: The programme’s effects on welfare: initial values of τ_i, 0.1 and 0.2

	$\frac{EV_1}{M_1} \times 100$		$\frac{EV_2}{M_2} \times 100$		$-\frac{\Delta\kappa_2}{\kappa_2} \times 100$	
	$\tau_i = 0.1$	$\tau_i = 0.2$	$\tau_i = 0.1$	$\tau_i = 0.2$	$\tau_i = 0.1$	$\tau_i = 0.2$
$\lvert\sigma\rvert = 1$:						
$\varepsilon = 0$	3.403	10.138	4.695	14.301	2.379	7.045
$\varepsilon = 0.2$	3.765	11.279	7.820	18.642	2.737	8.137
$\varepsilon = 0.5$	5.004	15.217	6.576	20.459	3.938	11.892
$\lvert\sigma\rvert = 0.5$:						
$\varepsilon = 0$	3.592	9.568	4.396	13.333	2.457	6.978
$\varepsilon = 0.2$	3.894	10.466	4.449	13.912	2.669	7.602
$\varepsilon = 0.5$	4.697	14.381	4.913	14.956	3.247	9.133

Note: $EV_k \equiv \Delta M_k$; good 1 at the farm gate is the numéraire.

Source: author’s choice of functional forms and parameter values, as discussed in the text.

In the light of all this, it seems sufficient for present purposes to derive the aggregate values of EV_k, which can be summed, albeit with reservations if distributive judgements are to be made. It should be recalled that EV_k also relates to changing numbers of residents in location k. A comparison of the columns reporting the proportional changes in EV_2 and κ_2 provides a helpful check in this regard: when $|\sigma| = 1$, a fall in τ_i leads to an increase in urban output and so induces more villagers to leave. Fuelled by this migration at zero opportunity cost in terms of rural output, the level of aggregate benefits, $EV_1 + EV_2$, responds quite strongly to agglomeration economies. In their absence, aggregate benefits, which can be calculated using the prices and incomes in Table 1, are 4.04 per cent of aggregate income when $\tau_i = 0.1$ initially. When ε takes the strong value 0.2, the corresponding enhancement is almost half as large again, at 5.78 per cent. When the initial value of τ_i is 0.2, the proportional enhancement is less dramatic, but still large, at 30 per cent.

The above results provide a good sense of the sort of numerical magnitudes associated with Proposition 2. As established in Section 3.1, however, there are dangers in attempting to draw general conclusions from results that rest on Cobb–Douglas preferences. In particular, goods are then rather good substitutes and the cross-price elasticities are zero. With just two goods, one of which is largely an aggregate of agricultural products and personal services, it is essential to investigate more limited substitutability, with both goods necessary in consumption. Consider, therefore, the preferences represented by

$$U_k = \frac{X_{1k} \cdot X_{2k}}{b_k X_{1k} + (1 - b_k) X_{2k}}, \; k = 1, 2,$$

for which the elasticity of substitution $\sigma = -0.5$. Income effects now take on a stronger role, which has potentially important implications when the agglomeration elasticity is substantial; for then demand is also more strongly in play. The associated Marshallian demand functions are

$$X_{1k} = \frac{M_k}{p_{1k} + (p_{1k} p_{2k})^{0.5} \cdot \xi_k}, \; X_{2k} = \frac{M_k}{p_{2k} + (p_{1k} p_{2k})^{0.5} / \xi_k}, \; k = 1, 2,$$

where $\xi_k = (b_k/(1 - b_k))^{0.5}$. The associated price indices are $\kappa_k(\mathbf{q}_k) = (p_{1k}^{1/2} + \xi_k p_{2k}^{1/2})^2$. In keeping with the values of the taste parameters when preferences are Cobb–Douglas, let $b_1 = 0.5$ and $b_2 = 0.6$, so that $\xi_1 = 1$ and $\xi_2 = 1.2247$.

The allocations with and without the rural roads programme are reported in the corresponding columns denoted by $|\sigma| = 0.5$ in Table 1. Now that the urban good is a rather poor substitute for the rural one, urban households devote more of their expenditure to the latter, thereby weakening demand for the former, and so raising its price, *ceteris paribus*, in the presence of agglomeration economies. In fact, this effect is outweighed by others stemming from the direct reductions in costs. The programme reduces the price of the urban good at the factory gate, p_{22}, a little more strongly when $\varepsilon = 0.2$ than when $\varepsilon = 0$; urban output also rises, and hence more villagers leave. Qualitatively, therefore, these results are much like those for the Cobb–Douglas case.

Where changes in welfare are concerned, the programme results in a decrease in the urban price index in the absence of agglomeration economies, so that native urban households are better off—indeed, slightly more so than under Cobb–Douglas preferences, with a sharper relative decline when ε takes the implausible value 0.5. In the presence of agglomeration economies, the proportional changes in EV_k are also smaller than in the corresponding Cobb–Douglas cases. Summing up, rather poor substitutability in consumption does not overturn the finding that, in this dual-economy setting, both villagers and townies benefit from a rural roads programme. It does, however, substantially temper the enhancing effects of agglomeration economies.

4 The open economy: one non-tradeable

The results obtained for such a closed dual economy might be viewed as placing a fairly tight upper limit on what can be expected when some goods are internationally tradeable. To test this intuitive conclusion from the findings in Sections 2 and 3, consider the hybrid case in which a non-tradeable is introduced into the framework of Section 2. Let labour be immobile, as in Section 2.1, so as to rule out the dominant role played by migration in Section 3. In conclusion, it will be shown that the argument also goes through when labour is fully mobile.

Let the third good be producible only in the city, yet transportable for use in production and consumption in the hinterland. The city is then, in principle, independent of the latter; goods 1 and 2 can be traded internationally at parametrically given prices, and good 3 can be produced in the city to supply any ensuing demand there. In the event that the city does not trade with its hinterland, it will have to produce and export good 2 in order to meet its needs for good 1. The hinterland, in contrast, is not independent of the city, for although villagers can export good 1 in exchange for imports of good 2 (both through the city), they must trade with the city in order to obtain good 3. In equilibrium, therefore, some of the city's demand for good 1 must be met from domestic production. Given that the overall setting is one in which the economy is still rather agrarian in nature, let the rural sector's endowment of the fixed factor be so large that, in equilibrium, good 1 is exported and good 2 imported, an assumption that does not necessarily rule out some domestic production of good 2.

Let the city and the port be one and the same, thus eliminating the trunk route. The prices of the traded goods are fixed by their respective world prices and the transport cost factors; the price of good 3 at the factory gate, p_3, is endogenous. Thus,

$$p_{11} = p_1^*/(1+\tau_1),\ p_{21} = (1+\tau_2)p_2^*,\ p_{31} = (1+\tau_3)p_3;\ p_{12} = p_1^*,\ p_{22} = p_2^*,\ p_{32} = p_3. \tag{19}$$

Let good 3 be produced by means of labour alone under constant returns to scale. Agglomeration economies arise from the domestic production of goods 2 and 3, whose input–output coefficients are then functions of Y_2 and Y_3. The urban wage rate is set, now endogenously, by the alternative of importing good 2 at price p_2^*:

$$w \geq (p_2^* - p_1^* a_{12}(Y_2,Y_3))/l_2(Y_2,Y_3), \tag{20}$$

which holds as an equality if good 2 is produced domestically. Let the latter condition hold. Then the urban wage rises with urban production in the presence of agglomeration economies, even though the world prices of goods 1 and 2 rule in the city. The price of good 3 is equal to its unit cost:

$$p_3 = wl_3(Y_2, Y_3) = (p_2^* - p_1^* a_{12}(Y_2, Y_3)) \cdot \frac{l_3(Y_2, Y_3)}{l_2(Y_2, Y_3)}, \tag{21}$$

whereby agglomeration economies make themselves felt through the inputs of good 1 in the production of good 2 and any differential effects of agglomeration on labour productivity in sectors 2 and 3.

The market-clearing conditions for goods are

$$Y_1 = X_{11} + (1 + \tau_1)(X_{12} + a_{12}(Y_2, Y_3) \cdot Y_2 + E_1),$$

$$Y_2 = X_{22} + (1 + \tau_2)(X_{21} - Y_{21}) + E_2,$$

and

$$Y_3 = (1 + \tau_3)(X_{31} - Y_{31}) + X_{32}. \tag{22}$$

By Walras' Law, the value of the domestic output of tradeables at world prices, adjusted for internal transport costs, is equal to the corresponding value of the total domestic absorption thereof:

$$\frac{p_1^* Y_1}{1 + \tau_1} + p_2^* Y_2 = p_1^* \left(\frac{X_{11}}{1 + \tau_1} + X_{12} + a_{12}(Y_2, Y_3) \cdot Y_2 \right) + p_2^*(X_{22} + (1 + \tau_2)(X_{21} - Y_{21})). \tag{23}$$

Turning to output, the domestic supply function of the rural good is $Y_1(\mathbf{p}_1)$, where $\mathbf{p}_1 = (p_{11}, p_{21}, p_{31})$. Villagers' aggregate income M_1 and their final demand vector $\mathbf{X}_1$ are likewise functions of $\mathbf{p}_1$, as are their derived demands for goods 2 and 3 in rural production, Y_{i1} $(i = 2, 3)$.

Facing a perfectly elastic demand for good 2 at the world price p_2^* but immobile labour, firms in the city are limited by urban households' labour supply, whose aggregate endowment $\bar{L}_2$ is offered completely inelastically. Hence, $l_2 Y_2 + l_3 Y_3 = \bar{L}_2$, aggregate urban income $M_2 = w\bar{L}_2$ and the aggregate final demand vector is $\mathbf{X}_2(\mathbf{p}_2, M_2)$. It is then seen from (19)–(21) that the system reduces to (22) and (23) in Y_2 and Y_3, given world prices and the values of the parameters associated with the technologies, tastes, and transport costs. Any positive pair (Y_2, Y_3) satisfying these two equations is an equilibrium in which good 2 is produced domestically. The effects of reductions in transport costs ramify throughout the entire system.

Matters are simple in the absence of agglomeration economies, since l_2, l_3, and a_{12} are then constants. Whereas rural producers face a favourable shift in prices as all τ_i fall, urban prices, including the wage rate, are then independent of changes in τ_i. Hence, urban income is likewise, and the rural roads programme confers no benefits on urban households, as in Section 2.1.

In the presence of agglomeration economies, the wage rate and urban incomes will depend on urban output:

$$M_2 = w\bar{L}_2 = (p_2^* - p_1^* a_{12}(Y_2, Y_3)) \cdot \bar{L}_2 / l_2(Y_2, Y_3).$$

Let such economies depend on the aggregate level of urban production, $Y_2 + Y_3$, and be iso-elastic in form. Then

$$l_i(Y_2, Y_3) = l_i \cdot (Y_2 + Y_3)^{-\varepsilon},\ i = 2, 3;\ a_{12}(Y_2, Y_3) = a_{12} \cdot (Y_2 + Y_3)^{-\varepsilon};$$

and hence

$$M_2 = (p_2^* \cdot (Y_2 + Y_3)^{\varepsilon} - p_1^* a_{12}) \cdot \bar{L}_2 / l_2 = \left[p_2^* \cdot (\bar{L}_2 / l_2 + (1 - l_3 / l_2) Y_3)^{\varepsilon} - p_1^* a_{12} \right] \cdot \frac{\bar{L}_2}{l_2}. \tag{24}$$

Thus, M_2 is increasing or decreasing in Y_3 according as $l_2 \gtrless l_3$, that is, according to which sector is the more labour-intensive. In less developed economies, labour productivity is normally higher in sector 2 $(l_2 < l_3)$. In that event, reductions in τ_i will induce an increase in the wage rate if and only if they induce a fall in the output of the non-tradeable. It is also seen from (21) that $l_2 < l_3$ implies that p_3 is decreasing in Y_3.

A preliminary step is to obtain the elasticity of M_2 with respect to Y_3. Differentiating (24) and rearranging terms, we have

$$\frac{Y_3}{M_2} \cdot \frac{dM_2}{dY_3} = \varepsilon(1 - l_3/l_2) \cdot \frac{Y_3/(Y_2 + Y_3)}{1 - p_1^* a_{12} \cdot (Y_2 + Y_3)^{-\varepsilon}/p_2^*}.$$

The denominator, $1 - p_1^* a_{12} \cdot (Y_2 + Y_3)^{-\varepsilon}/p_2^*$, is the cost-share of labour in the production of good 2. Since labour is the sole factor, its share would be at least two-thirds. In view of the general setting, with good 1 exported and allowing for $l_2 < l_3$, Y_3 should be at least twice Y_2. It is unlikely, however, that $l_3 > 2l_2$. Given these various limits, it follows that the absolute value of the elasticity of M_2 with respect to Y_3 is unlikely to exceed $\varepsilon/2$, whereby ε itself is at most 0.2 and might be as low as 0.1. In practice, therefore, M_2 will be rather insensitive to changes in urban production.

The roads programme now reduces $\tau_i\,(i = 1,2,3)$. As net producers of good 1, rural households are better off with any increase in the farm-gate price alone, and good 2 also becomes cheaper. Suppose the reduction in τ_3 at least outweighs any increase in p_3—in the event that Y_3 falls—so that p_{31} does not increase. Substitution effects then work to increase X_{21} and X_{31}, with the balance between them depending on the relative changes in τ_2 and τ_3 as well as substitutability in consumption. The derived demand for good 3 in rural production, $-Y_{31}$, will also increase if the fixed factor and goods 2 and 3 are fairly good substitutes in rural production. Given such assumptions, it follows that aggregate rural demand for good 3, $(1 + \tau_3)(X_{31} - Y_{31})$, will indeed increase as τ_3 falls if τ_1 and τ_2 fall at least as sharply, so that improvements in p_{11} and M_1 are comparably strong.

The programme's effect on the urban demand for good 3 depends heavily on the resulting change in the wage rate, since p_{12} and p_{22} are fixed at p_1^* and p_2^*, respectively. It is seen from (21) that changes in the aggregate level of urban production will have little effect on p_3 when the normalized cost element $p_1^* a_{12}$ is rather modest and the elasticity ε is small, as will hold in practice. These considerations point to the likelihood of a resultant increase in the total demand for, and output of, good 3, and hence, from $l_2 < l_3$ and (24), a fall in urban income.

An alternative argument proceeds from the condition $p_1^* E_1 + p_2^* E_2 = 0$. Suppose both Y_2 and urban income rise. The level of urban demand for good 1 will also increase if the change p_3 is small. Yet it is seen from (23) that a strong increase in Y_2 is almost surely ruled out. If the elasticity ε is small, as will hold in practice, the urban full-employment condition implies that an increase in exports of good 1 will induce a shift in employment in favour of sector 3 such that Y_2 falls. If $l_2 < l_3$, urban income will fall, thus making an increase in exports of good 1 certain and contradicting the hypothesis that the programme will lead to an increase in Y_2 and urban income. Summing up, given $l_2 < l_3$ and plausible assumptions about households' preferences and the value of ε, the rural roads programme will result in a fall in urban welfare.

The argument in Section 2.2 establishes that this conclusion also holds when labour is fully mobile; a rural roads programme that increases the opportunity cost of rural labour—urban agglomeration economies or no—will make migration in search of urban employment less attractive. With the urban wage determined by the level of urban output as well as world prices, as given by (20), and good 2 importable at p_2^*, a contraction in urban output will result.

4.1 A numerical example

A numerical example will serve to illustrate the various possibilities analysed above. For the purposes of comparison, it will be useful to adhere closely to one of the examples in Section 3.4. In view of the finding there, that in the presence of agglomeration economies, reductions in transport costs tend to induce a fall in urban output when there is limited substitutability in consumption, let preferences be Cobb–Douglas, in order to allow an escape through substantial flexibility.

One may normalize world prices to unity: $p_1^* = p_2^* = 1$. To maintain demand for good 3, let its (constant) cost-share in rural production, α_{31}, be 0.1, and let that for good 2, α_{21}, be likewise 0.1, as in Section 3.4; therefore, any reduction in τ_2 will induce some increase in rural production and incomes. The value of the scalar A_1 must be sufficiently large as to yield a surplus for export, but not so large as to make domestic production of good 2 unprofitable in equilibrium: $A_1 = 3$ satisfies these requirements. Let the value of rural households' taste parameter for good 1, β_{11}, remain at 0.5; and let $\beta_{21} = 0.2$ and $\beta_{31} = 0.3$, the balance in favour of good 3 corresponding to what is common in practice.

Turning to the city, let the agglomeration parameter ε take the alternative values 0 and 0.2, each with the associated coefficients $l_2 = 2/3$, $l_3 = 1$, and $a_{12} = 0.2$. The labour endowment $\bar{L}_2$ must be large enough to permit domestic production of good 2, but not so large as to induce imports of good 1: $\bar{L}_2 = 1.2$ satisfies these requirements. The value of the taste parameter β_{21} is retained at 0.4; let $\beta_{22} = 0.25$ and $\beta_{31} = 0.35$.

The calculations are straightforward in the absence of agglomeration economies. From (20) and (21), we obtain $w = p_{32} = 1.2$ and $M_2 = 1.44$ independently of τ_i. Hence, the vector of urban final demand $\mathbf{X}_2$ is likewise, and it is readily calculated from the associated demand functions (see Table 3). The only changes of real moment occur in the rural sector. When τ_i falls from 0.1 to 0.05, the improvement in producers' net terms of trade induces increases in Y_1 and M_1 of 2.32 and 7.22 per cent, respectively. The vector of rural final demand, $\mathbf{X}_1$, is also strictly larger, the increase in p_{11} being outweighed by the income effect. The upshot of all this is an increase in the levels of aggregate demand for good 3 and exports of good 1. There is a corresponding reallocation of urban labour, with employment in sector 3 rising at the expense of sector 2, albeit with no effects on urban prices or income. The changes are correspondingly larger when τ_i falls from 0.2 to 0.05, especially in EV_1.

In the presence of agglomeration economies, however, changes in urban production will indeed affect the wage rate and the price of good 3. It is seen from Table 3 that in the absence of such economies, the sum of Y_2 and Y_3 falls somewhat in response to the roads programme, as labour is shifted to the labour-intensive sector 3. This indicates that the programme may well result in a reduction in urban welfare, and this is indeed the outcome.[8] What happens in the rural sector is governed overwhelmingly by the direct reductions in τ_i, with any ensuing changes in the producer price of good 3 being very small. It follows that the levels of exports of good 1 and the aggregate demand for good 3 respond in very much the same way as in the absence of agglomeration economies, with the result that the wage rate, p_{32}, Y_2 and urban incomes all fall in response to the reductions in τ_i. The final demand vector $\mathbf{X}_2$ is strictly smaller: the corresponding reduction in EV_2 is about 0.6 per cent of income when the initial value of τ_i is 0.1, rising to 1.6 per cent when that value is 0.2. Rural households do slightly better in the presence of agglomeration economies, due to the slight fall in p_{32}. Their demand for good 1 actually falls very slightly, but this is overwhelmed by the increases in their consumption of goods 2 and 3. The sum of the EV_k is slightly smaller in the presence of agglomeration economies.

[8] The intrusion of the term $(Y_2 + Y_3)^{-0.2}$ renders the system non-recursive. Using $(Y_2 + Y_3)$ from the corresponding allocation with $\varepsilon = 0$ as a starting value, convergence is obtained in at most three iterations. The details are available on request.

Table 3: Allocations: two tradeables, one non-tradeable, various constellations

	$\tau_i = 0.2$		$\tau_i = 0.1$		$\tau_i = 0.05$	
	$\varepsilon = 0$	$\varepsilon = 0.2$	$\varepsilon = 0$	$\varepsilon = 0.2$	$\varepsilon = 0$	$\varepsilon = 0.2$
rural						
p_{11}	0.8333	0.8333	0.9091	0.9091	0.9524	0.9524
p_{21}	1.20	1.20	1.10	1.10	1.05	1.05
p_{31}	1.44	1.4622	1.32	1.3376	1.26	1.2751
Y_1	1.9811	1.9774	2.0694	2.0659	2.1179	2.1147
$-Y_{21}$	0.1376	0.1372	0.1710	0.1699	0.1921	0.1918
$-Y_{31}$	0.1146	0.1127	0.1425	0.1404	0.1601	0.1579
M_1	1.3208	1.3182	1.5050	1.5025	1.6136	1.6112
X_{11}	0.7925	0.7909	0.8278	0.8264	0.8472	0.8459
X_{21}	0.2201	0.2197	0.2736	0.2732	0.3074	0.3069
X_{31}	0.2752	0.2705	0.3420	0.3370	0.3842	0.3790
$(EV_1/M_1)\cdot 100$	22.175	22.339	7.218	7.263		
urban						
p_{12}	1.00	1.00	1.00	1.00	1.00	1.00
p_{22}	1.00	1.00	1.00	1.00	1.00	1.00
p_{32}	1.20	1.2185	1.20	1.2160	1.20	1.2144
w	1.20	1.2985	1.20	1.2848	1.20	1.2758
Y_2	0.4683	0.4678	0.3705	0.3466	0.2719	0.2390
Y_3	0.8878	0.9073	0.9530	0.9689	1.0187	1.0407
M_2	1.44	1.5582	1.44	1.5418	1.44	1.5310
X_{12}	0.576	0.6233	0.576	0.6167	0.576	0.6124
X_{22}	0.360	0.3896	0.360	0.3856	0.360	0.3827
X_{32}	0.420	0.4476	0.420	0.4438	0.420	0.4412
$(EV_2/M_2)\cdot 100$	0	−1.630	0	−0.613		

Notes: World prices at the port-city: $p_1^* = p_2^* = 1$. Urban labour endowment: $\bar{L}_2 = 1.2$.
$(EV_k/M_k)\cdot 100$ relative to $\tau_i = 0.05$.

Source: author's choice of functional forms and parameter values, as discussed in the text.

5 Conclusions

This paper has addressed the question: how do improvements in the rural road network affect economic activity and welfare in town and country in the presence of urban agglomeration economies? The answer depends, in particular, on the extent to which goods are internationally tradeable and labour is mobile. When all goods are so traded and rural–urban migration brings about equilibrium in the urban labour market, reductions in transport costs will, under quite weak assumptions, benefit villagers, but lead to a contraction in urban activity and income. In a closed, two-good, dual economy, with perfectly elastic supplies, at zero opportunity cost, of rural labour to urban production, both town and country benefit from a reduction in transportation costs, and these benefits are greater in the presence of empirically plausible agglomeration economies. The size of the latter's enhancing effect increases with the substitutability of rural and urban goods in final consumption.

Another factor comes into play when two tradeables are joined by a non-tradeable that is producible only in the city. If production of the non-tradeable is more labour-intensive than the urban tradeable, reductions in transport costs that directly benefit villagers will almost surely result in an increase in the aggregate demand for the non-tradeable, and hence in a fall in aggregate urban output. In the presence of agglomeration economies, the urban wage rate will then fall too, and with it, urban incomes and welfare—albeit only modestly when all goods are fairly good substitutes in consumption. A rural roads programme may indeed be socially profitable under the compensation principle; but it may also exacerbate the market failure stemming from external economies in the towns.

These findings urge caution when setting priorities in the task of estimating benefits for the purposes of programme evaluation. The enhanced benefits, in aggregate, that stem from empirically plausible agglomeration economies in the above variations are rather sensitive to the assumptions about the mobility of labour and the extent to which goods are internationally tradeable. In a closed dual economy that is set up so as to create highly favourable conditions for agglomeration economies to generate enhanced effects, the latter might amount to an extra 30 per cent or so; but when some goods are tradeable and the opportunity cost of rural labour increases in response to an improvement in the rural roads network, the presence of agglomeration economies could involve reductions in aggregate benefits, albeit modest in scale. In view of the difficulties and uncertainties of estimating the benefits generated by rural roads in the absence of urban agglomeration economies, it is far from clear that dealing with such externalities should be a pressing concern for practitioners.

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Appendix

Proof of Proposition 1. Let $Y_2 = G(L_2, Y_{12})$, and denote by G_i the partial derivative of G with respect to its ith argument, recalling that $Y_{12} \leq 0$. By assumption, all firms are identical, so that the first-order conditions for profit-maximization by urban firms may be written in aggregate form: $p_{22}G_1 = w$ and $p_{22}G_2 = -p_{21}$. Differentiating totally, noting that $\mathbf{p}_2$ is independent of the roads programme, and rearranging, we obtain $p_{22}(G_{11} - G_{12}^2/G_{22})dL_2 = dw$. Since G is strictly concave in the variable inputs alone, whereby $Y_{12} < 0$, $(G_{11} - G_{12}^2/G_{22}) < 0$, which implies that L_2 and w move in opposite directions.

Let $Y_1 = F(L_1, Y_{21})$, where F is strictly concave in the variable inputs. Since all rural households are identical, the first-order conditions may be written $p_{11}F_1 = w$ and $p_{11}F_2 = -p_{21}$. Differentiating totally, $F_1 dp_{11} + p_{11}(F_{11}dL_1 + F_{21}dY_{21}) = dw$ and $F_{12}dL_1 + F_{22}dY_{21} = -d(p_{11}/p_{21}) \equiv -\Delta$. Substituting and rearranging terms, we have $F_1 dp_{11} + p_{11}(F_{11} - F_{12}^2/F_{22})dL_1 - F_{22}\Delta = dw$. Since $dL_1 + dL_2 = 0$, substitution yields $\left(1 + \frac{p_{11}}{p_{22}} \cdot \frac{F_{11}F_{22} - F_{21}^2}{F_{22}} \cdot \frac{G_{22}}{G_{11}G_{22} - G_{12}^2}\right) dw = F_1 dp_{11} - F_{22}\Delta$. Since the programme leads to an increase in p_{11} and $\Delta > 0$, w will increase. □

Section 3.4 calculations. Let $\varepsilon = 0.2$ and $\tau_1 = \tau_2 = 0.1$. The calculations for $\varepsilon = 0$, when the system is recursive, are yet more straightforward; those for $\varepsilon = 0.5$ follow the same procedure as those for $\varepsilon = 0.2$.

Using (15) and (17), and given $\tau_1 = \tau_2 = 0.1$, we have $p_{22} = 1.22Y_2^{-0.2}$ and $0.55Y_1 = 0.62Y_2^{0.8}$, which yield $Y_1 = 2.49729/p_{22}^4$. From (18), $Y_1 = (1/1.1p_{22})^{1/9}$. Solving, we obtain $p_{22} = 1.26879, Y_1 = 0.96364$ and hence $Y_2 = 0.82197$. Turning to household incomes and consumption, $M_1 = (1 - \alpha_1)Y_1 = 0.86728$ and $p_{21} = 1.1 \times 1.26879 = 1.39567$. Given $\beta_1 = 0.5$, we obtain $X_{11} = 0.43364$ and $X_{21} = 0.43364/1.39567 = 0.31070$. Urban households' income is $M_2 = wl_2Y_2^{0.8} = 0.85484$, whence $X_{12} = 0.4M_2/1.1 = 0.31085$ and $X_{12} = 0.6M_2/1.26879 = 0.40425$. Upon completion of the roads programme, $\tau_1 = \tau_2 = 0.05$ and $p_{12} = 1.05$. Proceeding as before, the resulting allocation is reported in Table 1, along with those for $\varepsilon = 0$ and $\varepsilon = 0.5$.

EV_1 is calculated as follows. Substituting the values of X_{i1} with the programme ($\tau_i = 0.05$) from Table 1, we have $U_1 = 0.38875$. The lump-sum transfer in question, ΔM_1, must be such that rural households attain that level of utility when facing the prices ruling at $\tau_1 = \tau_2 = 0.1$. With their income thus augmented, they will consume $X_{11} = 0.5(0.87106 + \Delta M_1)$ and $X_{21} = 0.5(0.87106 + \Delta M_1)/(1.1 \times 1.26879)$. Hence, the required condition is $0.5(0.87106 + \Delta M_1)/(1.1 \times 1.26879)^{0.5} = 0.38875$, which yields $EV_1 \equiv \Delta M_1 = 0.02964$. That is to say, the transfer in question is $(0.02964/0.87106) \times 100 = 3.403$ per cent of aggregate rural income in the absence of the programme. Similar calculations yield the set of values of the (normalized) equivalent variation reported in Table 2, where it should be observed that M_1 and M_2 are approximately equal in all constellations.