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Measuring inequality of opportunity: recent empirical findings and some new theoretical results

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The plan of the presentation

1. Equality of opportunity: motivation
2. The theory of equal opportunity: the basic set up
3. Measuring inequality of opportunity (in the unidimensional case)
4. Recent empirical findings: www.equalchances.org
5. New theoretical results: measuring inequality of opportunity in a multidimensional setting

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1. Motivation

Equality of opportunity is not new in the economic literature:

- social mobility (origin independence)
- discrimination (e.g. gender)
- education (Heckman)
- inherited wealth in the long run (Piketty)

Beyond economics, very large consensus on the ideal of equality of opportunity.

Vagueness of the concept.

The “new” EOp literature: a unified approach, mostly devoted to measuring opportunity inequality, with a normative flavor (roots in political philosophy)

1. Motivation

i. **Inequality and distributive justice**: Rawls (1971), Sen (1980), Dworkin (1981), Arneson (1989), Cohen (1989), Roemer (1986, 1993). Distinguish «fair» and «unfair» inequalities.

ii. **Inequality and growth**: World Bank (2006), Marrero and Rodriguez (2013), Ferreira et al. (2014). Inconclusive empirical (macro) literature. Some micro evidence. Distinguish «good» and «bad» inequalities.

iii. **Social preferences**: Recent evidence in behavioral economics suggest that people are more willing to accept inequalities due to effort and ability than inequalities due to luck (Cappelen et al. 2007; Cappelen et al., 2010; Fong, 2001). This in turn affects the social preferences for redistribution.

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2. The theory of equal opportunities

- The economic models
  - The direct (Kranich, 1996)
  - <u>The indirect approach (Roemer 1993, Fleurbaey 1994)</u>

- The measurement of inequality of opportunity (Peragine 2002, World Bank 2006, Bourguignon et al. 2007)

- Recent surveys by Ferreira e Peragine (2015), Roemer e Trannoy (2015), Ramos e Van de Gaer (2015)

2. The theory of equal opportunities

$$x_i \equiv x(C_i, e_i; \varphi)$$

- Outcome: some"objective" measure of individual advantage
- Circumstances: variables outside the individual control
- Effort: e_i is defined so that $\frac{\partial x_i}{\partial e_i} \geq 0$
- Resources or policies

- Principle of compensation: "circumstances" should be compensated
- Principle of reward: "efforts" should be rewarded

- Compensation and reward are independent and, if g is not separable, incompatible

2. The theory of equal opportunities

Consider a finite population consisting of N = n x m individuals

Table 1

	e_1	e_2	e_3	…	e_m
C_1	x_{11}	x_{12}	x_{13}	…	x_{1m}
C_2	x_{21}	x_{22}	x_{23}	…	x_{2m}
C_3	x_{31}	x_{32}	x_{33}	…	x_{3m}
…	…	…	…	…	…
C_n	x_{n1}	x_{n2}	x_{n3}	…	x_{nm}

2. The theory of equal opportunities

Consider a finite population consisting of N = n x m individuals

A tranche

Table 1

	e_1	e_2	e_3	…	e_m
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C_3	x_{31}	x_{32}	x_{33}	…	x_{3m}
…	…	…	…	…	…
C_n	x_{n1}	x_{n2}	x_{n3}	…	x_{nm}

A type

Let a *type* consist of all individuals with identical circumstances

Let a *tranche* consist of all individuals with identical effort levels

2. The theory of equal opportunities

- Ex-ante approach (van de Gaer, 1993):
  - There is E.Op. if the individual opportunity sets have the same value
  - Need to evaluate opportunity sets
  - F(X|C) – that is a type distribution- interpreted as the opportunity set
  - Ex-ante Compensation: Focus on inequality between types
- Ex-post approach (Roemer, 1993; Fleurbaey 1995)):
  - There is E.Op. if all those who exert the same effort have the same outcome.
  - Need to observe (or deduce) the individual effort
  - Ex-post Compensation: Focus on inequality within tranches

2. The theory of equal opportunities

- Principle of Reward
  - Utilitarian (inequality neutral)
  - Agnostic
  - Inequality averse

2. The theory of equal opportunities

- Key results (Fleurbaey and Peragine, 2013):

  1. In general, ex-ante and ex-post compensation clash
  2. In general, ex-post compensation principle is inconsistent with reward
  3. Ex-ante compensation and reward principles are consistent.

- Either endorse an ex ante view, or look for a second best solution

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3. Measuring inequality of opportunity

- Testing for EOp (Lefranc et al. 2009)

- Partial orderings (Peragine 2002, 2004): dominance conditions

- Complete orderings: starting from distribution X
  i. construct a counterfactual distribution $\tilde{X}$ that (a) does not contain inequality due to e and (b) preserves fully the inequality due to c;
  ii. measure inequality in $\tilde{X}$.

3. Measuring inequality of opportunity

Partial orderings, ex ante approach

Peragine (2002) considers an increasing, additive and partially symmetric SWF:

$$W(X) = \sum_{i=1}^{n} \sum_{j=1}^{m} U_i(x_{i,j})$$

Imposes:
-inequality aversion between types (ex ante compensation)
-inequality neutrality within type (utilitarian reward)

Obtains the following condition: $$\sum_{i=1}^{k} \mu_i^X \geq \sum_{i=1}^{k} \mu_i^Y, \forall k = 1, ..., n$$

Generalized Lorenz dominance of the distributions of the type-means

3. Measuring inequality of opportunity

Complete orderings, Ex ante approach

- **Inequality between types** (Peragine 2002)
- *For all* j *and for all* i, $\tilde{x}_{i,j} = \mu_i$

$$\tilde{X}^{BT} = \begin{bmatrix} & e_1 & e_2 & e_3 \\ c_1 & \mu_1 & \mu_1 & \mu_1 \\ c_2 & \mu_2 & \mu_2 & \mu_2 \\ c_3 & \mu_3 & \mu_3 & \mu_3 \end{bmatrix}$$

- Fully consistent with reward and ex ante compensation, violates ex post compensation

3. Measuring inequality of opportunity

A number of methodological choices can affect the results of inequality of opportunity comparisons (in addition to usual problems related to total inequality):

- theoretical concept of EOp
- parametric vs. non parametric estimation
- choice of the inequality measure
- choice and treatment of circumstances (particularly when observable circumstances differ)

Caveat : A lower bound estimate

- Interpretation: *IEO (L or R)* is a lower-bound measure of inequality of opportunity
  - Omitted circumstances cannot lower it.
  - Whenever the dimension of the observed vector *C* is less than the dimension of the "true" vector *C**, then $\boldsymbol{I}(\widetilde{\boldsymbol{x}}_{\boldsymbol{BT}})$ is a lower-bound estimator of true inequality of opportunity: the inequality that would be captured by the same indices if the full vector *C** were observed.
  - Intuition: Suppose only race is observed (blacks, whites hispanics), but in reality gender also matters:

Table 2: Between-types inequality *(n=m=3)*

	e1	e2	e3
C1	μ_1	μ_1	μ_1
C2	μ_2	μ_2	μ_2
C3	μ_3	μ_3	μ_3

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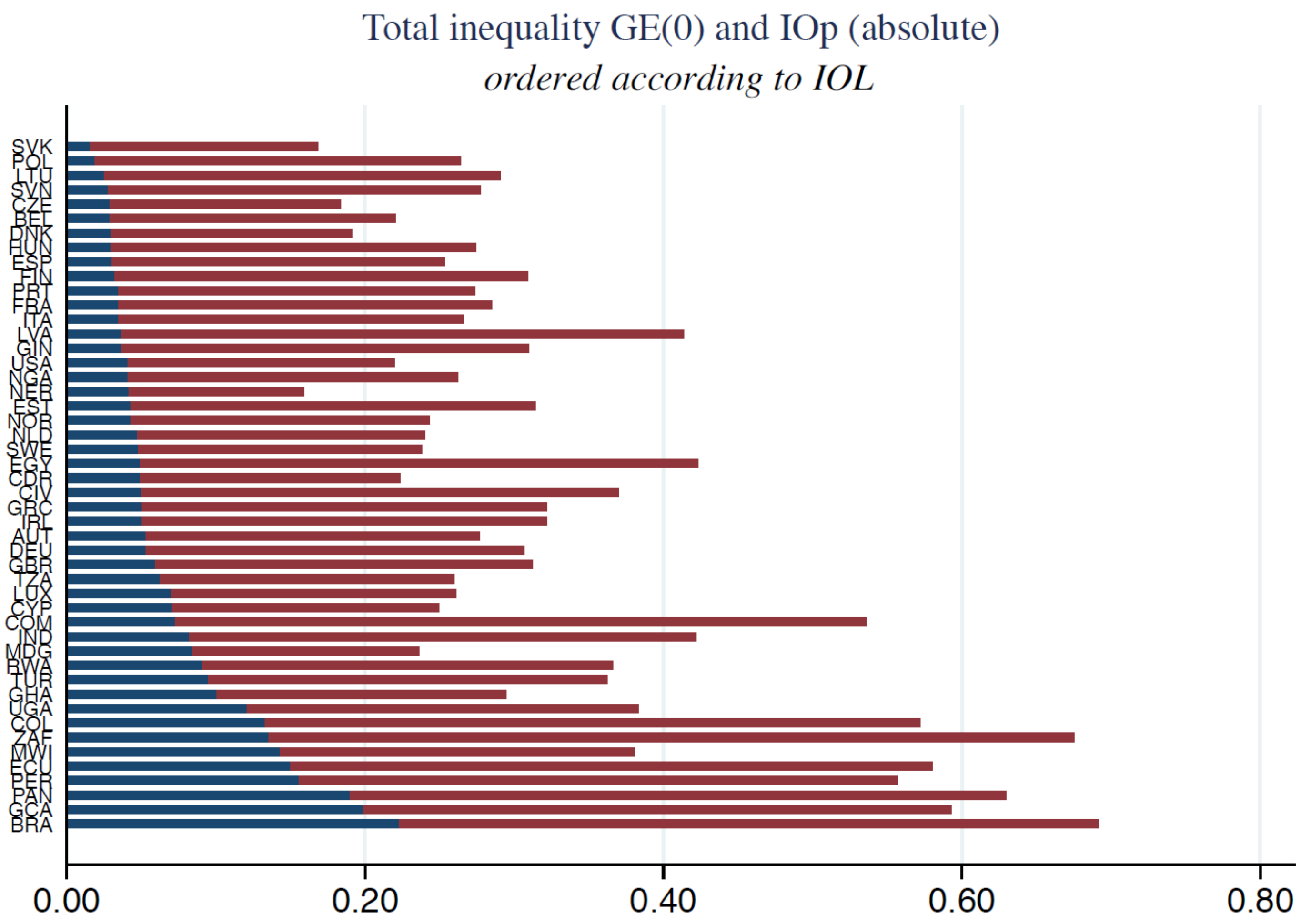
4. Recent empirical findings

	References	Countries	Data sources	Outcome	Method	Circumstances	Number of types
1	Checchi, Peragine, Serlenga (2016)	29 (Europe:an Couries	EU-Silc 2005 and 2011	post-tax individual equivalent incomes	Ex ante Parametric and non parametric	parental education, parental occupation, gender, nationality, age	96
2	Brunori, Palmisano, Peragine (2016)	Comoros, Democratic Republic of Congo, Ghana, Guinea, Madagascar, Malawi, Niger, Nigeria, Rwanda, Tanzania, and Uganda (Africa: 11)	Living Standard Measurement Surveys (LSMS), designed by the World Bank , for Malawi, Niger, Nigeria, Tanzania,Uganda. EIM for Comoros, GLSS for Ghana, EIBEP for Guinea, EPM for Madagascar, EICV for Rwanda.	per capita consumption	Ex ante Parametric and non parametric	father's occupation and education, region of birth, ethnicity	From 20 (Nigeria) to 64 (Malawi)
3	Ferreira and Gignoux (2011)	Brazil, Colombia, Ecuador, Guatemala, Panama, Peru	Brazil, PNAD 1996; Colombia, ECV 2003; Ecuador ECV 2006; Guatemala, ENCOVI 2000; Panama, ENV 2003; Peru, ENAHO 2001	household per capita income	Ex ante Parametric	gender, ethnicity, parental education, father's occupation, region of birth.	From to 108 (54 Peru)
4	Ferreira, Gignoux, Aran (2011)	Turkey	TDHS 2003-2004 and HBS 2003	imputed per capita consumption	Ex ante Parametric	urban/rural, region of birth, parental education, mother tongue, number of sibling	768
5	Hassine (2012)	Egypt	ELMPS 2006	total monthly earning	Ex ante Non parametric	gender, father's education, mother's education, father's occupation, region of birth.	72
6	Piraino (2012)	South Africa	NIDS 2008-2010	Individual gross income	Ex ante Parametric	race, father's education	24
7	Pistolesi (2009)	US	PSID 2001	individual annual earnings	Ex ante Semiparam etric	age, parental education, father's occupation, ethnicity, region of birth	7,680
8	Singh (2011)	India	IHDS 2004–2005	household per capita earnings	Ex ante parametric	father's education and occupation, caste, religion, location	108

4. Recent empirical findings



Source: Brunori, Ferreira, Peragine (2015)

Between types IOp

IOp: between 5% SVK and 40% Malawi.

Comparability problems:
- outcome: earnings, incomes, consumpion
- circumstances:
  - Western countries: mostly social and family background, gender
  - Africa: etnic groups, location

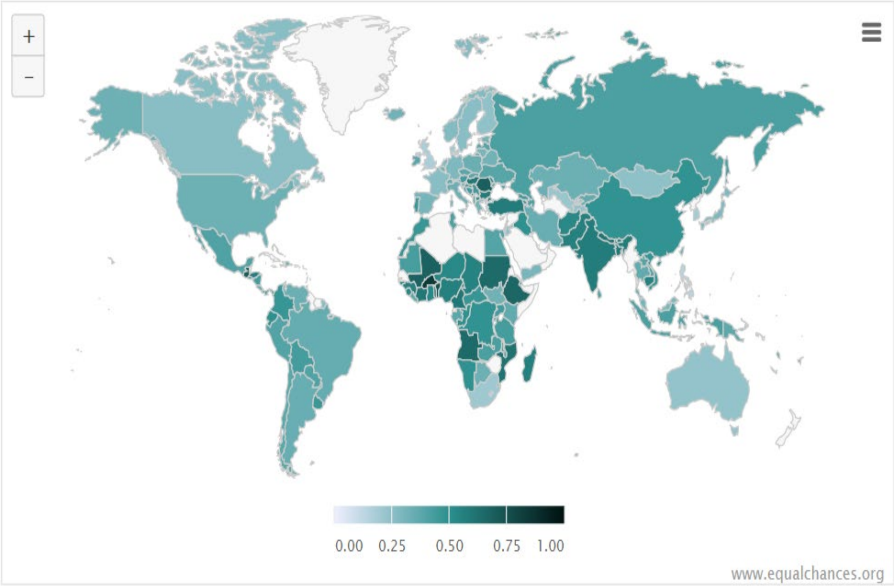
- Data quality is not independent of development

4. Recent empirical findings

www.equalchances.org

The World Database on Equality of Opportunity and Social Mobility

University of Bari and World Bank



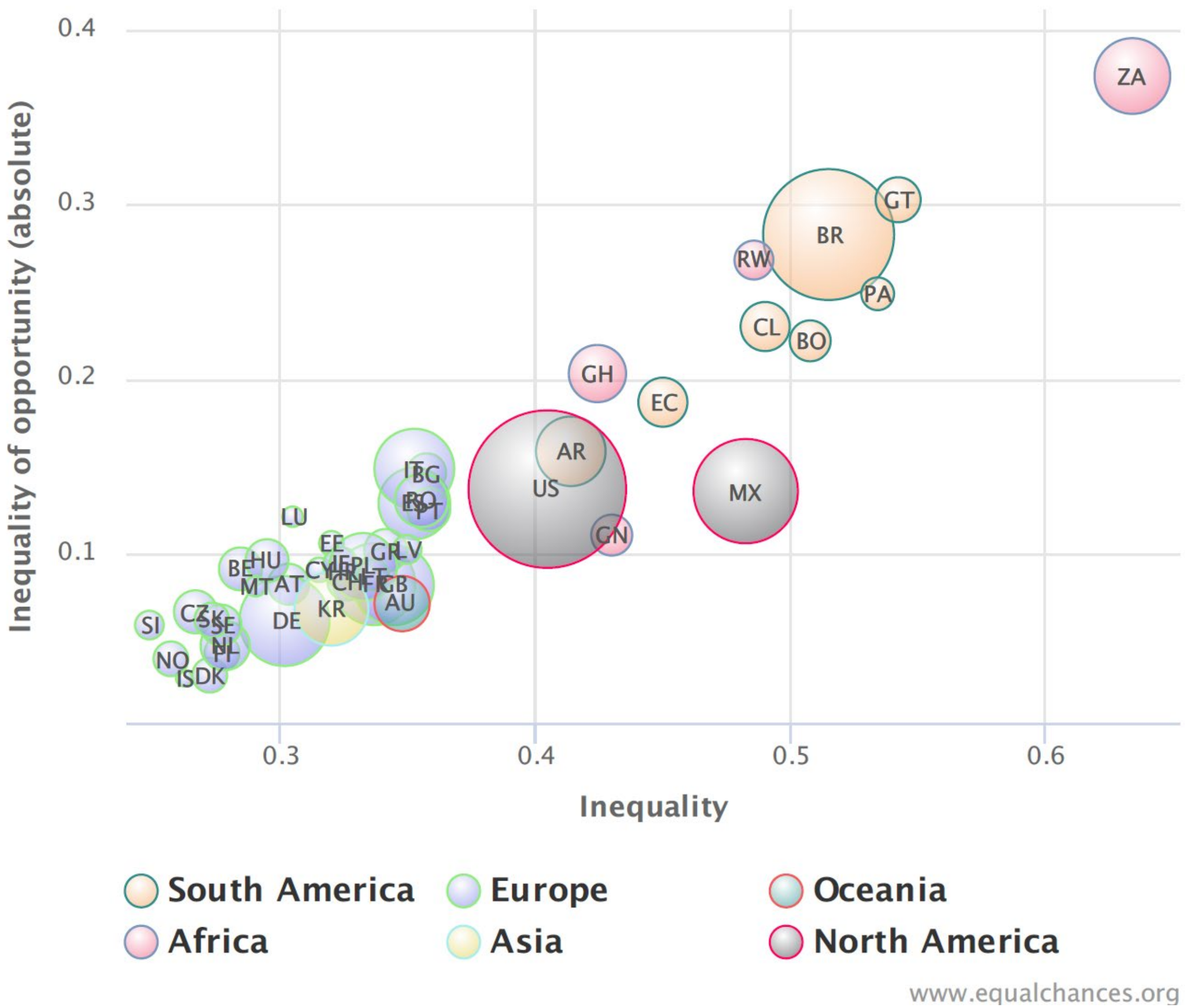
Maps

Intergenerational persistence is the coefficient from aregression of respondents' years of schooling on the highest years of schoolingof their parents.

The Equalchances.org database contains:

- measures of *inequality of opportunity* for 47 countries based on 124 surveys
- measures of *income mobility across generations* for 26 countries based on 52 surveys
- measures of the *intergenerational transmission of status* for 41 countries based on 288 surveys
- measures of *educational mobility across generations* for 148 countries based on 152 surveys

4. Recent empirical findings



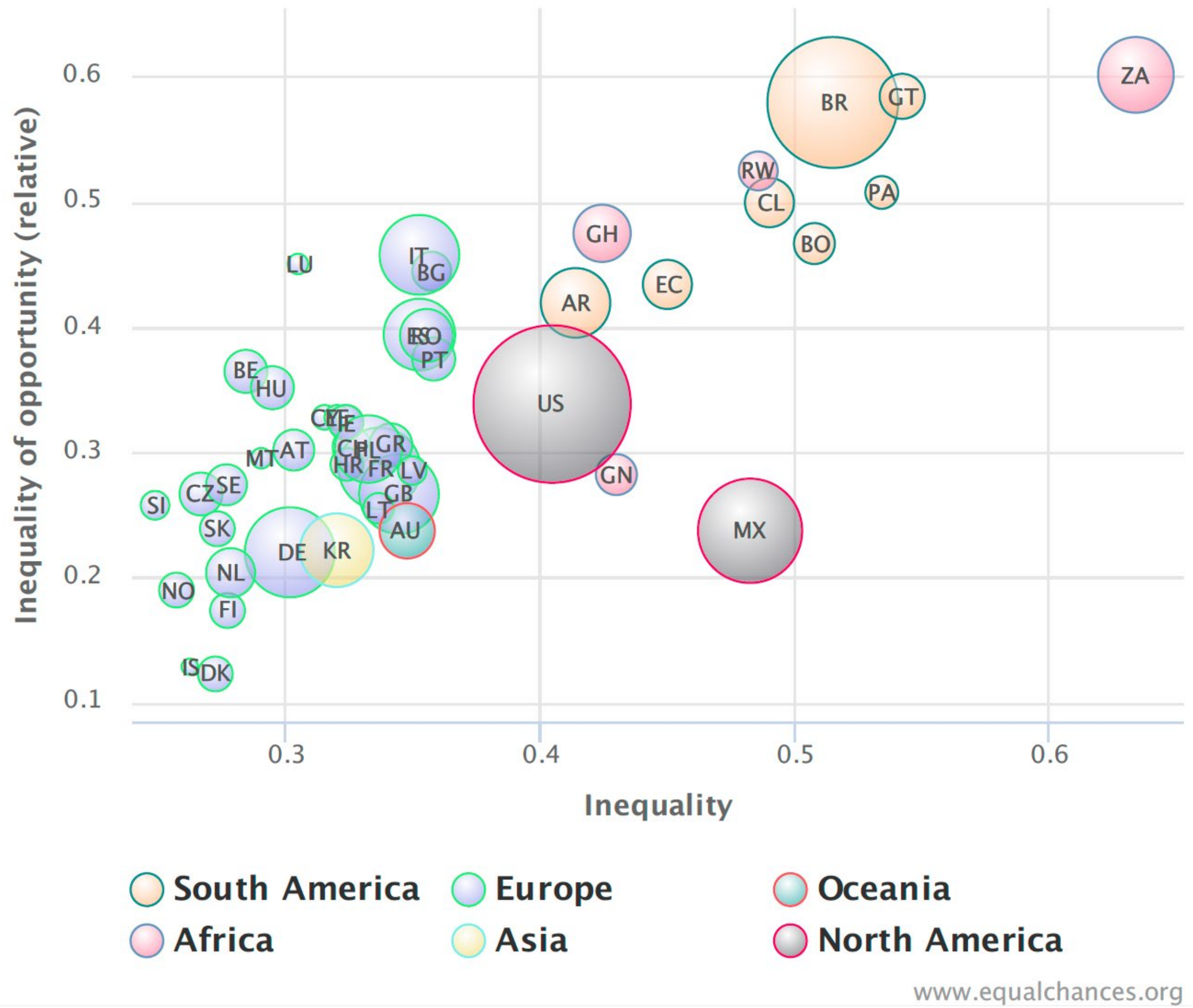
Ex ante «between types» inequality of opportunity.

Circumstances: parental education and occupation, origin.

Unfair inequality: between 0.02 and 0.3

Positive correlation between income inequality and inequality of opportunity

4. Recent empirical findings



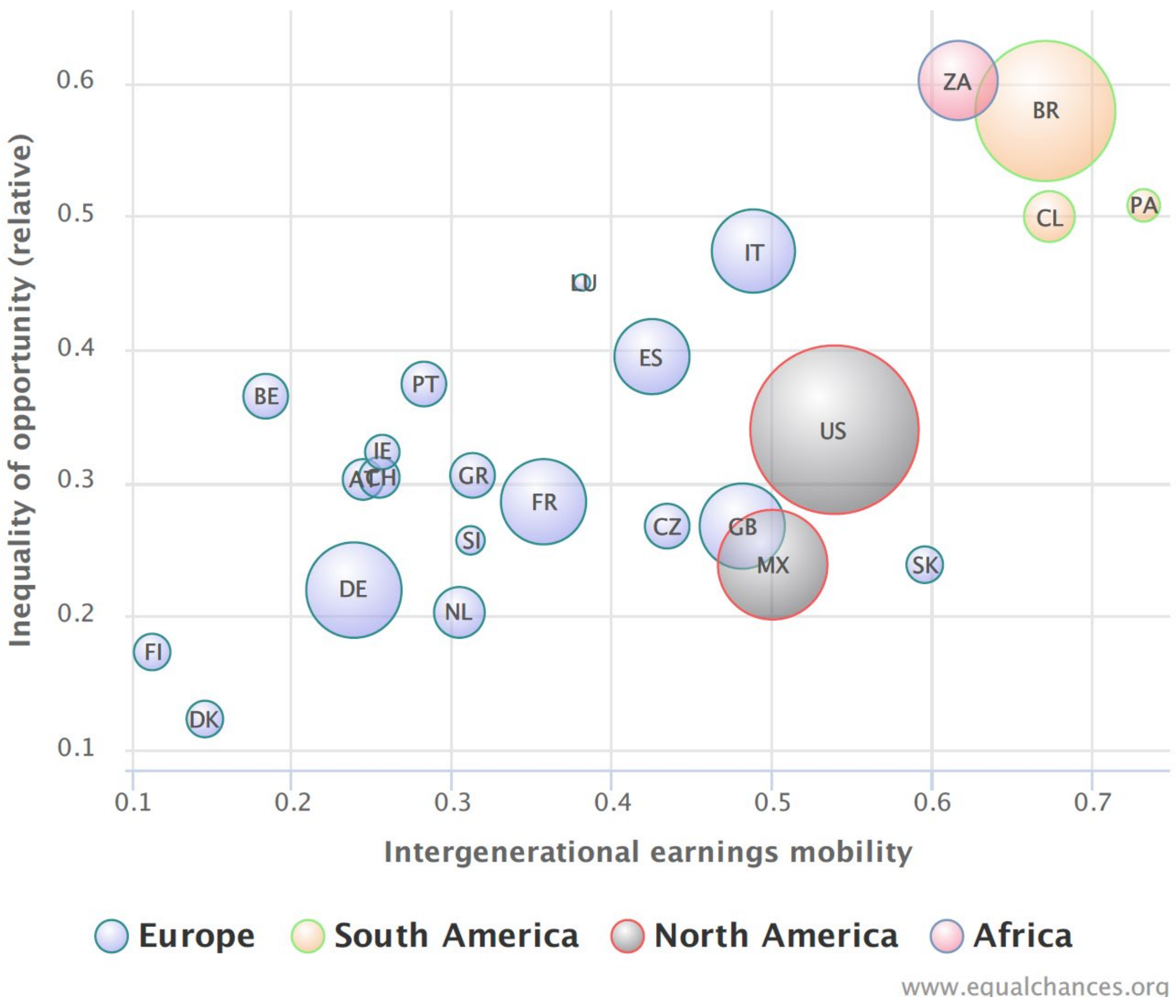
Ex ante «between types» <u>relative</u> inequality of opportunity.

Circumstances: parental education and occupation, origin.

Unfair inequality: between 10% and 60%

Positive correlation between income inequality and inequality of opportunity

4. Recent empirical findings



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A theory of multidimensional equality of opportunity

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The plan of the presentation

- Motivation & what we do
- The analytical framework
- Class (1, 2, 3) SEFs:
  - Axioms
  - Dominance results
  - IOp measures
- Conclusions

Motivation & what we do

- We extend EOP theory to the case in which individual **outcome is a multidimensional variable** (e.g. income, health, education).
- We focus on **ex-ante** approach.
- We have a multidimensional distribution of outcomes within each type.
- Types differ in marginal distributions and dependence. **What is the extent of unfair opportunities if one takes both into account?**
- How does considering joint distribution of outcomes affect measured IOP?

Motivation & what we do

- We order distributions of multidimensional outcomes X by a preference of an "opportunity egalitarian social decision maker" represented by a continuous social welfare function.
- We focus on three classes of social welfare functions, characterized by certain EOP axioms.
- All the classes satisfy **ex ante compensation.**
- The first class satisfies **utilitarian reward** (i.e., it is neutral to inequality within type)
- The second class satisfies **inequality-averse reward principle** (i.e., it is averse to inequality within type).
- The third class is **agnostic with respect to reward.**
- We adopt a dual perspective: welfare functions and inequality measures, which are **induced** by welfare functions.

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The analytical framework

- We have a society consisting of N individuals described by a vector of circumstances $O \in \mathcal{O}$ (ordered $O_i \prec O_{i+1}$) and effort (scalar) $w \in \Theta \subseteq \mathbb{R}_+$.
- Outcome is generated deterministically by $g : \mathcal{O} \times \Theta \to \mathbb{R}^k_+$.
- Let $D = \{X \in M_{N \times k}(\mathbb{R}_+) : g$ is monotone in $\mathcal{O}, \Theta\}$ denote the set of possible outcome profiles.
- $W : D \to \mathbb{R}$ is a continuous welfare function and $I : D \to [0, 1]$ is an inequality index.
- Notation: X^h_{ij} — i-th individual, j-th dimension, h type. X^h a distribution within a type h. X_μ rows are type-means on each dimension (equality *within* type but not *between* type). X^μ a matrix of population means (perfect equality).

IOp measures

- Inequality measure I_W is induced by a welfare function W if $I_W(X) = 1 - \delta(X)$ where $\delta(X) \in [0, 1]$ satisfies equation $W(X) = W(\delta(X)X^{\mu})$, where X^{μ} is a matrix of means (i.e. perfect equality within and between types).
- Then indices are normatively significant i.e. *under some restrictions e.g. equality of means* $W(X) \leq W(Y) \iff I(X) \geq I(Y)$.
- We care about inequality because we believe that it lowers welfare (Dalton 1920, Atkinson 1970).

Definitions

- A Pigou-Dalton Transfer in the multivariate context is a transfer between two individuals that simultaneously involves all attributes (but we admit different proportions).

PDT — between individuals i_1, i_2 from type h we make a transfer on each dimension j of potentially different amounts $X^h_{i_1 j} = Y^h_{i_1 j}\varepsilon_j + Y^h_{i_2 j}(1-\varepsilon_j)$ and $X^h_{i_2 j} = Y^h_{i_2 j}(1-\varepsilon_j) + Y^h_{i_1 j}\varepsilon_j$ where $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_k)$ with $\varepsilon_i \geq 0$ and at least one $\varepsilon_i > 0$.

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- A correlation-increasing transfer is an exchange of all attributes between two individuals after which one individual is left with the lowest endowment and the other with the maximum endowment of each attribute. Such an operation clearly increases correlation between dimensions.

CIT — correlation - increasing transfer (Tsui 1999). Individual i_1 is given $\left(\max X^h_{i_1 j}, X^h_{i_2 j}\right)$ and individual i_2 gets $\left(\min X^h_{i_1 j}, X^h_{i_2 j}\right)$ on each dimension j

Definitions

- All attributes are assumed to be transferable
  - Does it make much sense to talk of “transferring health”?

- Bosmans et al. (2009) study the implications of formulating a version of the Pigou-Dalton principle that applies only to transferable attributes

- Muller and Trannoy (2012) examine dominance conditions when attributes are asymmetric in the sense that one attribute (typically income) can be used to compensate for lower levels of other attribute(s) (e.g. needs, health, etc.).

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- Motivation & what we do
- The analytical framework
- Class (1, 2, 3) SEFs:
  - Axioms
  - Dominance results
  - IOp measures
- Conclusions

Class 1: Axioms

- **MONOTONICITY (MON)** $W : D \to \mathbb{R}$ is a monotone function
- **ADDITIVITY (ADD)** $W(X) = \sum_{h=1}^{n} \sum_{i=1}^{N_h^X} U^h(X_i^h)$ (*can be relaxed to adding utilities within type, but general aggregation between types)

Class 1: Axioms

Utilitarian Reward:

- **TYPE SYMMETRY (T-SYM)** W is invariant to permutation of individuals *within a type*
- **INEQUALITY NEUTRALITY WITHIN TYPES (INWT)** For all $X, Y \in D$, if $X^h = Y^h$ for all $h \neq l$, X^l is obtained from Y^l by PDT or CIT, then $W(X) = W(Y)$.

Class 1: Axioms

Utilitarian Reward:

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Ex ante Compensation:

- **INEQUALITY AVERSION BETWEEN TYPES (IABT)** For all $X, Y \in D$, X is obtained from Y via a PDT between two types and the ordering of types $\mathcal{O}$ is unchanged. Then $W(X) > W(Y)$.

Class 1: Definitions

- Altogether, Class 1 is

$$\mathcal{W}^{AOEN} = \{W|\mathrm{MON}, \mathrm{ADD}, \mathrm{T}-\mathrm{SYM}, \mathrm{INWT}, \mathrm{IABT}\}.$$

Class 1: Results

- **THEOREM 1** For $X, Y \in D$ we have

$$X_\mu \succeq_{CGLD} Y_\mu \iff W(X) \geq W(Y) \quad \forall_{W \in \mathcal{W}^{AOEN}},$$

  where CGLD is Generalized Lorenz Dominance applied to *each* dimension *separately* (C – component-wise).
- Because of neutrality, type distribution X^h can be summarized by type-means distribution X_μ.
- Necessary for a meaningful interpretation of CGLD: total sum on *each* dimension has to be higher in one type.
- A restrictive result - as a consequence of INWT utility functions are affine and W is of the form

$$W(X) = \sum_{h=1}^{n} \sum_{i=1}^{N_h^X} \sum_{j=1}^{k} a_j^h X_{ij}^h,$$

  with $a_j^h > a_j^{h+1}$ for all j.

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Class 2: Axioms and Definitions

Inequality Averse Reward

- **INEQUALITY AVERSION WITHIN TYPES (IAWT)** For all $X, Y \in D$, if $X^h = Y^h$ for all $h \neq l$, X^l is obtained from Y^l by PDT or CIT, then $W(X) < W(Y)$.
- We keep **IABT, MON, ADD, T-SYM**
- Altogether, Class 2 is

$$\mathcal{W}^{AOEA} = \{W | \mathrm{MON}, \mathrm{ADD}, \mathrm{T-SYM}, \mathrm{IAWT}, \mathrm{IABT}\}.$$

Class 2: results

- **THEOREM 3** For X, $Y \in D$ we have

$$X \succeq_{LD(\mathcal{U}^{ICL})} Y \iff W(X) \geq W(Y) \quad \forall_{W \in \mathcal{W}^{AEOA}},$$

where

$$X \succeq_{LD(\mathcal{ICL})} Y \iff \sum_{h=1}^{l} u_h^X \geq \sum_{h=1}^{l} u_h^Y \quad \forall_{l=1,\ldots,n} \forall_{U^h \in \mathrm{ICL}},$$

and

$$\mathcal{U}^{ICL} = \{U | \mathrm{Increasing}, \mathrm{Type} - \mathrm{Concave}, \mathrm{Submodular}\}.$$

Definition 4. ***Type-Concavity*** *Function* $U^h : \mathbb{R}^k \to \mathbb{R}$ *is type-concave if its first derivate decrease with respect to a type i.e. the better the type the lower the first derivative. Formally,* $dU^h/dX > dU^{h+1}/dX > 0$.

Definition 5. ***Submodularity*** *Function* U^h *is submodular, if* $U^h(X_p^h) + U^h(X_q^h) > U^h(X_p^h \wedge X_q^h) + U^h(X_p^h \vee X_q^h)$ *where* $X_p^h \wedge X_q^h$ *is a vector of elements* $\max\{X_{pj}^h, X_{qj}^h\}$ *and* $X_p^h \wedge X_q^h$ *of* $\min\{X_{pj}^h, X_{qj}^h\}$

Submodularity reflects that association between dimensions matters, and if there is more of it the utility is lower.

Class 2: Results

- LD first aggregates individual utilities within type (note that an individual utility is a function of many attributes), thereby obtaining a value of type opportunity set; and then compares partial sums of such aggregate utility vectors.

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Class 3

Agnostic with respect to reward

$$\mathcal{W}^{AOEAG} = \{W|\text{MON}, \text{ADD}, \text{T} - \text{SYM}, \text{IABT}\}$$

$$\mathcal{U}^{ICLA} = \{U|\text{Increasing}, \text{Type} - \text{Concave}\}.$$

Theorem 3. $X \succeq_{LD(\mathcal{U}^{ICAL})} Y \iff W(X) \geq W(Y) \quad \forall_{W \in \mathcal{W}^{AEOAG}}$

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IOp measures

Definition

I is inequality measure if it satisfies the following properties

1. I is continuous,
2. $I(X^{\mu}) = 0$,
3. $I(X) = I(\sigma(X))$ for any permutation σ satisfying T-SYM i.e. *within* types only,
4. $I(Y) < I(X)$ if Y is PDT *between types* of X <- aversion to inequality *between* types.

Measure is ***relative*** if additionally $I(XC) = I(X)$ for diagonal matrix C. If $I(XC) = I(X)$ for C diagonal with equal elements, then I is ***weakly relative***.

Class 1 IOp Measures: Results

THEOREM 2

1. I_W is given by $1 - \frac{W(X_\mu)}{W(X^\mu)} = 1 - \frac{\sum_{h=1}^{n} N_h^X \sum_{j=1}^{k} a_j^h (X_\mu)_{1j}^h}{\sum_{h=1}^{n} N_h^X \sum_{j=1}^{k} a_j^h (X^\mu)_{1j}^h}$,
2. Here $a_j^h > a_j^{h+1}$: better off type gets lower weight
3. I_W is a weakly relative inequality measure.

- The index related to the class 1 is one minus the weighted sum of type-means for each dimension normalized by the highest amount of welfare achievable

- It is a weakly relative measure: it does not change when all attributes are scaled by the same factor, but it is not invariant when each attribute is scaled by its mean.

Class 2 IOp measures

- Further restriction on the Class 2
  RATIO SCALE INVARIANCE (RSI) (Tsui 1995)
  $W(X) = W(Y) \iff W(XC) = W(YC)$ for C diagonal

Class 2 IOp Measures

THEOREM 4

1. $I_W(X)$, is a relative inequality measure,
2. Utility functions U^h are of the form $a_h \prod_{j=1}^{k} (X_{ij}^h)^{r_j}$, $r_j \in (0,1]$,
3. $I_W(X)$ is given by

$$I_W(X) = 1 - \left(\sum_{h=1}^{n} w_h \frac{U^h((X_\mu)_1^h)}{U^h((X^\mu)_1^h)} \right)^{\frac{1}{\sum_{j=1}^{k} r_j}}$$

where $w_h = \frac{\delta_h(X) N_h^X}{N}$ for $\delta_h(X)$ of form

$$\delta_h(X) = \left[\frac{1}{N_h^X} \sum_{i=1}^{N_h^X} \prod_{j=1}^{k} \left(\frac{X_{ij}^h}{(X_\mu)_{1j}^h} \right)^{r_j} \right].$$

Class 2 measures

- Inequality indices related to Class 2 are weighted sums of normalized types' utilities, where weights are Tsui (1995) inequality indices computed within type.

- Two components: the distribution of utilities *between* types $\left(\frac{U^h\left((X_\mu)_1^h\right)}{U^h\left((X^\mu)_1^h\right)}\right)$ and the distribution of attributes *within* type $(\delta_h(X)$.
- Weights w_h are Tsui (1995) inequality indices computed *within* a type – sensitivity to attributes' dependence.
- Due to concavity of I_W a more equal distribution of $U^h((X_\mu)_1^h)$ is preferred – inequality aversion between types' welfare.
- For $r_j = r$ for all j, an increase in r results in a decrease of inequality. r_j are dimensions' weights.

- Parameters r are dimensions weights. The higher r the higher the degree of concavity in a given dimension and the higher inequality weight attached to this dimension.

Conclusions

- We have incorporated multidimensionality of outcomes in the canonical model of EOp, and characterized dominance conditions which pays attention to it.

- We also characterized classes of “induced” IOP measures

- Further research:
  (i) domain extensions (type partitions)
  (ii) allowing for transferable and non transferable attributes
  (iii) different degrees of inequality aversion for circumstances-based and effort-based inequalities
  (iv) extension of the ex post approach, in all its variants (see Roemer, 1998, Fleurbaey 2008 and Fleurbaey et al. 2017);
  (v) empirical analysis.

Thank you.