Untimely Destruction: Pestilence, War and Accumulation in the Long Run

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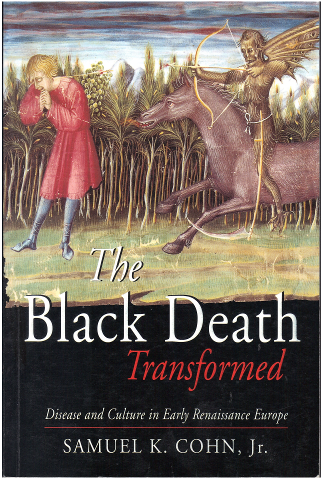

The
Black Death
Transformed
Disease and Culture in Early Renaissance Europe
SAMUEL K. COHN, Jr.

Pestilence

High-frequency: The Black Death, Spanish influenza.
Low-frequency (and endemic): TB, AIDS, malaria.

Age-specific and socio-economic patterns of mortality.

The Black Death: all and sundry (indiscriminate).
Spanish influenza, TB, AIDS: mostly young and prime-aged adults.
Malaria: young children.

Wars

WWI: 15-20 million deaths, majority young males.
Limited physical destruction, especially in the West.
WWII: 50-60 million deaths, at least one-half non-combatants.
Wholesale razing of cities and infrastructure in all theatres.

Table: 1. Life expectancy and HIV prevalence, 2005 (WHO, UNAIDS)

Country	E_0	E_{20}	${}_{20}q_{20}$	${}_{30}q_{20}$	HIV[1]	[low - high estimates]
Botswana	41.5	28.3	0.414	0.642	24.1	[23.0 - 32.0]
Kenya	51.3	39.7	0.209	0.344	6.1	[5.2 - 7.0]
Mozambique	45.6	34.5	0.305	0.471	16.1	[12.5 - 20.0]
Namibia	51.8	35.9	0.305	0.478	19.6	[8.6 - 31.7]
Nigeria	47.6	40.8	0.171	0.291	3.9	[2.3 - 5.6]
South Africa	51.0	35.8	0.286	0.434	18.8	[16.8 - 20.7]
Tanzania	49.1	37.9	0.216	0.372	6.5	[5.8 - 7.2]
Uganda	49.7	39.4	0.199	0.335	6.7	[5.7 - 7.6]
Zambia	40.4	31.0	0.364	0.546	...	[15.9 - 18.1]
China	72.4	54.9	0.024	0.053	0.1	[$<$0.2]
India	63.0	49.2	0.065	0.123	0.9	[0.5 - 1.5]
Germany	79.3	59.9	0.012	0.033	0.1	[0.1 - 0.2]
Japan	82.2	62.7	0.012	0.029	$<$0.1	[$<$0.2]
US	77.9	58.8	0.022	0.050	0.6	[0.4 - 1.0]

The questions.

- How large can these hazards be without calling into existence a poverty trap?
- How large can they be without ruling out the possibility of steady-state growth?
- In what settings are both secular, low-level stagnation and steady-state growth possible equilibria?
- Is balanced growth possible when parents are moved by altruism?
- If so, is stronger altruism conducive to faster steady-state growth?

Literature.

(i) General relationship between health and economic activity.
Barro and Sala-I-Martin (1995); Bloom et al. (2001).

(ii) The long-run effects of HIV/AIDS.
Corrigan, Glomm and Méndez (2004, 2005). A two-generation OLG framework, capital accumulation, no role for expectations.
Young (2005). Solovian model with endogenous schooling and fertility, constant savings rate.
Bell, Devarajan and Gersbach (2006). A two-generation OLG model, central role for expectations, none for physical capital.

(iii) Theoretical contributions, mortality at centre stage.
Chakraborty (2004). OLG framework with endogenous mortality.
Boucekkine and Laffargue (2010). OLG, heterogeneous mortality.
Bell and Gersbach (2013). 2-generation OLG, 2-period shocks to mortality.

(iv) Balanced growth once more. Grossman at al. (2016).

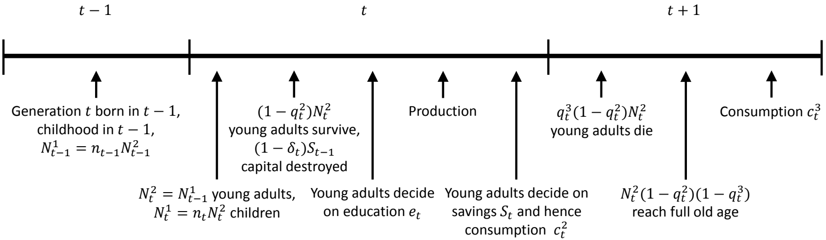
Three overlapping generations, numerous large extended families.

Demography.
NRR: n_t (exogenous).
The numbers of young adults and offspring reaching maturity:

$$N_t^2 = n_{t-1} N_{t-1}^2 \text{ and } N_t^1 = n_t N_t^2.$$

The numbers of young and old adults making claims on output:

$(1 - q_t^2) N_t^2$ young adults survive to raise all children, and
$(1 - q_t^3) N_t^3 = (1 - q_t^3)(1 - q_{t-1}^2) N_{t-1}^2$ old adults survive to full old age.


$t-1$
t
$t+1$
Generation t born in $t-1$, childhood in $t-1$, $N_{t-1}^1 = n_{t-1}N_{t-1}^2$
$N_t^2 = N_{t-1}^1$ young adults, $N_t^1 = n_t N_t^2$ children
$(1-q_t^2)N_t^2$ young adults survive, $(1-\delta_t)S_{t-1}$ capital destroyed
Young adults decide on education e_t
Production
Young adults decide on savings S_t and hence consumption c_t^2
$q_t^3(1-q_t^2)N_t^2$ young adults die
$N_t^2(1-q_t^2)(1-q_t^3)$ reach full old age
Consumption c_t^3

Social rules.
1. Each child consumes βc_t^2 $(\beta < 1)$ when each surviving young adult consumes c_t^2 of the aggregate good.
2. Surviving old adults obtain together the share ρ of the family's current 'full income', $\bar{Y}_t$.

Each surviving old adult consumes

$$c_t^3 = \frac{\rho \bar{Y}_t}{(1 - q_t^3) N_t^3} \, . \qquad (1)$$

Since the family is very large, only the idiosyncratic risk of mortality at the start of old age remains.

Production.

Output produced by means of labour (measured in efficiency units) and capital, which is made of the same stuff as output.

Each young adult possesses λ_t efficiency units, each child γ units.

Each fully educated child ($e_t = 1$) requires $w\,(< 1)$ young adults as teachers.

Total endowment of the surviving young adults' human capital is $\Lambda_t \equiv (1 - q_t^2)N_t^2\lambda_t$.

Labour supplied to the production of the aggregate good is

$$L_t \equiv [(1 - q_t^2 - wn_te_t)\lambda_t + n_t\gamma(1 - e_t)]N_t^2. \qquad (2)$$

The capital stock available for current production is $K_t = \sigma_t S_{t-1}$

The current levels of aggregate output and full income are

$$Y_t = F(L_t, \sigma_t S_{t-1}). \qquad (3)$$

$$\bar{Y}_t \equiv Y(e_t = 0) = F(\Lambda_t + \gamma N_t^1, \sigma_t S_{t-1}).$$

The budget constraint.

$$P_t c_t^2 + S_t + \rho \bar{Y}_t = Y_t, \tag{4}$$

where $P_t \equiv [(1 - q_t^2) + \beta n_t] N_t^2$ is effectively the price of c_t^2 in terms of output, the numéraire.

The formation of human capital.

The human capital attained by a child on reaching adulthood is given by

$$\lambda_{t+1} = z_t h(e_t) \lambda_t + 1. \tag{5}$$

$h(e_t)$ represents the educational technology,
with efficiency factor z_t and a given w.

Let h be increasing and differentiable on $[0, 1]$,
with $h(0) = 0$ and $\lim_{e \to 0+} h'(e) < \infty$.

Preferences and choices.

Young adults make all allocative decisions.

They must forecast mortality and destruction rates in period $t+1$.

They are assumed to have sharp priors and
additively separable preferences over $(c_t^2, c_{t+1}^3, \lambda_{t+1})$:

$$V_t = u(c_t^2) + \delta(1 - q_{t+1}^3)u(c_{t+1}^3) + \frac{b(1 - q_{t+1}^2)}{(1 - q_t^2)} n_t v(\lambda_{t+1}), \quad (6)$$

Their decision problem:

$$\max_{(c_t^2, e_t, S_t)} V_t \;\text{ s.t. } (1) - (5),\; c_t^2 \geq 0, e_t \in [0,1], S_t \geq 0. \quad (7)$$

The current state variables: $(N_t^1, N_t^2, N_t^3, q_t^2, q_t^3, \lambda_t, K_t)$.

The variables to be forecast: $(n_{t+1}, q_{t+1}^2, q_{t+1}^3, \sigma_{t+1})$.

Evolution of the economy.
Let (c_t^{20}, e_t^0, S_t^0) solve (7), where

$$e_t^0 = e_t^0(\lambda_t, K_t, \mathbf{N}_t, \mathbf{q}_t \,; n_{t+1}, \mathbf{q}_{t+1}, \sigma_{t+1}; \beta, \rho, w, \gamma, \delta, b),$$
$$S_t^0 = S_t^0(\lambda_t, K_t, \mathbf{N}_t, \mathbf{q}_t \,; n_{t+1}, \mathbf{q}_{t+1}, \sigma_{t+1}; \beta, \rho, w, \gamma, \delta, b),$$

The system's behaviour is governed by the pair of equations

$$\lambda_{t+1} = z_t h(e_t^0)\lambda_t + 1 = H(\lambda_t, K_t, \mathbf{N}_t, \mathbf{q}_t \,; n_{t+1}, \mathbf{q}_{t+1}, \sigma_{t+1}; \cdot)$$
$$K_{t+1}^0 = \sigma_{t+1} S_t^0 = G(\lambda_t, K_t, \mathbf{N}_t, \mathbf{q}_t \,; n_{t+1}, \mathbf{q}_{t+1}, \sigma_{t+1}; \beta, \rho, w, \gamma, \delta, b).$$

Define

$$\chi_t \equiv \delta(1 - q_{t+1}^3) \text{ and } \nu_t \equiv \frac{b(1 - q_{t+1}^2)n_t}{(1 - q_t^2)} .$$

Normalisation.

Let $l_t \equiv L_t / N_t^2$ and $s_t \equiv S_t / N_t^2$.
Then, (1) and (4) can be written as

$$c_{t+1}^3 = \frac{\rho n_t}{(1 - q_{t+1}^3)(1 - q_t^2)} \cdot F\left[(1 - q_{t+1}^2)\lambda_{t+1}(e_t) + n_{t+1}\gamma, \frac{\sigma_{t+1} s_t}{n_t}\right] \tag{8}$$

and

$$[(1 - q_t^2) + \beta n_t] c_t^2 + s_t + \rho F\left[(1 - q_t^2)\lambda_t + n_t \gamma, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right] = F\left(l_t, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right). \tag{9}$$

The stock of physical capital available to each surviving young adult in period t is

$$k_t \equiv \frac{\sigma_t s_{t-1}}{(1 - q_{t-1}^2) n_{t-1}}.$$

Normalised output is

$$y_t \equiv F\left((1 - q_t^2 - w n_t e_t)\lambda_t + n_t \gamma (1 - e_t), \frac{\sigma_t s_{t-1}}{n_{t-1}}\right).$$

Analogously, normalised full income is

$$\bar{y}_t \equiv F\left(\bar{l}_t, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right).$$

$\bar{l}_t \equiv \bar{L}_t / N_t^2$ denotes the normalised endowment of labour at t.

Stationary environment: $n_t, \mathbf{q}_t$ and z_t are constant.
Labour-augmenting technical progress only.
Output per head can increase only if λ_t increases.
Questions.
For any given stationary environment:

1. Is $e_t^0 = 0 \ \forall t$, and hence $\lambda_t = 1 \ \forall t$ (*'backwardness'*), an equilibrium?
2. If so, is it locally stable?
3. Do there exist equilibria in which $e_t^0 > 0$ for all t?
4. Are both backwardness and steady-state growth equilibria?

Q3 and Q4: Focus on fully educated ($e_t = 1$) generations.
Unbounded growth of output per head is possible only if λ_t can grow without bound.
Recalling (5), we then require $zh(1) > 1$.

Conditions for a poverty trap.

$\lambda_t = 1 \; \forall t$ is a steady state iff, $\forall t$,
young adults in t, expecting $e_{t+1} = 0$, choose $e_t = 0$.
Rewrite V_t as a function of the decision variables:

$$V_t = u(c_t^2) + \chi_t u\left(\frac{\rho n_t \bar{y}_{t+1}}{(1 - q_{t+1}^3)(1 - q_t^2)}\right) + \nu_t v(z_t h(e_t)\lambda_t + 1). \tag{10}$$

The associated Lagrangian is

$$\Phi_t = V_t + \mu_t[y_t - [(1 - q_t^2) + \beta n_t]c_t^2 - s_t - \rho \bar{y}_t]. \tag{11}$$

Along a steady-state path with $e_t^0 = 0$,

$$\bar{y}_t = y_t(e_t^0 = 0) = F\left[(1 - q_t^2) + n_t\gamma, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right] \; \forall t,$$

since $\lambda_t = zh(0)\lambda_{t-1} + 1 = 1 \; \forall t$.

When $e_t = 0$, (9) specialises to

$$[(1-q^2)+\beta n]c^2 + s = (1-\rho)F[(1-q^2)+\gamma n, \sigma s/n], \tag{12}$$

and (8) to

$$c^3 = \frac{n\rho}{(1-q^2)(1-q^3)} \cdot F[(1-q^2)+\gamma n, \sigma s/n]. \tag{13}$$

Substituting for c^2 and c^3 from (12) and (13) in the f.o.c. w.r.t. c_t and s_t, we obtain an equation in s.
Denote by $s^b = s^b(n, q^2, q^3, \sigma)$ the smallest positive solution.

The final step is to examine the f.o.c. w.r.t. e_t, with $e_t = 0\ \forall t$. Substituting for μ_t, using the other f.o.c. and rearranging terms, we obtain

$$\left((\gamma + w) - \frac{1-q^2}{\sigma} \cdot \frac{zh'(0)}{F_2(\bar{l}, \frac{\sigma s}{n})} \right) u'(c^2) F_1(\bar{l}_t, \frac{\sigma s}{n}) \\ \geq \left[(1-q^2) + \beta n \right] bv'(1) zh'(0), \tag{14}$$

where F and its derivatives are evaluated at the arguments $((1-q^2) + \gamma n, \sigma s^b / n)$.

(14) holds as a strict inequality at $e_t^0 = 0$ only if

$$\sigma F_2\left[(1-q^2)+\gamma n, \sigma s^b/n\right] > \frac{1-q^2}{\gamma+w} \cdot zh'(0). \qquad (15)$$

$(\gamma + w)$ is the sum of the opportunity and direct costs of educating a child when $\lambda = 1$.
At $e_t = 0$, a marginal investment yields $zh'(0)$ units of HK.
The fraction $1 - q^2$ of all children will survive early adulthood.

Remark. If h is concave, $zh'(0) \geq zh(1)$, with equality only if h is proportional to e_t (by assumption, $h(0) = 0$): that is, $h'(0) \geq 1$. Since v is strictly concave, however, h may be weakly convex without violating the requirement that V_t be concave over the whole feasible set.

σF_2 is the opportunity cost of investing a little in education, considering only old-age provision.

For any input bundle $((1-q^2)+\gamma n, \sigma s^b/n)$, F_2 will be large if TFP is large.

Such a property is quite separate from $zh(1) > 1$.

Hence, (15) can be satisfied if F is sufficiently efficient and both h' and $|h''|$ are sufficiently small.

By inspection, if (15) holds, then it will do likewise for all values of λ_t sufficiently close to 1.

Result.
In the absence of altruism ($b = 0$), condition (15) is also sufficient to ensure the existence of a locally stable state of backwardness.

It does not, however, rule out $zh(1) > 1$, and hence the possible existence of a steady-state path along which output per head grows without limit.

If condition (15) holds strongly, then by continuity, the same conclusions will also hold if the altruism motive is weak, since the latter implies that the r.h.s of (14) will be small.

If, however, altruism is strong, such a low-level equilibrium may well not exist.

Conclusion.

Conditions (14) and $zh(1) > 1$ are compatible, especially if altruism is not too strong and the survival rates for investments in both forms of capital are similar.

If the former condition holds as a strict inequality, there will be a poverty trap.

If both hold, escape from the trap can be followed by an asymptotic approach to a steady-state growth path along which output per head increases without bound.

Functional conditions allowing growth as an alternative.

The following general conditions must be satisfied.

(i) $Z(e_t) \equiv v[zh(e_t)\lambda_t + 1]$ is concave $\forall e_t \in [0, 1]$.
This ensures that V_t is concave over the feasible set.

(ii) Condition (15) holds, so that 'backwardness' can be an equilibrium.

(iii) $zh(1) > 1$, to allow unbounded growth when $e_t^0 = 1 \ \forall t$.

We explore these conditions in detail and examine whether they can be met simultaneously.

Condition (i).

We now assume functional forms.

A1. Let $v(\lambda_{t+1})$ be iso-elastic:

$v \equiv (\lambda_{t+1} - 1)^{1-\eta}/(1-\eta), \quad \eta > 0.$

Then,

$$Z'' = v' z \lambda_t \left[-\eta \cdot \frac{h'^2}{h} + h'' \right] \equiv H(e) \cdot z \lambda_t v',$$

so that $sgn\ Z'' = sgn\ H(e_t)$.

A2. Let $h(e_t) = a_1 e_t + a_2 e_t^2/2 - a_3 e_t^3/3, \quad (a_1, a_2, a_3) \gg \mathbf{0}$.

Then $h'' \gtreqless 0$ according as $e_t \gtreqless a_2/2a_3$.

If, further, $h'(0) = a_1 > 0$ and $h'(1) = a_1 + a_2 - a_3 > 0$, then $h'(e_t) > 0 \quad \forall e_t \in [0, 1]$.

Under *A2*, we have

$$H(e_t) = -\eta \cdot \frac{(a_1 + a_2 e_t - a_3 e_t^2)^2}{(a_1 + a_2 e_t/2 - a_3 e_t^2/3) e_t} + (a_2 - 2a_3 e_t).$$

Since $a_1 > 0$, $\lim_{e_t \to 0} H(e_t) = -\infty$.

Hence, there exists a measurable set $S^h = \{\mathbf{a}, \eta : \mathbf{a} \gg \mathbf{0}, a_1 + a_2 > a_3, \eta > 0\}$ s.t. $H(e_t) < 0 \, \forall e_t \in [0, 1]$.

Condition (ii).
In condition (15), s^b is chosen at $e_t = 0$,
but the exact form of $h(e_t)$ has no effect on s^b provided that form is also compatible with a growth path along which $e_t^0 = 1\ \forall t$.
It follows that (15) will be satisfied if a_1 is sufficiently close to zero.

Condition (iii).
Are there members of S^h satisfying both (15) and $zh(1) > 1$?

$$z(a_1 + a_2/2 - a_3/3) > 1. \tag{16}$$

Using this inequality in the r.h.s. of (15), we have

$$\frac{1-q^2}{\gamma + w} \cdot z\, a_1 > \frac{1-q^2}{\gamma + w} \cdot \frac{a_1}{a_1 + a_2/2 - a_3/3}.$$

Choose **a** s.t. (16) just holds, i.e., the growth rate $g \equiv zh(1) - 1$ is barely positive, so that the r.h.s. of (15) is arbitrarily close to

$$\frac{1-q^2}{\gamma + w} \cdot \frac{a_1}{a_1 + a_2/2 - a_3/3}.$$

Consider

$$a_1 = 0.1,\ a_2 = 2,\ a_3 = 1;\ \frac{a_1}{a_1 + a_2/2 - a_3/3} = \frac{3}{23}.$$

The r.h.s. of (15) is barely larger than
$((1 - q^2)/(\gamma + w))\frac{3}{23}$,
which is surely smaller than the l.h.s. of (15) if $F(\cdot)$ is fairly productive and the destruction rate $1 - \sigma$ is sufficiently low.

Conclusion. If v is iso-elastic, F is sufficiently productive and $1 - \sigma$ is sufficiently small, then there exists a measurable subset of the family satisfying *A2* s.t. conditions (i), (ii) and (iii) are satisfied.

Does there exist a path with $e_t^0 = 1 \; \forall t$? If so,
λ_t, and hence (c_t^2, c_t^2, s_t), grow at the rate $g = zh(1) - 1 > 0$.

The pairwise MRTs are obtained from the budget constraint (9).

$F_1\left[l_t, \frac{\sigma s_{t-1}}{n}\right]$ will be constant along the hypothesised path.

Total differentiation of (10) yields the MRS.

$F_1(\bar{l}_{t+1}, \frac{\sigma s_t}{n})$ and $F_2(\bar{l}_{t+1}, \frac{\sigma s_t}{n}))$ are constant along the path $e_t^0 = 1$.

The ratio c_t^3/c_t^2 will be constant, its value is denoted by κ.

A3. Let u be iso-elastic: $u = c^{1-\xi}/(1-\xi)$.
Then, along the hypothesised path,
$u'(c_t^2)/u'(c_{t+1}^3) = [\kappa(1+g)]^{\xi}$.
Since $c_t^{20} > 0$ and $s_t^0 > 0$, it follows from a comparison of MRT_{cs} and MRS_{cs} that

$$(1-q^2) + \beta n = \frac{(1-q^2)[\kappa(1+g)]^{\xi}}{\delta\rho\sigma F_2\left(\bar{l}_{t+1}, \frac{\sigma s_t}{n}\right)}. \tag{17}$$

Given that steady-state growth has been long established, $F_2\left(\bar{l}_{t+1}, \frac{\sigma s_t}{n}\right)$ depends only on the ratio
$(1-q^2)\lambda_{t+1}/(\sigma s_t/n) = \lambda_{t+1}/k_{t+1}$,
which is a constant along the path in question.

Result. Given $(n, q^2, q^3, \sigma; \beta, \delta, \rho)$ and the technologies zh and F, (17) yields the (unique) steady-state value of λ_t/k_t.

Further steps.

1. Compare MRT_{ce} and MRS_{ce} along the path $e_t^0 = 1$.

Differentiate (9) and the expression for MRS_{ce} totally, noting that

$$ds_t/s_t = dk_t/k_t = d\lambda_t/\lambda_t = dc_t^2/c_t^2. \tag{18}$$

2. Differentiate a term Q_t in MRS_{cs} totally, noting (18) and *A1*.
3. Steps 1 and 2 yield, after some manipulation,

$$\frac{dQ_t}{d\lambda_t} \cdot \frac{\lambda_t}{Q_t} = -\frac{\xi A + \eta B(1+g)^{-(\eta-\xi)}}{A + B(1+g)^{-(\eta-\xi)}}\,, \tag{19}$$

where A and B are positive constants.

4. The following expression arises:

$$M \equiv \frac{\left[(1-q^2-wn)F_1\left(l_t, \frac{\sigma s_{t-1}}{n}\right) - \rho F_1\left(\bar{l}_t, \frac{\sigma s_{t-1}}{n}\right)\right]\lambda_t - s_t}{[(1-q^2)+\beta n]c_t^2} \,. \quad (20)$$

This is a constant, since c_t, s_t and λ_t are growing at the rate g. Recalling (9) and that F is homogeneous of degree one, we have $M \gtrless 1$ according as

$$\rho F_2\left(\bar{l}_t, \frac{\sigma s_{t-1}}{n}\right) = \rho F_2((1-q^2)\lambda_t + \gamma n, (1-q^2)k_t) \gtrless F_2[(1-q^2-wn)\lambda_t, (1-q^2)k_t)].$$

In practice, $wn \leq 0.1$ and $\rho \leq 1/3$.
Hence, M is almost surely less than, but plausibly rather close to, 1. For

$$(1-q^2)\left(F_2\left(l_t, \frac{\sigma s_{t-1}}{n}\right) - \rho F_2\left(\bar{l}_t, \frac{\sigma s_{t-1}}{n}\right)\right) k_t/[(1-q^2)+\beta n]c_t$$

involves a *difference* in capital's share as the numerator, but the combined consumption of a young adult and children as denominator.

The final step. Does $e_t^0 = 1$, once attained, remain optimal?
$|MRT_{ce}|$ goes to zero at the rate g, so that

$$\lim_{\lambda_t \to \infty} \left[\frac{d(\log |MRT_{ce}|)}{d(\log \lambda_t)} \right] = -1.$$

To maintain the optimality of $e_t = 1$, however, $|MRS_{ce}| = R_t$ must fall at least as fast as $|MRT_{ce}|$ as λ_t grows.
From steps 1 and 2, we obtain

$$\frac{dR_t}{d\lambda_t} \frac{\lambda_t}{R_t} = -\xi M + \frac{\xi A + \eta B(1+g)^{-(\eta-\xi)}}{A + B(1+g)^{-(\eta-\xi)}} - 1.$$

It follows that the required condition is

$$\xi M \geq \frac{\xi A + \eta B(1+g)^{-(\eta-\xi)}}{A + B(1+g)^{-(\eta-\xi)}} . \qquad (21)$$

When $M < 1$, (21) holds only if $\xi M \geq \eta$,
though sufficiency is also ensured only when $(2M - 1)\xi \geq \eta$.
If M is close to 1, then ξ may be close to η.

Intuition. If there is only human capital, the required condition is $\xi \geq \eta$, with equality as the limiting case.
Old-age provision though saving renders educating the children less pressing in this regard, so that altruism has to work that much harder to maintain $e_t^0 = 1$; for
λ_{t+1} is an argument of $v(\cdot)$, but $k_{t+1}\,(= \sigma s_t / n(1 - q^2))$ is not.

Result. v must be less strongly concave than u if steady-state growth with a fully educated population is to be possible.

Suppose parents are perfectly selfish ($\eta = 0$).
Then condition (21) specialises to

$$M \geq \frac{A}{A + B(1+g)^{\xi}},$$

which is more easily satisfied, the r.h.s. being clearly less than one. This rather paradoxical result stems from the assumption that altruism is expressed only through investment in education, parents making transfers of the aggregate good neither *inter vivos* nor as bequests.

Concluding points.

- Unremitting warfare and communicable diseases in the absence of public health measures will surely suffice to bring about a Hobbesian existence, even when productive technologies are available.
- Yet there are stationary constellations of war losses and premature adult mortality such that both backwardness and steady growth with a fully educated population are possible equilibria. The associated poverty trap is thereby precisely characterised.
- Parents' altruism makes backwardness less likely; but, rather paradoxically, it can also be an obstacle to attaining steady-state growth.
- It remains to analyse environments in which cohort mortality rates and war losses are stochastic.