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Risks and Parental Investment in the Human Capital of Children

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Abstract

This paper develops a theoretical model to analyze the effects of parental income risk on the human capital investment of its child, when the human capital investment is risky. It finds that the effects of these two risks on the human capital investment depends on whether bequest constraint is binding. Bequests provide an instrument through which parents and children can share risks. If the bequest constraint is not binding, increasing parental income risk has a positive effect on the human capital investment, but increasing human capital investment risk has a negative effect on the human capital investment. However, if the bequest constraint is binding, the effects are reversed. Quantitative analysis shows that the parental income risk has a larger negative effect on the human capital investment of high ability children. On the other hand, the human capital investment risk has a larger negative effect on the human capital investment of low ability children.

Keywords: human capital investment, parental endowment income risk, human capital investment risk, bequest, borrowing constraint

JEL Code: J22, I20, D60

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1 Introduction

Parents finance a significant part of educational cost of their children both in the developing and the developed countries. Empirical evidence shows that the parental financial support has a significant effect on the educational attainment of their children (e.g. Orazem and King 2006, Brown, Scholz and Seshadri 2012). Evidence also suggests that a significant proportion of parents make transfers to their adult children, after they finish their schooling (e.g. Gale and Scholz 1994, Altonji, Hayashi and Kotlikoff 1997, Albertini et. al. 2007, Cox and Fafchamps 2008, Brown, Scholz and Seshadri 2012).

While making human capital investment decisions, parents face various kinds of risks. By very nature human capital investment is risky. For example, parents may have imperfect information about the ability of their children and the future labor market conditions. However, at the time of making decisions, the future income of the parents may also be risky. While a literature has emerged examining the effects of the human capital investment risk on the human capital investment, very little is known both theoretically and empirically about the effects of the parental future income risks on the investment of human capital of their children.[1]

The main aim of the paper is to study the effects of the parental future income risk on the parental investment in the human capital of their children and its interaction with the human capital investment risk. It develops a model to incorporate the parental future income risk and the human capital investment risk using the unitary household framework (Becker 1991).

The distinction between the parental future income risk and the human capital investment risk faced by children is important for number of reasons. Firstly, these two risks may have different effects on the parental human capital investment decisions. The human capital is an asset. Risky human capital investment is likely to affect the parental human capital investment decisions both through the (positive) precautionary motive and the (negative) substitution effect, similar to the effects of capital income risk on saving (Sandmo 1970, Eeckhoudt and Schlesinger 2008). However, the parental future income risk is likely to affect the parental decisions only through the precautionary motive. Secondly, occupations, abilities, and locations of parents and children may differ and thus may have different risk profiles.

[1] Hartog and Diaz-Serrano (2013, 2015) provide a thorough review of empirical evidence on the effects of risks and attitude towards risk on schooling decisions.

In the model, there are two periods. A family consists of a parent and a child. The parent is altruistic and its utility depends not only on its own consumption, but also on the utility enjoyed by its child. The parent chooses its own consumption, saving, and the human capital investment and the amount of bequest for the child. A higher level of human capital investment in the first period leads to higher earnings for the child next period. While making these decisions, the parent faces different kinds of uninsurable idiosyncratic risks. In particular, I assume that the future (second period) parental endowment income and the productivity of human capital investment are random.

There are three key aspects of the model: (i) The bequest plays a dual role in the model. First, it reduces the consumption inequality between the parent and the child, a role explored in the human capital models without risk (e.g. Becker 1991, Brown, Scholz and Seshadri 2012, Kumar 2013). Second, it allows the parent and the child to share and diversify their risks. As shown below, the diversification opportunity significantly alters the effects of risks on the human capital investment. (ii) As discussed above, the parental endowment income risk affects the parental decisions through the precautionary motive. But, the human capital investment risk affects the parental decision through both the precautionary motive and the substitution effect. (iii) The parent faces a version of the portfolio allocation problem. It can increase its future utility both by increasing the human capital investment of child and saving.

The paper has two parts. In the first part, I examine the effects of small risks on the human capital investment.[2] I analytically derive following main results. Firstly, the effects of risk on the human capital investment depends on the type of risks and whether the bequest and the borrowing constraints bind. When the parent is *unconstrained*, an increase in the parental endowment income risk has a positive effect on the human capital investment. However, an increase in the human capital investment risk has a negative effect on the human capital investment.

When the parent faces binding bequest constraint, the link between the future incomes of the parent and the (adult) child is broken. In this case, an increase in the parental endowment income risk reduces the human capital investment. However, an increase in the human capital investment risk increases the human capital investment, if the precautionary motive (the

[2] In particular, I assume that the parent either gives bequest for all realizations of random variables or it does not give bequest for any realization of random variables.

relative-risk prudence) is relatively large.

When the parent faces binding borrowing constraint, a rise in the parental endowment income risk increases human capital investment. An increase in the human capital investment risk also increases human capital investment if the precautionary motive (the relative-risk prudence) is relatively large.

In the second part, I numerically examine the effects of large risks on the human capital investment.[3] For the numerical analysis, I choose the values of parameters to match salient features of educational expenditure and intergenerational transfers in the United States. The human capital investment risk is proxied by the variance of (log) wages in the United States.

Results show that the effects of risks depend crucially on the ability of child. The parental income risk has a larger negative effect on the human capital investment of high ability child. On the other hand, the human capital investment risk has a larger negative effect on the human capital investment of low ability child. In addition, for the parent facing binding borrowing constraint, the parental income risk has a positive effect on the human capital investment of low ability child.

Quantitative analysis also shows that providing income subsidy to parents which is financed by future (lump-sum) taxes on parents has little effect on the human capital investment, except for the parents facing binding borrowing constraint. However, if such income subsidy is financed by future (lump-sum) taxes on adult children, it has a large positive effect on the human capital investment.

This paper directly relates to two strands of theoretical literature on human capital investment and risks. Firstly, it relates to the theoretical literature which analyzes the effects of human capital investment risk on the human capital investment (e.g. Levhari and Weiss 1974, Williams 1979, Kodde 1986, Snow and Warren 1990, Gould, Moav, and Weinberg 2001, Hogan and Walker 2007). These models analyze the effects of the human capital investment risk(s) in which an individual undertakes human capital investment to increase its future income.[4]

By design these models do not distinguish between the parental income risk and the human capital investment risk. These models do not examine

[3] The parent can give bequest for some realizations of random variables and not in others.

[4] The empirical literature also has largely focussed on studying the effects of an individual's attitude towards risk and/or labor market risks faced by it on its own schooling decisions (Hartog and Diaz-Serrano 2013, 2015).

the interaction among risks, the bequest and the borrowing constraint. This is despite the fact that both the theoretical models of the human capital investment without risks (e.g. Becker 1991, Brown, Scholz and Seshadri 2012, Kumar 2013) and the empirical literature (e.g. Orazem and King 2007) suggest that the bequest transfers and financial development have significant effect on the human capital investment.

Secondly, it relates to Loury (1981) who develops an OLG model to examine the role of parental investment in education of their children in the evolution of inter-generational earnings distribution. In his model, parents face uncertainty about ability of children (human capital risk) and they are assumed neither to save or to give bequest. Aiyagri et. al. (2002) in a related model study the issue of efficiency of parental investment in education of their children when there is uncertainty about the ability of (grand) children.[5]

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 analytically analyzes the effects of parental endowment and human capital investment (small) risks on the human capital investment and saving. In section 4, I quantitatively study the effects of (large) risks on the human capital investment and saving and discuss the policy implications. Section 5 concludes the paper. All proofs are contained in the appendix.

2 Model

There are two periods, $t = 1, 2$. Consider a household consisting of a parent and a child. Parent and child live in both periods. The parent is altruistic and its utility depends not only on its own consumption but also on the utility of the child. The parental expected utility is given by

$$E[U(c_1^p) + U(c_2^p) + \delta U(c)] \tag{2.1}$$

where function $U()$ is the period utility function and E is the expectation operator. $U()$ is a strictly increasing and concave function of consumption,

[5] There is emerging literature which has examined the effects of borrowing constraint and public policies (e.g. tuition subsidy, public expenditure on education, taxes) on the parental investment in education of their children in quantitative-theoretic framework in risky environment (e.g. Cancutt and Kumar 2003, Restuccia and Urrutia 2004, Cancutt and Lochner 2012, Krebs et. al. 2015). However, these studies do not analyze the effects of risk *per-se*, which is focus of this paper.

with $U_c() > 0$ and $U_{cc}() < 0$. I also assume that the third and the fourth derivatives of the period utility function exist and $U_{ccc}() > 0$ & $U_{cccc}() < 0$.[6] c_t^p is the consumption of the parent in period $t = 1, 2$ and c is consumption of child in period 2. Parameter $0 < \delta < 1$ measures the degree of parental altruism. For analytical simplicity, I set the discount rate to one.

Denote the parental endowment income in period $t = 1, 2$ by y_t. The parent chooses human capital investment, s, and bequest, $b \geq 0$, for its child and its own consumption and saving, $k \geq 0$.[7] The human capital investment is made in the first period and the bequest is given in the second period.[8]

The human capital investment by the parent in the first period increases the human capital or the earnings of the child next period. The earnings (human capital) function of the child is given by, $\phi h(s)$, where ϕ is the productivity parameter.[9] Suppose that $h(0) > 0$, $h_s(s) > 0$ & $h_{ss}(s) < 0$.

While making its decisions, the parent faces different kinds of uninsurable idiosyncratic risks. In particular, suppose that the second period parental endowment income, y_2, and the human capital productivity parameter, ϕ, are random variables with strictly positive and finite support. Let $\overline{y}_2$ and $\overline{\phi}$ denote means and $\sigma_{y_2}^2$ and σ_ϕ^2 variances of the second period parental endowment income and the human capital productivity parameter respectively. Let $\sigma_{y_2,\phi}$ denote the co-variance between y_2 and ϕ. Assume that the rate of interest on saving, R, is risk-free.

In terms of timing, the first period decisions (c_1^p, s, k) are made before the values of random variables are known. The second period decisions (c_2^p, b) are made after the values of random variables are known. Given the assumptions, the budget constraints faced by the parent and the child can be written as

$$c_1^p + k + s = y_1; \tag{2.2}$$

[6]Throughout the paper, for any function $F(x, z)$, $F_x(x, z)$, $F_{xx}(x, z)$, $F_{xxx}(x, z)$, $F_{xxxx}(x, z)$ and $F_{xz}(x, z)$ denote the first, the second, the third, the fourth and the cross partial derivative respectively.

[7]$b \geq 0$ implies that the parent cannot leave debt to (adult) children or enter into binding contract with (minor) children in the first period. This is consistent with the idea that such an arrangement cannot be legally enforced. In many developing countries, social norms may be such that parents can leave debt to their children. One can allow for limited amount of negative bequest ($b \geq -\underline{b}$). Also one can allow for limited amount of borrowing by the parent.

[8]One can also assume that the human capital function depends on the time devoted by the parent. In that case, s can be interpreted as the forgone earnings by the parent.

[9]ϕ can also be interpreted as wage per-effective labor.

$$c_2^p + b = y_2 + Rk \ \& \tag{2.3}$$

$$c = b + \phi h(s). \tag{2.4}$$

2.1 Optimal Strategies

The parental optimization problem is to

$$\max_{s,b,k} E \sum_{t=1}^{2} U(c_t^p) + \delta U(c)$$

subject to the budget constraints (2.2)-(2.4).

Since the focus of the paper is on the effects of risks on the human capital investment, throughout the analysis I assume that the parameter values are such that there is an interior solution for the human capital investment, $s > 0$. Consumptions of the parent and the child are given by the budget constraints (2.2)-(2.4). The first order conditions for the optimal choices are as follows:

$$s: \ U_c(c_1^p) = \delta E U_c(c) \phi h_s(s); \tag{2.5}$$

$$b: \ U_c(c_2^p) = \delta U_c(c), \text{ if } b > 0; \tag{2.6}$$

$$b: \ U_c(c_2^p) \geq \delta U_c(c), \text{ if } b = 0; \tag{2.7}$$

$$k: \ U_c(c_1^p) = E U_c(c_2^p) R, \text{ if } k > 0 \ \& \tag{2.8}$$

$$k: \ U_c(c_1^p) \geq E U_c(c_2^p) R, \text{ if } k = 0. \tag{2.9}$$

(2.5) equates the marginal cost of human capital investment to its expected marginal benefit. An additional unit of human capital investment reduces the utility of parents by $U_c(c_1^p)$ in the first period. On the other hand, an increase in the human capital investment increases the earnings of the child next period by $\phi h_s(s)$, which enhances the utility of parents by $\delta U_c(c) \phi h_s(s)$.

(2.6) equates the marginal cost of bequest to its marginal benefit. An additional unit of bequest reduces the utility of parents by $U_c(c_2^p)$ in the second period. At the same time, it increases the utility of parents by $\delta U_c(c)$ in the second period. If the marginal cost of bequest exceeds the marginal benefit, then the parent will not give any bequest to the child. (2.7) characterizes this condition. This case can arise when the parental income in the second period is low or it puts small weight on the utility of its child, or the income of child is relatively high in the second period. Note that since bequest is given after the realization of the values of random variables, the parent does not face any risk while giving bequest.

(2.8) equates the marginal cost of saving with its expected marginal benefit. The marginal cost of saving is the loss in the utility by having to consume one unit less in the first period. One unit of saving increases income by R next period, the expected value of which is $EU_c(c_2^p)R$. If the marginal cost of saving exceeds its expected marginal benefit, then the parent will not save. (2.9) characterizes this condition. This can occur if the first period income is low relative to the expected future income.

From (2.3), (2.4), and (2.6), it follows that

$$\frac{db}{dy_2} \;\&\; \frac{db}{dR} > 0 \;\&\frac{db}{d\phi} < 0. \tag{2.10}$$

A higher second period parental endowment income and the rate of return on saving increases and a higher productivity of the human capital investment reduces bequest from the parent for a given level of human capital investment. The reason is that a higher endowment income and return on saving reduces the marginal cost of bequest, while a higher productivity of the human capital investment reduces the marginal benefit of bequest.

These predicted relationship among transfers between parents and children and incomes of parents and children are supported by empirical evidence. Risk-sharing is a significant explanatory factor explaining inter-vivo transfers and remittances (see Laferrere and Wolf 2006 and Cox and Fafchamps 2008 for a thorough review of literature on inter-vivo transfers and Rapoport and Docquier 2008 on migration and remittances).

In the model, bequest provides a direct link between the second period consumption of the parent and the child. It plays a dual role: (i) it reduces the consumption inequality between the parent and the child, a role explored in the human capital models without risks (e.g. Becker 1991, Kumar 2013) and (ii) and it allows the parent and the child to share risks.

2.2 Certainty Case

As a benchmark, I first analyze the optimal level of human capital investment when there is no uncertainty. Assume that in the certainty case, $y_2 = \overline{y}_2$ and $\phi = \overline{\phi}$. Using the first order conditions, it is straightforward to show that when b & $k > 0$, the optimal level of human capital investment is given by (Becker 1991, Kumar 2013)

$$\overline{\phi} h_s(s) = R. \tag{2.11}$$

The parent equates the rate of return from the human capital investment to the rate of return on saving. (2.11) also characterizes the efficient level of human capital investment in the sense that a social planner, who can smooth the idiosyncratic risks across large population through lump-sum transfers, will choose the same level of human capital investment.

From the first order conditions, it is easy to show that either when $b = 0$ or $k = 0$,

$$\overline{\phi} h_s(s) \geq R. \tag{2.12}$$

When the parental income is too low to provide bequest to the child and/or it faces binding borrowing constraint, the human capital investment is inefficiently low.

3 Effects of Risks

To analyze the effects of risks analytically, I distinguish among three cases: (i) k & $b > 0$ for all the realizations of the random variables; (ii) $k > 0$ & $b = 0$ for all the realizations of the random variables and (iii) $k = 0$ & $b \geq 0$.

These three cases are important for both the empirical and the theoretical reasons. From the theoretical point of view, most of the existing models of human capital investment with risks either assume that $k > 0$ & $b > 0$ (e.g. Levhari and Weiss 1974, Kodde 1986, Snow and Warren 1990)[10] or $k = 0$ & $b = 0$ (Loury 1981), or $k > 0$ & $b = 0$ (e.g. Aiyagari et.al. 2002). As discussed earlier, the models of human capital investment with

[10]These models analyze the effects of the human capital risk in which an individual undertakes human capital investment to increase its future income. As discussed in footnote 13, these models are a special case of my model.

certainty suggest that whether bequest or borrowing constraint binds or not has important implication with regard to the human capital investment.

Empirical evidence on private inter-vivo transfers across different countries show that while these transfers are wide-spread among households, it is by no means universal. Albertini et. al. (2007) using the Survey of Health and Ageing and Retirement in Europe (SHARE) 2004, find that 21% of parents provided financial support to their adult children. The incidence of transfer from parent to children varied greatly from country to country (32% in Sweden to just 9% in Spain). Similarly, evidence from the U.S. suggests that 30% of parents provided support to their adult children (Gale and Scholz 1994). Brown, Scholz and Seshadri (2012) using the Health and Retirement study of the U.S. find that about 50% of parents provide financial support to their children after graduation.

There is a large empirical literature, which has examined the effects of the borrowing constraint on the human capital investment both in the context of developing (Orazem and King 2007) and developed countries (e.g. Keane and Wolpin 2001, Kane 2006, Belly and Lochner 2007, Brown, Scholz and Seshadri 2012, Lochner and Cancutt 2012). Overall the evidence suggests that the borrowing constraint can be one of the most important factors inhibiting the human capital investment particularly in the developing countries.

Case I: $k > 0$ & $b > 0$ **for all the realizations of the random variables**

I first consider the case in which the parent saves and expects to provide bequest for all the realizations of the random variables. Using (2.5), (2.6), and (2.8), it is straightforward to show that the human capital investment is characterized by

$$EU_c(c_2^p)[\phi h_s(s) - R] = 0. \tag{3.1}$$

Using the co-variance decomposition, (3.1) can be written as

$$[\overline{\phi} h_s(s) - R]EU_c(c_2^p) = -h_s(s)Cov(U_c(c_2^p),\ \phi). \tag{3.2}$$

Proposition 1: Suppose that $b > 0$ in all states:

(I) The human capital investment is inefficiently low

$$\overline{\phi} h_s(s) > R \tag{3.3}$$

when the human capital investment is risky and the $Cov(U_c(c_2^p),\ \phi) < 0$.

(II) The human capital investment is at the efficient level, $\overline{\phi} h_s(s) = R$, if the parent only faces endowment income risk ($\sigma_{y_2}^2 > 0$ & $\sigma_\phi^2 = 0$).

(3.2) shows that the level of human capital investment depends on the sign and the size of the covariance of the parental marginal utility of consumption in the second period with the rate of return on human capital investment. The parent can increase its second period utility either by increasing human capital investment or saving. Since, their marginal costs are same, if the parent chooses strictly positive values of both the human capital investment and saving, it must be the case that their expected marginal benefits are same.

The negative correlation between the second period parental marginal utility of consumption and ϕ reduces the expected marginal benefit of human capital investment relative to saving and makes saving more attractive for the parent. The reason is that the rate of return on the human capital is high when the parental marginal utility of consumption is low. This induces the parent to choose an inefficiently low of the human capital investment.

On the other hand, if they are positively correlated, the human capital investment becomes more attractive relative to saving. In this case, the rate of return on the human capital is high when the parental marginal utility of consumption is high. This incentivizes the parent to choose an inefficiently high level of human capital investment. The additional human capital investment can be interpreted as the risk-premium the parent pays to reduce the variability of its own consumption and that of the child.

To further develop the intuition of these results, assume that the period utility function is homothetic, so that the parental consumption and consumption of child are constant proportion of the total family income in the second period.

Assumption 1: When $b > 0$,

$$c = (1 - M)(y_2 + Rk + \phi h(s)) \ \&\ c_2^p = M((y_2 + Rk + \phi h(s)) \tag{3.4}$$

where $M \in [0, 1]$[11]

[11] For example, if $U(c) = \frac{c^{1-\alpha}}{1-\alpha}$, then $M = \frac{\delta^{-1/\alpha}}{1+\delta^{-1/\alpha}}$, for $U(c) = -\exp^{-\alpha c}$, $M = \frac{\delta}{1+\delta}$ and for $U(c) = c^\alpha$, $M = \frac{\delta^{\frac{1}{\alpha-1}}}{1+\delta^{\frac{1}{\alpha-1}}}$.

Then taking the second order Taylor approximation around $(\overline{y}_2,\ \overline{\phi})$ and normalizing $\overline{\phi} = 1$, I have

$$Cov(U_c(c_2^p),\ \phi) \approx MU_{cc}(c_2^p)[h(s)\sigma_\phi^2 + \sigma_{y2,\phi}]. \tag{3.5}$$

From (3.5), it is clear that the sign and the size of $Cov(U_c(c_2^p),\ \phi)$ in part depends on the sign and the size of the covariance between y_2 and ϕ, $\sigma_{y2,\phi}$. In particular, if $\sigma_{y2,\phi} > 0$, the negative effect of the shocks on ϕ is reinforced by the shocks on y_2. In this case, the risky parental endowment income further reduces the human capital investment.

(3.5) also shows that if the parent faces only the human capital investment risk or it is independent of the parental endowment income risk, $\sigma_{y2,\phi} = 0$, the human capital investment will be inefficiently low. The reason is that when $\sigma_{y2,\phi} \geq 0$, a higher human capital investment increases the variability of the consumption of both the parent and the child.

On the other hand, if $\sigma_{y2,\phi} < 0$, the negative effect of the shocks on ϕ is counteracted (at least partly) by the shocks on y_2. In this case, it is possible that a higher human capital investment reduces the variability of the consumption of both the parent and the child. The risky parental endowment income may encourage the human capital investment. For example, if y_2 and ϕ are negatively correlated, it is possible that $Cov(U_c(c_2^p),\ \phi) > 0$. In this case, the parent will undertake inefficiently high level of human capital investment.

For example, suppose that y_2 and ϕ have the same variance and are perfectly negatively correlated, $\sigma_\phi^2 = \sigma_{y2}^2 = -\sigma_{y2,\phi}$. Then, it is straightforward to show that the human capital investment will be inefficiently high (low) if $h(s) < (>) 1$.[12]

The result that the human capital investment risk can reduce the parental human capital investment relates to Levhari and Weiss (1974), who analyze the effect of human capital investment risk on an individual's human capital investment in the one risk-framework.[13] Our analysis shows that the

[12]This suggests that in the case of negative co-variance, a parent of a child with low productivity earnings function (e.g. low-ability child) or a parent with relatively high earnings may undertake inefficiently high level of human capital investment. For example, suppose that $y_2 = \gamma\hat{y}_2$, where $\hat{y}_2$ is the human capital of the parent and γ is the random productivity parameter of the parental human capital. Suppose that $\sigma_\phi^2 = \sigma_\gamma^2 = -\sigma_{\phi,\gamma}$. Then the human capital investment will inefficiently high if $\tilde{y}_2 > h(s)$.

[13]Their model is a special case of my model with $M = 1$ and no parental endowment income risk, $\sigma_{y_2} = 0$.

negative effect of the human capital investment risk depends crucially on its co-variability with the parental endowment income risk.

From (3.5), it is also clear that when the parent faces only the endowment income risks, the human capital investment will be at the efficient level and also be independent of the uncertainty about the parental endowment income. The parental endowment income risk does not directly affect the human capital investment. The parent just adjusts its savings. Under the assumption that $U_{ccc}(c_2^p) > 0$, i.e. the parent is prudent, it is straightforward to show that it will save more than the certainty case due to precautionary saving motive.

Comparative Static

Now, I consider the effects of increasing the first period parental income, y_1, the human capital investment risk, σ_ϕ^2, and the parental endowment income risk, $\sigma_{y_2}^2$. To analyze their effects, I take the second order Taylor approximation of the (2.8) and (3.1) around $(\overline{y}_2,\ \overline{\phi})$ with $\overline{\phi} = 1$. Then,

$$V_s = EU_c(c_2^p)[\phi h_s(s) - R] \approx (h_s(s) - R)[U_c(c_2^p) + \frac{M^2}{2} U_{ccc}(c_2^p)\{\sigma_{y_2}^2 + 2h(s)\sigma_{y_2,\phi} + h^2(s)\sigma_{\phi^2}\}]$$

$$+ MU_{cc}(c_2^p)h_s(s)[\sigma_{y_2,\phi} + h(s)\sigma_{\phi^2}] = 0. \tag{3.6}$$

$$V_k = -U_c(c_1^p) + EU_c(c_2^p)R \approx$$

$$-U_c(c_1^p) + R[U_c(c_2^p) + \frac{M^2}{2} U_{ccc}(c_2^p)\{\sigma_{y_2}^2 + 2h(s)\sigma_{y_2,\phi} + h^2(s)\sigma_\phi^2\}] = 0. \tag{3.7}$$

Using the Cramer's rule, the effect of a change in an exogenous variable, z, on the human capital investment can be derived as follows

$$\frac{ds}{dz} = \frac{V_{sk}V_{kz} - V_{kk}V_{sz}}{D} \tag{3.8}$$

where $D \equiv V_{ss}V_{kk} - V_{sk}^2$. For the maximum to exist, it must be the case that $D > 0$.

From (3.6) and (3.7), it is clear the effect of a change in an exogenous variable will depend on the sign and size of the co-variance term $\sigma_{y_2,\phi}$ and how this change affects it. To develop the intuition, I analyze three cases: (i) $\sigma_{y_2,\phi} = 0$; (ii) $\sigma^2_{y_2} = \sigma^2_{\phi} = \sigma_{y_2,\phi}$; and (iii) $\sigma^2_{y_2} = \sigma^2_{\phi} = -\sigma_{y_2,\phi}$. In the first case, y_2 and ϕ are independent of each other. With this assumption, one can analyze the effect of increasing one risk independent of another risk.

In the second case, they are perfectly positively correlated. This can arise if both the parent and the child are expected to work in a similar occupation and region. In the third case, they are perfectly negatively correlated. This is expected to arise if both are expected to work in different occupations or regions. This case provides the maximum opportunity to share risks to the parent. Note that in cases II and III, an increase in the variance increases income risk for both the parent and the child symmetrically. One may think of these two cases as increasing risk at the joint household level or the market level rather than at the individual level. In the rest of the paper, in the cases II and III an increase in the variance is referred to as increasing joint household earnings risk.

Proposition 2: *The effect of increase in* y_1:

(I) Either when $\sigma_{y_2,\phi} = 0$ or $\sigma^2_{y_2} = \sigma^2_{\phi} = \sigma_{y_2,\phi}$, or there is only the human capital investment risk, $\sigma^2_{y_2} = 0$, then an increase in y_1 increases the human capital investment.

(II) In the case, $\sigma^2_{y_2} = \sigma^2_{\phi} = -\sigma_{y_2,\phi}$, then an increase in y_1 increases the human capital investment, only if $h(s) > 1$. If $h(s) < 1$ then an increase in y_1 reduces the human capital investment.

The effect of a higher y_1 on the human capital investment depends on whether the initial human capital investment is inefficiently low or high. If the initial human capital investment is inefficiently low, then a higher y_1 induces the parent to increase the human capital investment.

However, if the initial human capital investment is inefficiently high then a higher y_1 induces the parent to reduce the human capital investment. In this case, a higher y_1 induces the parent to increase its saving more and then compensate the child by increasing the bequest. Essentially, an increase in y_1 increases the wealth of the parent inducing them to bear more risks. Thus, it is willing to pay less risk-premium leading to reduction in the human capital investment.[14]

[14]If there is fixed cost in attending school (e.g. school fee), then any decrease in the fixed cost of schooling is akin to increase in y_1.

Proposition 3: *The effect of increasing risk of the parental endowment income,* $\sigma^2_{y_2}$*:*

(I) If there is only parental endowment income risk i.e. $\sigma^2_\phi = 0$, then an increase in $\sigma^2_{y_2}$ has no effect on the human capital investment.

(II) If both the parental endowment income and the human capital investment are risky, but are independently distributed, $\sigma_{y_2,\phi} = 0$, then an increase in $\sigma^2_{y_2}$ increases the human capital investment.

The first part follows from the fact that when only y_2 is risky, the human capital investment is at the efficient level. As the parental endowment income risk increases, the parent increases its saving due to the precautionary saving motive.

When there is also the human capital investment risk, the human capital investment is inefficiently low. In this case, as the parental endowment risk rises, the precautionary saving motive induces the parent to increase the human capital investment.

Now I consider the effect of an increase in the human capital productivity risk, σ^2_ϕ.

Assumption 2: Suppose that the *absolute risk-prudence* of the parent, $\rho(c_2^p) \equiv -\frac{U_{ccc}(c_2^p)}{U_{cc}(c_2^p)}$ is decreasing ($\rho_c(c_2^p) \leq 0$) in consumption.[15]

Proposition 4: *The effect of increasing risk of the human capital investment* σ^2_ϕ: Either if there is only the human capital investment risk or both the parental endowment income and the human capital investment are independently distributed, $\sigma_{y_2,\phi} = 0$, then an increase in σ^2_ϕ reduces the human capital investment.

The reason is that a rise in σ^2_ϕ increases the negative covariance between the parental marginal utility of consumption in the second period and the human capital investment risk making the human capital investment less attractive. As the human capital investment risk increases, the risk-prudent parent reduces the human capital investment in order to reduce its exposure to this risk.

Now I consider the cases when y_2 and ϕ are perfectly correlated and the joint household earnings risk increases.

[15]See Kimball (1989) for the appropriateness of this assumption.

Proposition 5:

(I) Suppose that the parental endowment income and the human capital investment risk are perfectly *positively* correlated, $\sigma_{y2}^2 = \sigma_\phi^2 = \sigma_{y2,\phi}$, then an increase in the joint household earnings risk, σ_ϕ^2, reduces the human capital investment.

(II) However, if the parental endowment income and the human capital investment risk are perfectly *negatively* correlated, $\sigma_{y2}^2 = \sigma_\phi^2 = -\sigma_{y2,\phi}$, then an increase in the joint household earnings risk, σ_ϕ^2, reduces the human capital investment only if $h(s) > 1$. In the case, $h(s) < 1$, an increase in σ_ϕ^2 increases the human capital investment.

Proposition 5 shows that whether greater joint household earnings risk reduces or increases the human capital investment depends on whether the initial human capital investment is inefficiently low or high. Only in case the initial human capital investment is inefficiently low, greater joint household earnings risk reduces the human capital investment. The reason is that in this case a rise in σ_ϕ^2 increases the negative covariance between the parental marginal utility of consumption in the second period and the human capital investment risk. However, if the initial human capital is inefficiently high, then an increase in σ_ϕ^2 increases the positive covariance between the parental marginal utility of consumption in the second period and the human capital investment risk, making human capital investment more attractive.

Case II: $k > 0$ & $b = 0$ **for all the realizations of the random variables**

From (2.5) and (2.8) it follows that the optimal human capital investment is given by

$$\delta EU_c(c)\overline{\phi}h_s(s) - EU_c(c_2^p)R = -\delta h_s(s)Cov(U_c(c),\ \phi). \tag{3.9}$$

(3.9) shows that unlike the previous case, the human capital investment depends on the co-variability of marginal utility of consumption of the child (as perceived by the parent in the second period) with the rate of return on human capital and not with its co-variability with marginal utility of consumption of the parent in the second period. Since the parent cannot give bequest to child in any state, the link between consumption of the parent and the child in the second period is broken, and the bequest cannot be used as a risk-sharing device.

Taking the second order Taylor approximation around $(\overline{y}_2,\ \overline{\phi})$ with $\overline{\phi} = 1$ (and $b = 0$ in case of certainty), I have

$$\delta h_s(s) Cov(U_c(c),\ \phi) \approx \delta h_s(s) U_{cc}(c) h(s) \sigma_\phi^2 < 0 \tag{3.10}$$

Proposition 6: When $b = 0$ for all the realizations of the random variables, the human capital investment is inefficiently low

$$\overline{\phi} h_s(s) > R \tag{3.11}$$

if the human capital investment is risky.

Intuitively, a negative $Cov(U_c(c),\ \phi)$ reduces the expected marginal benefit of human capital investment relative to saving and makes saving more attractive to the parent. Thus, the parent chooses inefficiently low level of human capital investment.

Note that as the second period consumptions of the parent and child are independent, the human capital investment does not depend on the covariance between the parental endowment risk and the human capital investment risk. This implies that unlike the previous case ($b > 0$), the human capital investment will be inefficiently low even if the human capital investment provides the opportunity to diversify the income risks. In addition, as $EU_c(c_2^p)R \geq \delta EU_c(c)\phi h_s(s)$, a corollary is that if the parent faces only endowment income risk or $Cov(U_c(c),\ \phi) = 0$, the human capital investment will still be inefficiently low.

The following proposition summarizes the effects of changes in y_1, $\sigma_{y_2}^2$, and σ_ϕ^2.

Proposition 7:

(I) An increase in y_1 increases the human capital investment.

(II) An increase in $\sigma_{y_2}^2$ reduces the human capital investment.

(III) An increase in σ_ϕ^2 increases (decreases) the human capital investment if the *relative risk-prudence* of the child

$$r\rho(c) \equiv -\frac{U_{ccc}(c)}{U_{cc}(c)} c > (<) 2. \tag{3.12}$$

As discussed earlier, an increase in y_1 reduces the marginal cost of human capital investment. Thus, the parent increases the human capital investment. An increase in $\sigma^2_{y_2}$ induces the parent to save more due to the precautionary motive. Since the parent cannot adjust its bequest, the parent also reduces the human capital investment. This result is in contrast to the previous case ($b > 0$), in which a higher $\sigma^2_{y_2}$ increases the human capital investment.

Finally, increasing riskiness in the human capital investment affects the parental decision in two ways: (i) the precautionary motive induces higher human capital investment and (ii) the investment itself becomes risky as one extra unit of investment does not increase earnings of the child for sure, which reduces the investment. This is akin to the negative substitution effect on saving identified in the precautionary saving literature, where riskiness of the rate of interest reduces saving (e.g. Sandmo 1970, Eeckhoudt and Schlesinger 2008). Thus, the increasing riskiness of human capital investment can have a positive or a negative effect on the human capital investment similar to the result on the effect of the capital income risk on the capital investment (e.g. Sandmo 1970, Eeckhoudt and Schlesinger 2008).

When the *relative risk-prudence* is high, the precautionary motive dominates the substitution effect. The result is that a higher σ^2_ϕ increases the human capital investment. When the relative risk-prudence is low, a higher σ^2_ϕ reduces the human capital investment.

The comparison of proposition 7 (II & III) with propositions 3 and 4 shows that when the bequest constraint is binding, the effects of increasing parental endowment income and the human capital investment risks on the human capital investment are reversed. These propositions highlight the crucial effects inter-vivo transfers/bequest can have on the human capital investment. The inter-vivo transfers/bequests not only affect the level of human capital investment (the issue emphasized in the models of human capital investment without risks), but also the response of parents to various risks.

Case III: $k = 0$

The main difference from the previous cases is that due to the binding borrowing constraint, a parent can adjust its inter-temporal utility only by choosing the human capital investment. I first consider the case when the bequest cannot be given for any realization of the random variables. The optimal human capital investment is given by (2.5). Also (2.5), (2.7), and

(2.9) imply that

$$\delta EU_c(c)\overline{\phi}h_s(s) - EU_c(c_2^p)R \geq 0. \tag{3.13}$$

Then using the co-variance decomposition it is straightforward to show that the human capital investment will be inefficiently low, $\overline{\phi}h_s(s) > R$, as in the certainty case.

Below I summarize the effects of changes in y_1 and risks.

Proposition 8: Suppose $b = 0$.

(I) An increase in y_1 increases the human capital investment.

(II) The parental endowment risk has no effect on the human capital investment.

(III) An increase in σ_ϕ^2 increases (decreases) the human capital investment if the (perceived) relative risk-prudence of the child

$$\equiv -\frac{U_{ccc}(c)}{U_{cc}(c)}c > (<)2. \tag{3.14}$$

The intuition for these results are as follows. As discussed earlier, a higher y_1 reduces the marginal cost of the human capital investment encouraging more human capital investment. Also since $b = 0$, the link between the second period consumption of the parent and the child is broken. Thus, the human capital investment becomes independent of the parental endowment risk.

Finally, as discussed earlier, the increasing riskiness in the human capital investment can have a positive or a negative effect on the human capital investment. When the relative risk-prudence of the child as perceived by the parent is high the precautionary motive dominates the substitution effect. The result is that a higher σ_ϕ^2 increases the human capital investment. When the relative risk-prudence is low, a higher σ_ϕ^2 reduces the human capital investment.

Now I consider the case that $b > 0$ for all the realizations of the random variables. In this case, the optimal human capital is characterized by

$$U_c(c_1^p) = EU_c(c_2^p)\phi h_s(s). \tag{3.15}$$

Taking the second order Taylor approximation of the RHS around $(\overline{y}_2,\ \overline{\phi})$ with $\overline{\phi} = 1$, I have

$$U_c(c_1^p) \approx h_s(s)[U_c(c_2^p) + \frac{M^2}{2}U_{ccc}(c_2^p)[\sigma_{y_2}^2 + 2h(s)\sigma_{y_2,\phi} + \sigma_\phi^2 h^2(s)]+$$

$$MU_{cc}(c_2^p)[\sigma_{y_2,\phi} + \sigma_\phi^2 h(s)]]. \tag{3.16}$$

Proposition 9: Suppose $b > 0$.

(I) An increase in y_1 increases the human capital investment.

(II) Either if there is only the parental endowment income risk or the parental endowment income risk and the human capital investment risk are independently distributed, $\sigma_{y_2,\phi} = 0$, then an increase in $\sigma_{y_2}^2$ increases the human capital investment.

(III) Either if there is only the human capital investment risk or the parental endowment income risk and the human capital investment risk are independently distributed, $\sigma_{y_2,\phi} = 0$, then an increase in σ_ϕ^2 increases (reduces) the human capital investment only if

$$-\frac{U_{ccc}(c_2^p)}{U_{cc}(c_2^p)}Mh(s) > (<)2. \tag{3.17}$$

(IV) If the parental endowment income risk and the human capital investment risk are perfectly positively correlated, $\sigma_{y_2}^2 = \sigma_\phi^2 = \sigma_{y_2,\phi}$, then an increase in the joint household earnings risk, σ_ϕ^2, increases (reduces) the human capital investment only if

$$-\frac{U_{ccc}(c_2^p)}{U_{cc}(c_2^p)}M(h(s)+1) > (<)2. \tag{3.18}$$

(V) If the parental endowment income risk and the human capital investment risk are perfectly negatively correlated, $\sigma_{y_2}^2 = \sigma_\phi^2 = -\sigma_{y_2,\phi}$, then an increase in the joint household earnings risk, σ_ϕ^2, increases (reduces) the human capital investment only if

$$-\frac{U_{ccc}(c_2^p)}{U_{cc}(c_2^p)}M(h(s)-1) > (<)2. \tag{3.19}$$

As before, a higher y_1 reduces the marginal cost of the human capital investment encouraging the parent to invest more. A higher $\sigma_{y_2}^2$ increases the human capital investment due to precautionary motive. Finally, (3.17)-(3.19) show the conditions under which the precautionary motive is stronger than the substitution effect, and an increase in the human capital investment risk and the joint household earnings risk can increase the human capital investment.

The comparison of the propositions 4 and 5 with the proposition 9 (III) and (IV) shows that the response of the parents to the human capital investment risk and household earnings risk can differ substantially depending on whether the parents are facing binding borrowing constraint (but with non-binding bequest constraints). As discussed earlier, parents with non-binding borrowing constraint reduce the human capital investment when human capital investment or household earnings risks increase (at least when the human capital investment is inefficiently low). However, parents facing the binding borrowing constraint may increase the human capital investment when the relative risk-prudence is high. However, proposition 3 and 9(II) show that their response to the parental endowment risks is qualitatively similar.

On the other hand, the comparison of the propositions 7 (II) and (III) with the proposition 9 (II) and (III) shows that the response of the parents to the human capital investment risk are qualitatively similar whether the parents are facing binding borrowing constraint or binding bequest constraint. However, their response to the parental endowment risk differs.

Tables 1-3 summarize the main results regrading the effects of changes in income and risks on the human capital investment. Results show that the effects of risks on the human capital investment and saving depend on the type of risks, their co-variance, the risk-attitude of parents, possibility of bequests, and whether the borrowing constraint binds or not.

4 Quantitative Analysis

In this section, I quantitatively analyze the effects of risks on human capital investment and the response of parents to policy changes. It also allows us to examine the effects of large risks. In particular, parents can give bequests for some of the realizations of the random variables and not in others. The model is extended to include public expenditure on education and taxes and transfers. Let the utility function be

$$U(c_1^p) + \beta E[U(c_2^p) + \delta U(c)]$$

where $\beta \in (0,1)$ is the discount factor. Let τ be the proportional wage tax rate, tr_1 and tr_2 be the lump-sum transfers received by a parent in periods 1 and 2 respectively, and tc_2 be the lump-sum transfers received by the child in period 2. Suppose that the human capital function depends on both the private human capital investment, s, and the public human capital investment, g. With these modifications, the budget constraints are given by

$$c_1^p + k + s = (1-\tau)y_1 + tr_1; \tag{4.1}$$

$$c_2^p + b = (1-\tau)y_2 + Rk + tr_2 \ \& \tag{4.2}$$

$$c = b + \phi(1-\tau)h(s,g) + tc_2. \tag{4.3}$$

For the quantitative analysis, I Assume that the period utility function is of the CRRA form

$$U(x) = \frac{x^{1-\alpha}}{1-\alpha}. \tag{4.4}$$

As in Becker (1991) and Restuccia and Urrutia (2004), the human capital investment function is specialized to

$$\phi h(s,g) = a \exp^{\zeta}(s+g)^{\mu}. \tag{4.5}$$

where a is the ability level of the child and $\exp^{\zeta}$ is the wage of (adult) child per-unit of human capital. Wages are assumed to be *log-normally distributed* with mean zero and variance σ_ζ^2. The parental endowment income in the second period is assumed to be

$$y_2 = \exp^{\lambda} \tilde{y}_2 \tag{4.6}$$

where $\exp^{\lambda}$ is the wage of the parent per-unit of its human capital. Assume that the parental wage is also *log-normally distributed* with mean zero and variance σ_λ^2.

Note that since the wage of the child is a convex function of ζ, a rise in σ_ζ^2 increases not only the variance of its wage, but also the average wage.[16] Thus, an increase in σ_ζ^2 will affect the parental human capital investment both by increasing the variance of wages and the expected rate of return on the human capital investment. Other things remaining the same, an increase in the average wage will affect the parental human capital investment through income and substitution effects. The relative strength of these two effects will depend on the degree of the relative risk-aversion.

Similarly, an increase in σ_λ^2 increases not only the variance of the parental wage, but also its mean and thus expected parental future endowment income. Other things remaining the same, a higher expected future endowment income will reduce saving and increase the incidence of bequest.

For the quantitative analysis, a subset of the parameter values are taken from Restuccia and Urrutia (2004) and Cancutt and Lochner (2012), who develop quantitative theoretic models to examine the role of liquidity constraint and parental human capital investment in explaining the inter-generational persistence of earnings in the United States. Rest of the parameter values are chosen to match some salient features of educational expenditure and inter-generational transfers in the United States.

I set the time period to be 25 years and assume that the discount factor $\beta = \frac{1}{R} = 0.50$. The implied annualized discount factor is 0.972. Following Restuccia and Urrutia (2004), I set the coefficient of the relative risk-aversion $\alpha = 1.5$ and the elasticity of human capital with respect to the human capital investment $\mu = 0.24$. Restuccia and Urrutia (2004) estimate the variance of the cross-section earnings in the U.S. to be 0.36. Accordingly, I set $\sigma_\zeta^2 = 0.36$. I also set $\sigma_\lambda^2 = 0.36$. In the benchmark model, I assume that ζ and λ are independently distributed, $\sigma_{\zeta,\lambda} = 0$. Cancutt and Lochner (2012) provide an estimate of the degree of parental altruism. Accordingly, I set $\delta = 0.67$. In the benchmark model, I set lump-sum transfers tr_1, tr_2, and tc_2 to be zero.

Rest of the parameters, y_1, $\tilde{y}_2$, a, g, and τ are chosen simultaneously such that: (i) The ratio of g to y_1 equals the ratio of public expenditure on education to the US GDP (0.039); (ii) The average earnings of the adult child

[16] This suggests that it is important to take into account the forms of probability distribution in interpreting the results. For example, in the human capital function both the ability level and wages enter multiplicatively. Suppose that the ability of child is random and is uniformly distributed as assumed in Cancutt and Kumar (2003). Then an increase in the variance of ability can have different effect than an increase in the variance of (log) wages both qualitatively as well as quantitatively.

equals the first period earnings of the parent; (iii) The child receives bequest for 30% of the realizations of random variables which is consistent with Gale and Scholz (1994), who estimate that 30% of parents provide transfers to their adult children in the United Sates; (iv) The wage tax receipt covers the public expenditure on education; (v) The parent saves and does not provide bequest in the deterministic case. The parameter values are reported in Table 4.

With these parameter values, in the benchmark model the parental human capital investment, $s = 0.1823$ (Table 5). By way of comparison, in the deterministic case with the same parameter values, the parental human capital investment is 0.2289. Adding risks reduces the parental human capital investment by 20.4% relative to the deterministic case.

Since heterogeneity in the academic ability of children has received considerable attention in the literature, I also simulate the model for two other ability levels of child which are 50% higher or lower than the base level ($a = 8.45$ medium ability child), $a = 12.675$ (high ability child) and $a = 4.225$ (low ability child). Differences in the ability level of child affect the parental human capital investment in three ways. Firstly, it affects the rate of return from the human capital investment, which can lead to a positive substitution effect and a negative income effect. If the substitution effect dominates the income effect, the parental human capital investment will rise with ability level. Secondly, it affects the incidence of bequest. Other things remaining the same, the high ability child is less likely to receive bequest than the low ability child. This will have negative effect on the human capital investment of high ability child. Finally, an increase in σ_ζ^2 will increase the mean and the variance of earnings of high ability child more than the low ability child.

Simulation shows that in the case of high ability child, the parental human capital investment, $s = 0.136$ (Table 5), which is 90.2% of the deterministic case. The parent gives bequest for only 5% of the realizations of the random variables. In the case of low ability child, the parental human capital investment, $s = 0.1081$, which is 84.2% of the deterministic case and the parent gives bequest for the 92% of the realizations of the random variables.

These simulations show that for the given set of parameter values, the parental human capital investment is lower for both the high ability and the low ability child relative to the medium ability child, suggesting a highly non-linear relationship between the two.[17]

[17]Despite lower human capital investment for the high ability child relative to the

4.1 Effects of Increasing Variability of the Parental Endowment Income and the Return from the Human Capital Investment

First, I increase σ_λ^2 by 50% over the base level to 0.54, keeping σ_ζ^2 at the base level (0.36). The results are reported in Table 5. As discussed earlier, a rise in σ_λ^2 not only increases the variance of the future parental endowment income, but also the expected future parental endowment income.

The results show that a higher σ_λ^2 reduces the human capital investment for all types of children. It has a larger negative effect on the human capital investment of children with higher ability with the implied elasticity being around -0.30 compared to the low ability child (-0.28).

Next, I increase σ_ζ^2 by 50% over the base level to 0.54, keeping σ_λ^2 at the base level (0.36). As discussed earlier, a higher σ_ζ^2 not only increases the variance of return from the human capital investment, but also the expected return.

The results show that a higher σ_ζ^2 reduces the human capital investment for all types of children (Table 5). Qualitatively, the effects are similar to that of an increase in σ_λ^2. However, there are important quantitative differences across different types of child. An increase in σ_ζ^2 has a smaller effect on the human capital investment in the case of high and medium ability children compared to an increase in σ_λ^2. However, in the case of low ability child its effect on the human capital investment is larger compared to an increase in σ_λ^2. Secondly, the negative effect of σ_ζ^2 on the human capital investment is decreasing in the ability level of child. Overall, results suggest that an increase in σ_ζ^2 has a larger negative effect on the human capital investment of low ability child compared to child with higher ability, with implied elasticity being -0.1130 for the high ability child compared to -0.5050 for the low ability child.

medium ability child, the average wage earnings of the high ability child (10.21) is higher than the average wage earnings of the medium ability child (7.00). The average wage earnings of the low ability child is 3.35.

4.2 High Parental Future Income and Borrowing Constraint

Now I analyze the response of a parent to risks with relatively high expected second period endowment income. I reduce $y_1(= 5.6)$ by 20% over the baseline case, and increase $\tilde{y}_2(= 8.75)$, by 40% over the baseline case. With these changes, the present value of the expected parental endowment income remains (approximately) the same as in the baseline case. With this endowment income profile, the parent faces the binding borrowing constraint.

Results show that the borrowing constraint reduces the human capital investment significantly (Table 6). In the deterministic case, the human capital investment for the medium ability child is just 52% of the human capital investment of a parent with baseline endowment profile. Also, the negative effect of the borrowing constraint on the human capital investment is increasing in the ability level of child. The reason is that low y_1 increases the marginal cost of human capital investment which induces the parent to reduce the human capital investment. Since for a given level of human capital investment, the incomes of higher ability children are larger, the parents reduce the human capital investment of higher ability children relatively more.

Results also show that adding risks has significant negative effect on the human capital investment of children with higher ability. For example, the human capital investment for the medium ability child is just 56.5% of the deterministic case. However, for a low ability child adding risk has a large positive effect on the human capital investment. The parental human capital investment at 0.0176 is 213.6% of the deterministic case.

The reason for the differential effects of risks is as follows. Adding risks increases the expected income of both the parent and the child. However, it increases the income of higher ability children relatively more. This induces the parent to reduce the human capital investment of higher ability children but increase the human capital investment of low ability children.

Results show that increasing σ_λ^2 has a large negative effect on the human capital investment for medium and high ability child. However, it has a large positive effect on the human capital investment of the low ability child, unlike the previous cases. A higher σ_λ^2 increases the expected future parental income which encourages the parent to reduce the human capital investment and increase its first period consumption. At the same time as discussed earlier (Proposition 9), a higher σ_λ^2 induces the parent to increase the human capital investment. In the case of children with higher ability the first effect dominates the second leading to lower human capital investment.

Results also show that increasing σ_ζ^2 has a large negative effect on the human capital investment of children. In addition, the negative effect is much larger in comparison to the previous cases. A rise in σ_ζ^2 has two conflicting effects on the human capital investment of children. Other things remaining the same, greater variance should have positive effect on the human capital investment. However, a higher variance also increases the expected earnings of children, which reduces the human capital investment. In this example, the second effect dominates the first.

4.3 Effects of Co-variability Between Parental Endowment Income and Human Capital Investment Risks

Now I allow for the fact that parental endowment income and human capital investment risks may be correlated. The conditional mean and variance of the (log) wage of the child for a given $\lambda = \lambda_i$ are given by

$$E(\zeta|\lambda = \lambda_i) = \rho\frac{\sigma_\zeta}{\sigma_\lambda}\lambda_i \; \& \tag{4.7}$$

$$Var(\zeta|\lambda = \lambda_i) = \sigma_\zeta^2(1-\rho^2) \tag{4.8}$$

respectively where $\rho = \frac{\sigma_{\lambda,\zeta}}{\sigma_\lambda\sigma_\zeta}$.

First I assume that $\sigma_{\lambda,\zeta} = 0.18$. The rest of the parameters remain the same as in the baseline exercise. For the quantitative exercise, I change σ_λ^2 or σ_ζ^2 as before, assuming that this change does not affect the co-variance, $\sigma_{\lambda,\zeta}$. Results are reported in Table 7. Results show that adding risks reduces the human capital investment relative to the deterministic case much more compared to the baseline case. Also, the negative effect of an increase in σ_λ^2 on the human capital investment of the high ability child becomes much larger.

Table 8 reports the results when risks are negatively correlated, with $\sigma_{\lambda,\zeta} = -0.18$. In this case, adding risks reduces the human capital investment relative to the deterministic case relatively less compared to the baseline case. Results also show that the parental response to increase in σ_ζ^2 is relatively smaller.

4.4 More Risk-Averse Parent

Now I consider the effects of risks when the parent is more risk-averse. I set $\alpha = 2.5$. Rest of the parameter values remain the same as in the baseline case ($\sigma_{\lambda,\zeta} = 0$). Results are reported in Table 9. Qualitatively, results are similar to the case when $\alpha = 1.5$. Adding risks incentivizes parents to reduce human capital investment. However, the negative effect of risks on the human capital investment is much larger. In addition, risks reduce the human capital investment of higher ability children relatively more.

Results also show that increasing σ_λ^2 or σ_ζ^2 reduces the human capital investment. Also, the negative effect of increasing σ_λ^2 on human capital investment increases with the ability level of child.

These results show that for a given (base) level of σ_λ^2 and σ_ζ^2, there is a negative relationship between the degree of relative risk-aversion and the human capital investment, which is consistent with the empirical evidence on the relationship between the parental risk-attitude and educational outcomes of children. There is a nascent empirical literature which has examined the effect of parental risk-attitude on the educational outcomes of children. Brown, Ortiz-Nuez, and Taylor (2012) using the 1996 US Panel Study of Income Dynamics (PSID) data, that a parent's degree of risk aversion is significantly negatively related to both the academic test scores of their children and their probability of attending college post high school. Similarly Wolfel and Heineck (2012) using German data find that the parental risk attitude has a negative effect on the probability of their children attending college post high school.

4.5 Effects of Lump-Sum Income Transfers

Since, the human capital investment is inefficiently low, I consider the issue of whether providing income assistance to parents will encourage human capital investment. First I consider the case in which the parents receive lump-sum transfer in the first period which is financed by lump-sum tax on them in the second period. I assume that parents receive income transfer equal to 5% of their income in the first period, $tr_1 = 0.35$, which is financed by taxing parents equal to $tr_2 = -0.70$ in the second period.[18] Such tax and transfer scheme does not affect the life-time income of parents. Results are reported in Table 10 (Panel A). These results show that such tax-transfer scheme has

[18] I am assuming that the government can lend and borrow at the risk-free rate R.

virtually no effect on the human capital investment of parents except for the parents facing borrowing constraint. For the parent facing the borrowing constraint, it raises the human capital investment substantially. However, for other parents such a scheme just affects their savings.

Next I consider the case in which the first period income transfer is financed by taxing adult children equal to $tc_2 = -0.70$. Results are reported in Table 10 (Panel B). These results show that such tax-transfer scheme has a significant positive effect on the human capital investment of parents, including the parents facing the borrowing constraint. The reason is that such tax-transfer scheme increases the life-time income of parents. In addition, taxing children reduces their net income, which induces altruistic parents to increase the human capital investment of children.[19]

5 Conclusion

This paper developed a theoretical model to analyze the effects of parental income risk on the human capital investment of its child, when the human capital investment is risky. It finds that the effects of these risks on the human capital investment depends on whether bequest and borrowing constraints bind. Bequest provides an instrument through which parents and children can share risks. If the bequest and borrowing constraints do not bind, increasing parental income risk has a positive effect on the human capital investment, but increasing human capital investment risk has a negative effect on the human capital investment. However, if the bequest constraint is binding, the effects of increasing these risks are reversed. The paper also finds that a parent facing binding borrowing constraint may increase human capital investment in response to an increase in the parental endowment risk and the human capital investment risk.

The quantitative analysis shows that the parental income risk has a larger negative effect on the human capital investment of high ability children. On the other hand, the human capital investment risk has a larger negative effect on the human capital investment of low ability children. It also finds that

[19] I also consider the effects of other policy changes (not reported). Results show that an increase in the public expenditure on education, g, has virtually no effect on the overall human capital investment. In response to an increase in the public expenditure, the private expenditure on education falls almost one to one. An increase in the wage tax, τ, reduces the human capital investment.

providing income subsidy to parents which is financed by future (lump-sum) taxes on parents has little effect on the human capital investment, except for the parents facing binding borrowing constraint. However, if such income subsidy is financed by future (lump-sum) taxes on adult children, it has a large positive effect on the human capital investment.

Appendix I

Proofs of Propositions 1

Follows from the discussion in the text.

Proofs of Propositions 2-5, $b, k > 0$

I first consider the case where $\overline{k}$ & $\overline{b} > 0$. Taking the second order Taylor approximation around the certainty case, $\overline{y}_2$ & $\overline{\phi}$, with $\overline{\phi} = 1$, I have

$$EU_c(c_2^p)\phi \approx U_c(c_2^p)+\frac{M^2}{2}U_{ccc}(c_2^p)[\sigma_{y_2}^2+h^2(s)\sigma_\phi^2+2h(s)\sigma_{y_2,\phi}]+MU_{cc}(c_2^p)[\sigma_{y_2,\phi}+h(s)\sigma_\phi^2]; \tag{1}$$

$$EU_c(c_2^p) \approx U_c(c_2^p) + \frac{M^2}{2}U_{ccc}(c_2^p)[\sigma_{y_2}^2 + h^2(s)\sigma_\phi^2 + 2h(s)\sigma_{y_2,\phi}] \text{ \&} \tag{2}$$

$$Cov(U_c(c_2^p), \phi) \approx MU_{cc}(c_2^p)[\sigma_{y_2,\phi} + h(s)\sigma_\phi^2]. \tag{3}$$

respectively. Then

$$V_s = EU_c(c_2^p)[\phi h_s(s)-R] \approx (h_s(s)-R)[U_c(c_2^p)+\frac{M^2}{2}U_{ccc}(c_2^p)\{\sigma_{y_2}^2+2h(s)\sigma_{y_2,\phi}+h^2(s)\sigma_{\phi^2}\}]$$

$$+MU_{cc}(c_2^p)h_s(s)[\sigma_{y_2,\phi} + h(s)\sigma_{\phi^2}] = 0. \tag{4}$$

$$V_k = -U_c(c_1^p) + EU_c(c_2^p)R \approx$$

$$-U_c(c_1^p) + R[U_c(c_2^p) + \frac{M^2}{2}U_{ccc}(c_2^p)\{\sigma_{y_2}^2 + 2h(s)\sigma_{y_2,\phi} + h^2(s)\sigma_{\phi^2}\}] = 0. \tag{5}$$

Differentiating (4) and (5) and evaluating the expressions at $h_s(s) = R$, I have

$$V_{sk} = M^2RU_{ccc}(c_2^p)h_s(s)[\sigma_{y_2,\phi} + h(s)\sigma_\phi^2]; \tag{6}$$

$$V_{kk} = U_{cc}(c_1^p) + R^2[MU_{cc}(c_2^p) + \frac{M^3}{2}U_{cccc}(c_2^p)\{\sigma_{y_2}^2 + 2h(s)\sigma_{y_2,\phi} + h^2(s)\sigma_{\phi^2}\}] < 0; \tag{7}$$

and

$$V_{ss} = h_{ss}(s)[[U_c(c_2^p) + \frac{M^2}{2}U_{ccc}(c_2^p)\{\sigma_{y_2}^2 + 2h(s)\sigma_{y_2,\phi} + h^2(s)\sigma_\phi^2\}]] +$$

$$M^2U_{ccc}(c_2^p)h_s^2(s)[\sigma_{y_2,\phi} + h(s)\sigma_\phi^2] + MU_{cc}(c_2^p)[h_{ss}(s) + h_s^2(s)]\sigma_\phi^2. \tag{8}$$

Note that $V_{sk} > 0$ for any $\sigma_{y_2,\phi} \geq 0$. However, $V_{sk} < 0$ if $\sigma_{y_2,\phi} = -\sigma_\phi^2$ and $1 > h(s)$. Note also that $V_{sk} = 0$ when there is no human capital investment risk. Since, $V_{kk} < 0$, it must be the case that $V_{ss} < 0$ for maximization.

Differentiating (4) and (5) w.r.t. y_1, I have

$$V_{s,y_1} = 0 \ \& \ V_{k,y_1} = -U_{cc}(c_1^p) > 0. \tag{9}$$

Using Crammer's rule it is straightforward to show that

$$\frac{ds}{d_{y_1}} > 0 \text{ if } \sigma_{\lambda,\phi} \geq 0, \ \frac{ds}{d_{y_1}} < 0 \text{ if } \sigma_{y_2,\phi} < 0 \ \& \ 1 > h(s) \tag{10}$$

which proves proposition 2.

Suppose now that $\sigma_{y_2,\phi} = 0$. Then, Differentiating (4) and (5) w.r.t. σ_ϕ^2, I have

$$V_{s,\sigma_{y_2}^2} = 0 \ \& \ V_{k,\sigma_{y_2}^2} = \frac{M^2}{2}U_{ccc}(c_2^p) > 0. \tag{11}$$

Then

$$\frac{ds}{d_{\sigma_{y_2}^2}} > 0 \text{ if } \sigma_\phi^2 > 0, \ \frac{ds}{d_{\sigma_{y_2}^2}} = 0 \text{ if } \sigma_\phi^2 = 0 \tag{12}$$

this proves proposition 3.

Differentiating (4) and (5) w.r.t. $\sigma_{y_2}^2$, when $\sigma_{y_2,\phi} = 0$, I have

$$V_{s,\sigma_\phi^2} = MU_{cc}(c_2^p)h(s)h_s(s) < 0 \ \& \ V_{k,\sigma_\phi^2} = R\frac{M^2}{2}U_{ccc}(c_2^p)h^2(s) > 0. \tag{13}$$

$$\frac{ds}{d_{\sigma^2_\phi}} = \frac{V_k\sigma^2_\phi V_{sk} - V_s\sigma^2_\phi V_{kk}}{D}. \tag{14}$$

Since $D > 0$, the sign of $\frac{ds}{d_{\sigma^2_\phi}}$ depends on the sign of the numerator of (14).

$$V_{k\sigma^2_\phi}V_{sk} - V_{s\sigma^2_\phi}V_{kk} = -[U_{cc}(c^p_1) + R^2MU_{cc}(c^p_2) + \frac{R^2M^3}{2}U_{cccc}(c^p_2)\sigma^2_{y_2}]V_s\sigma^2_\phi$$

$$+\frac{R^2M^4}{2}[-U_{cc}(c^p_2)U_{cccc}(c^p_2) + U^2_{ccc}(c^p_2)]h^3(s)h_s(s)\sigma^2_\phi < 0 \tag{15}$$

when we have decreasing absolute risk-prudence. This proves proposition 4.

Now suppose that $\sigma^2_{y_2} = \sigma^2_\phi = \sigma_{y_2,\phi}$. Then, differentiating (4) and (5) w.r.t. σ^2_ϕ

$$V_{s\sigma^2_\phi} = MU_{cc}(c^p_2)(1 + h(s))h_s(s) < 0 \ \& \tag{16}$$

$$V_{k\sigma^2_\phi} = R\frac{M^2}{2}U_{ccc}(c^p_2)(1 + h(s))^2 > 0. \tag{17}$$

In this case,

$$V_{k\sigma^2_\phi}V_{sk} - V_{s\sigma^2_\phi}V_{kk} = -[U_{cc}(c^p_1) + R^2MU_{cc}(c^p_2)]V_s\sigma^2_\phi$$

$$+\frac{R^2M^4}{2}[-U_{cc}(c^p_2)U_{cccc}(c^p_2) + U^2_{ccc}(c^p_2)](1 + h(s))^3h_s(s)\sigma^2_\phi < 0 \tag{18}$$

when we have decreasing absolute risk-prudence.

Similarly, when $\sigma^2_{y_2} = \sigma^2_\phi = -\sigma_{y_2,\phi}$, then

$$V_{s\sigma^2_\phi} = MU_{cc}(c^p_2)(h(s) - 1)h_s(s) < 0 \ \& \tag{19}$$

$$V_{k\sigma^2_\phi} = R\frac{M^2}{2}U_{ccc}(c^p_2)(h(s) - 1)^2 > 0 \tag{20}$$

if $h(s) > 1$. In this case,

$$V_{k\sigma^2_\phi}V_{sk} - V_{s\sigma^2_\phi}V_{kk} = -[U_{cc}(c^p_1) + R^2MU_{cc}(c^p_2)]V_s\sigma^2_\phi$$

$$+\frac{R^2M^4}{2}[-U_{cc}(c_2^p)U_{cccc}(c_2^p)+U_{ccc}^2(c_2^p)](h(s)-1)^3h_s(s)\sigma_\phi^2<0 \tag{21}$$

when we have decreasing absolute risk-prudence.

However, when $h(s)<1$, $V_{s\sigma_\phi^2}>0$ and

$$V_{k\sigma_\phi^2}V_{sk}-V_{s\sigma_\phi^2}V_{kk}=-[U_{cc}(c_1^p)+R^2MU_{cc}(c_2^p)]V_s\sigma_\phi^2$$

$$+\frac{R^2M^4}{2}[-U_{cc}(c_2^p)U_{cccc}(c_2^p)+U_{ccc}^2(c_2^p)](h(s)-1)^3h_s(s)\sigma_\phi^2>0 \tag{22}$$

when we have decreasing absolute risk-prudence. 5(I) and 5(II) follow from (18), (21), and (22).

Proof of Proposition 6

Follows from the discussion in the text.

Proof of Proposition 7 ($b=0,\ k>0$)

$$EU_c(c)\phi\approx U_c(c)+[U_{ccc}(c)h^2(s)+2U_{cc}h(s)]\sigma_\phi^2\ \& \tag{23}$$

$$EU_c(c_2^p)\approx U_c(c_2^p)+U_{ccc}(c_2^p)\sigma_{y_2}^2. \tag{24}$$

$$V_s\approx\delta h_s(s)[U_c(c)+[U_{ccc}(c)h^2(s)+2U_{cc}h(s)]\sigma_\phi^2]-R[U_c(c_2^p)+U_{ccc}(c_2^p)\sigma_{y_2}^2]\ \& \tag{25}$$

$$V_k\approx=-U_c(c_1^p)+R[U_c(c_2^p)+U_{ccc}(c_2^p)\sigma_{y_2}^2]. \tag{26}$$

Then,

$$V_{sk}=-R^2[U_c(c_2^p)+U_{ccc}(c_2^p)\sigma_{y_2}^2]<0; \tag{27}$$

$$V_{kk}=U_{cc}(c_1^p)+R^2[U_{cc}(c_2^p)+U_{cccc}(c_2^p)\sigma_{y_2}^2]<0\ \& \tag{28}$$

$$V_{ss}=\delta h_{ss}(s)[U_c(c)+[U_{ccc}(c)h^2(s)+2U_{cc}h(s)]\sigma_\phi^2]$$

$$+\delta h_s^2(s)[U_{cc}(c)+[U_{cccc}(c)h^2(s)+4U_{ccc}h(s)+2U_{cc}(c)]\sigma_\phi^2]. \tag{29}$$

Proof of 7(I)

For maximization $V_{ss}<0$. Now,

$$V_{sy_1}=0. \tag{30}$$

Then , from Crammer's rule, it follows that $\frac{ds}{dy_1}>0$.

Proof of Proposition 7(II) and 7(III)

To derive the effect of increasing risks, it is convenient to write the f.o.c of s in the following form.

$$V_s=-U_c(c_1^p)+\delta EU_c(c)\phi h_s(s)=0. \tag{31}$$

Then,

$$V_s\approx -U_c(c_1^p)+\delta h_s(s)[U_c(c)+[U_{ccc}(c)h^2(s)+2U_{cc}(c)h(s)]\sigma_\phi^2]. \tag{32}$$

V_k continues to be given by (26). Now

$$V_{sk}=U_{cc}(c_1^p)<0\ \& \tag{33}$$

$$V_{ss}=U_{cc}(c_1^p)+\delta h_{ss}(s)[U_c(c)+[U_{ccc}(c)h^2(s)+2U_{cc}(c)h(s)]\sigma_\phi^2]$$

$$+\delta h_s^2(s)[U_{cc}(c)+[U_{cccc}(c)h^2(s)+4U_{ccc}h(s)+2U_{cc}(c)]\sigma_\phi^2] \tag{34}$$

with $V_{ss}<0$ for maximization.

$$V_{s\sigma_{y_2}^2}=0,\ V_{k\sigma_{y_2}^2}=RU_{ccc}(c_2^p)>0,\ V_{k\sigma_{y_2}^2}=0\ \& \tag{35}$$

$$V_{s\sigma_\phi^2}=\delta h(s)h_s(s)[U_{ccc}(c)h(s)+2U_{cc}(c)]. \tag{36}$$

The sign of $V_{s\sigma_\phi^2}$ depends on the sign of the expression $U_{ccc}(c)h(s)+2U_{cc}(c)$. Proposition 7(II) follows from (33)-(36) and Cremmer's rule. Note that

$$V_{s\sigma_\phi^2} > 0 \text{ if } \rho(c) > 2 \text{ \& } V_{s\sigma_\phi^2} \leq 0 \text{ otherwise.} \tag{37}$$

Proposition 7(III) follows from (33-37) and Cremmer's rule.

Proof of Proposition 8 b & $k = 0$

The optimal level of s is given by

$$U_c(c_1^p) = h_s(s)EU_c(c_2^p)\phi. \tag{38}$$

Then

$$\frac{ds}{dy_1} = \frac{U_{cc}(c_1^p)}{h_{ss}(s)EU_c(c_2^p)\phi + h_s^2(s)EU_{cc}(c_2^p)\phi^2} > 0 \tag{39}$$

which proves 8(I). 8(II) follows from the discussion in the text.
The second derivative of $U_c(c_2^p)\phi$ with respect to ϕ is given by

$$\Big[\frac{U_{ccc}(c)c}{U_{cc}(c)} + 2\Big]U_{cc}(c)h(s). \tag{40}$$

From (40) it follows that $U_c(c_2^p)\phi$ is a convex function of ϕ when $r\rho(c) > 2$. This proves 8(III).

Proof of Proposition 9

Let

$$V_s \approx -U_c(c_1^p) + h_s(s)[U_c(c_2^p) + \frac{M^2}{2}U_{ccc}(c_2^p)[\sigma_{y_2}^2 + 2h(s)\sigma_{y_2,\phi} + \sigma_\phi^2 h^2(s)] +$$

$$MU_{cc}(c_2^p)[\sigma_{y_2,\phi} + \sigma_\phi^2 h(s)]]. \tag{41}$$

For the maximum to exist, it must be the case that $V_{ss} < 0$. Taking the derivative of (41) w.r.t. y_1 I have

$$\frac{ds}{dy_1} = \frac{U_{cc}(c_1^p)}{V_{ss}} > 0$$

which proves 9(I). Taking the derivative of (41) w.r.t. $\sigma^2_{y_2}$ I have,

$$\frac{ds}{d\sigma^2_{y_2}} = -\frac{M^2}{2}\frac{U_{ccc}(c_2^p)}{V_{ss}} > 0$$

which proves 9(II). Similarly, when $\sigma_{y_2,\phi} = 0$

$$\frac{ds}{d\sigma^2_{\phi}} = -\frac{Mh(s)}{2V_{ss}}[U_{ccc}(c_2^p)Mh(s) + 2U_{cc}(c_2^p)]. \tag{42}$$

When $\sigma_{y_2,\phi} = \sigma^2_{\phi}$

$$\frac{ds}{d\sigma^2_{\phi}} = -\frac{Mh(s)}{2V_{ss}}[U_{ccc}(c_2^p)(1 + Mh(s)) + 2U_{cc}(c_2^p)]. \tag{43}$$

Finally, when $\sigma_{y_2,\phi} = -\sigma^2_{\phi}$

$$\frac{ds}{d\sigma^2_{\phi}} = -\frac{Mh(s)}{2V_{ss}}[U_{ccc}(c_2^p)(1 - Mh(s)) + 2U_{cc}(c_2^p)]. \tag{44}$$

Propositions 9(III), 9(IV), and 9(V) follow from (42), (43), and (44) respectively.

Table 1
Effects of Increasing Parental Income (y_1)

	$\sigma_{y_2,\phi} = 0$	$\sigma_{y_2,\phi} = \sigma^2_\phi$	$\sigma_{y_2,\phi} = -\sigma^2_\phi$
	s	s	s
b & $k > 0$	+	+	+ if $h(s) > 1$
$b = 0$ & $k > 0$	+	+	+
$b = 0$ & $k = 0$	+	+	+
$b > 0$ & $k = 0$	+	+	+

Table 2
Effects of Increasing Risks ($\sigma_{y_2,\phi} = 0$)

	$\sigma^2_{y_2}$	σ^2_ϕ
	s	s
b & $k > 0$	+	-
$b = 0$ & $k > 0$	-	+ if $r\rho(c) > 2$
$b = 0$ & $k = 0$	0	+ if $r\rho(c) > 2$
$b > 0$ & $k = 0$	+	+ if $r\rho^*(c^p) > 2$

Note: $r\rho(c) \equiv -\frac{U_{ccc}(c)}{U_{cc}(c)}c$ and $r\rho^*(c^p) \equiv -\frac{U_{ccc}(c_2^p)}{U_{cc}(c_2^p)}Mh(s)$.

Table 3
Effects of Increasing Joint Household Earnings Risk
($\sigma_{y_2,\phi} = \sigma^2_\phi$ or $\sigma_{y_2,\phi} = -\sigma^2_\phi$ and $h(s) > 1$)

	s
b & $k > 0$	-
$b > 0$ & $k = 0$	+ if $r\rho^{**}(c^p) > 2$

Note: $r\rho^{**}(c^p) \equiv -\frac{U_{ccc}(c_2^p)}{U_{cc}(c_2^p)}M(h(s)+1)$ if $\sigma_{y_2,\phi} = \sigma^2_\phi$ and $r\rho^{**}(c^p) \equiv -\frac{U_{ccc}(c_2^p)}{U_{cc}(c_2^p)}M(h(s)-1)$ if $\sigma_{y_2,\phi} = -\sigma^2_\phi$.

Table 4
Parameter Values

Parameter	Values
α	1.5
$\beta = \frac{1}{R}$	0.50
δ	0.67
y_1	7.00
$\tilde{y}_2$	6.25
μ	0.24
$\overline{\zeta}$	0.00
σ_{ζ}^2	0.36
$\overline{\lambda}$	0.00
σ_{λ}^2	0.36
Medium Ability a	8.45
High Ability a	12.675
Low Ability a	4.225
g	0.27
τ	0.02
tr_1	0.00
tr_2	0.00
tc_2	0.00

Table 5
Effects of Increasing Risks ($\sigma^2_{\lambda,\zeta} = 0$)

Child	Medium	Ability	High	Ability	Low	Ability
$Variance$	s	$Elas^{**}_s$	s	$Elas^{**}_s$	s	$Elas^{**}_s$
$\sigma^2_\lambda = 0,\ \sigma^2_\zeta = 0$	0.2289		0.1507		0.1284	
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$	0.1823		0.1360		0.1081	
$\sigma^2_\lambda = .54,\ \sigma^2_\zeta = .36$	0.1524	-0.3280	0.1156	-0.3000	0.0928	-0.2830
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .54$	0.1600	-0.2450	0.1283	-0.1130	0.0808	-0.5050

Note: $Elas^{**}_s$ is calculated as the ratio of the % change in s to the % change in σ^2_λ or σ^2_ζ relative to s when $\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$.

Table 6
Effects of Increasing Risks ($\sigma^2_{\lambda,\zeta} = 0$):
Parent Facing Borrowing Constraint

Child	Medium	Ability	High	Ability	Low	Ability
$Variance$	s	$Elas^{**}_s$	s	$Elas^{**}_s$	s	$Elas^{**}_s$
$\sigma^2_\lambda = 0,\ \sigma^2_\zeta = 0$	0.1192		0.0596		0.0076	
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$	0.065		0.0456		0.0176	
$\sigma^2_\lambda = .54,\ \sigma^2_\zeta = .36$	0.0545	-0.3230	0.0364	-0.4040	0.0248	0.8180
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .54$	0.0497	-0.4710	0.0360	-0.4210	0.0107	-0.7840

Note: $Elas^{**}_s$ is calculated as the ratio of the % change in s to the % change in σ^2_λ or σ^2_ζ relative to s when $\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$.

Table 7

Effects of Increasing Risks: Positively Correlated Risks ($\sigma^2_{\lambda,\zeta} = 0.18$)

Child	Medium	Ability	High	Ability	Low	Ability
$Variance$	s	$Elas_s^{**}$	s	$Elas_s^{**}$	s	$Elas_s^{**}$
$\sigma^2_\lambda = 0,\ \sigma^2_\zeta = 0$	0.2289		0.1507		0.1284	
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$	0.1518		0.1050		0.0881	
$\sigma^2_\lambda = .54,\ \sigma^2_\zeta = .36$	0.1251	-0.3520	0.0811	-0.4554	0.0791	-0.2044
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .54$	0.1320	-0.2608	0.0931	-0.2266	0.0735	-0.3316

Note: $Elas_s^{**}$ is calculated as the ratio of the % change in s to the % change in σ^2_λ or σ^2_ζ relative to s when $\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$.

Table 8

Effects of Increasing Risks: Negatively Correlated Risks ($\sigma^2_{\lambda,\zeta} = -0.18$)

Child	Medium	Ability	High	Ability	Low	Ability
$Variance$	s	$Elas_s^{**}$	s	$Elas_s^{**}$	s	$Elas_s^{**}$
$\sigma^2_\lambda = 0,\ \sigma^2_\zeta = 0$	0.2289		0.1507		0.1284	
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$	0.1955		0.1411		0.1181	
$\sigma^2_\lambda = .54,\ \sigma^2_\zeta = .36$	0.1711	-0.2966	0.1158	-0.3586	0.1046	-0.2288
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .54$	0.1811	-0.1474	0.1355	-0.0801	0.0988	-0.3270

Note: $Elas_s^{**}$ is calculated as the ratio of the % change in s to the % change in σ^2_λ or σ^2_ζ relative to s when $\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$.

Table 9
Effects of Increasing Risks ($\sigma^2_{\lambda,\zeta} = 0$):
High Relative Risk Aversion ($\alpha = 2.5$)

Child	Medium	Ability	High	Ability	Low	Ability
$Variance$	s	$Elas_s^{**}$	s	$Elas_s^{**}$	s	$Elas_s^{**}$
$\sigma^2_\lambda = 0,\ \sigma^2_\zeta = 0$	0.214		0.0865		0.1284	
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$	0.1497		0.0716		0.1037	
$\sigma^2_\lambda = .54,\ \sigma^2_\zeta = .36$	0.1128	-0.4930	0.0460	-0.7150	0.0891	-0.2820
$\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .54$	0.1251	-0.3290	0.0678	-0.1060	0.0763	-0.5280

Note: $Elas_s^{**}$ is calculated as the ratio of the % change in s to the % change in σ^2_λ or σ^2_ζ relative to s when $\sigma^2_\lambda = .36,\ \sigma^2_\zeta = .36$.

Table 10
Effects of Lump-Sum Income Transfers ($\sigma^2_{\lambda,\zeta} = 0$):
Panel A ($tr_1 = 0.35$ & $tr_2 = -0.70$)

Child	Medium	Ability	High	Ability	Low	Ability
Income Profile	s	s/s^*	s	s/s^*	s	s/s^*
Base Line	0.1822	0.9990	0.1357	0.9980	0.1080	0.9990
High Future Income	0.1048	1.6123	0.0757	1.6601	0.0751	4.267

Panel B ($tr_1 = 0.35$ & $tc_2 = -0.70$)

Base Line	0.2890	0.1.5853	0.2027	1.4904	0.3300	3.0527
High Future Income	0.1631	2.5092	0.1130	2.4781	0.1721	9.7784

Note: s^* refers to human capital investment, when there is no transfer, $tr_1,\ tr_2,$ & $tc_2 = 0$.

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